# Slant Frames

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A need exists to have downward continuation techniques not only for waves which are near to vertically up- or down-going waves, but also for waves which are near to some slanted propagation path. Figure 1 illustrates application to downward continuing head waves and Figure 2 illustrates study of salt dome flanks by "forward-looking" or "backward-looking" techniques.

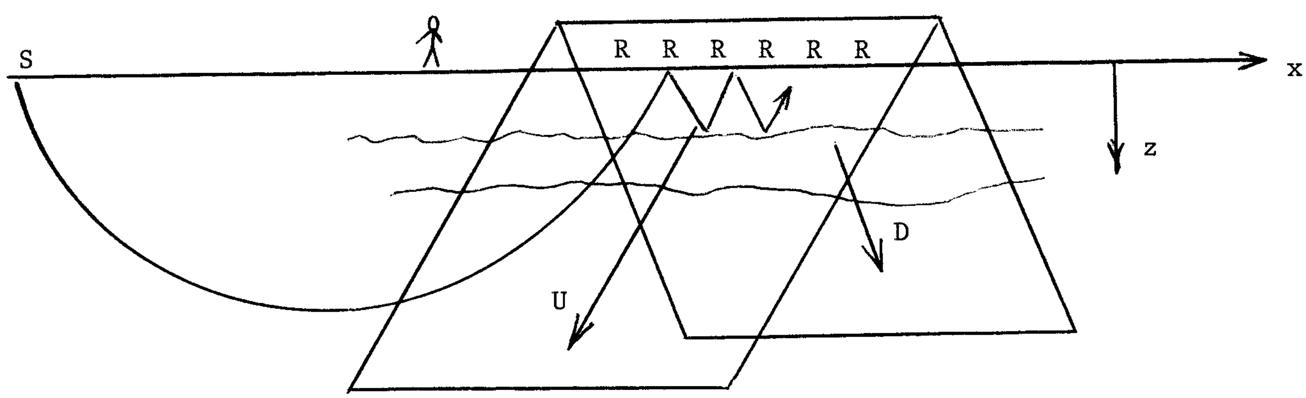


Figure 1. Use of slant frames to downward continue refraction early arrivals. Reflection coefficients may be estimated by coherence of up- and down-going waves. The arrow alongside U indicates the direction of extrapolation (which is opposite to the direction of propagation). Clearly the downward continuation cannot go on very far before the up and down going frames no longer overlap. Also, it is only near the first breaks where elastic wave propagation is well approximated with the scalar wave equation. Later on shear waves must develop.

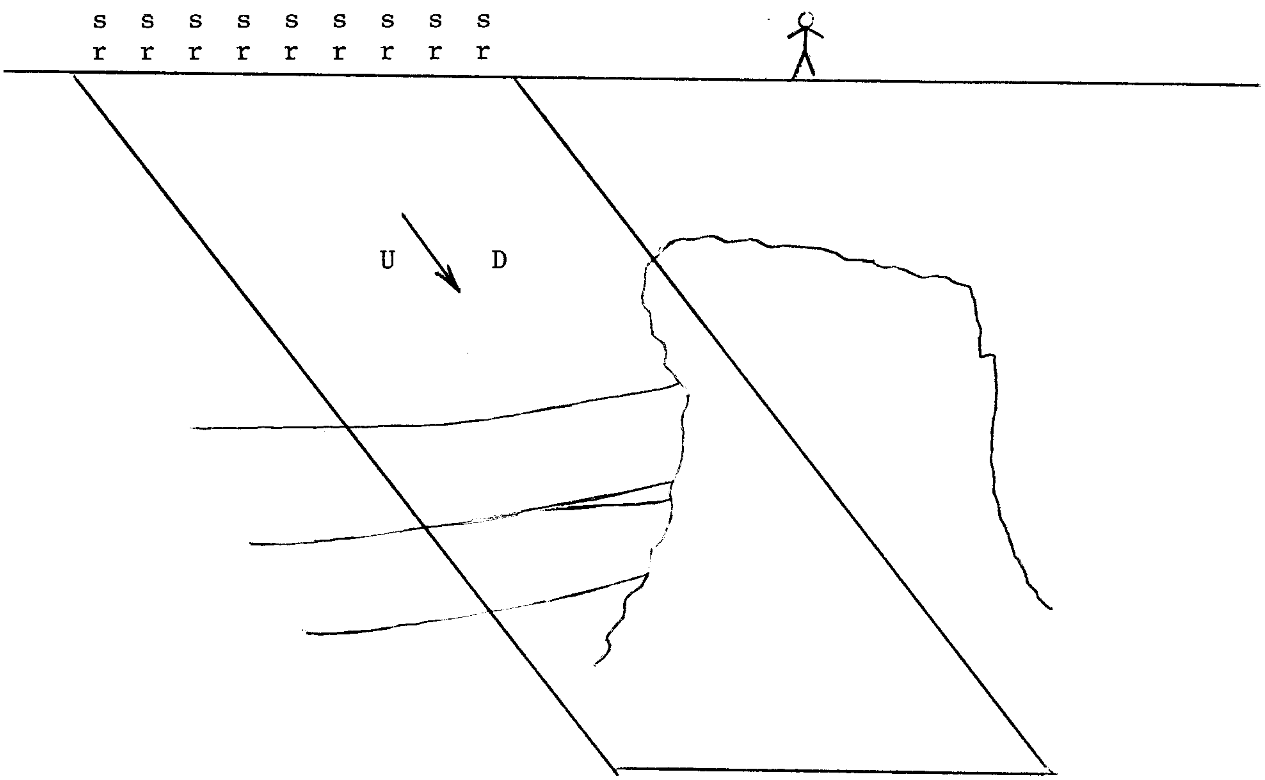


Figure 2. Use of slant frames to downward continue reflection data into salt dome flanks without utilizing waves which have propagated through the confused dome top.

Before proceding to the proposed technique it is instructive to look at an obvious approach and the reasons why we will discard it. The obvious idea is to do a perturbation about a slanted wave, i.e.

$$P(x,z) = Q(x,z)\, e^{im(z\cos\theta + x\sin\theta)} \tag{1}$$

This substitution into the Helmholz equation $P_{xx} + P_{zz} = m^2 P$ along with the parabolic approximation $Q_{zz} = 0$ gives

$$Q_{xx} + 2im(Q_z \cos\theta + Q_x \sin\theta) = 0 \tag{2}$$

For waves near rays in the $\theta$ direction our choice of (1) implies that $Q_{xx}$ should be small in (2) . For such waves (2) reduces to

$$Q_z = Q_x \tan \theta \qquad (3)$$

But (3) is merely a convection or an interpolation equation, that is if

$$Q( x,z_0 ) = f(x) \quad \text{then} \quad Q( x,z_0 + \Delta z ) = f( x + \Delta z \tan \theta ) .$$

To solve (3) or anything like it would be wasting effort because in extrapolating $Q$ in the z-direction one merely is translating an x-dependent function along the x-axis. This should be done by the coordinate transformation, not the differential equation. Not only is it inefficient use of the computer but also the non-vanishing $Q_z$ in (3) will press against the approximation $Q_{zz} \approx 0$ .

Wave extrapolation equations which handle only diffraction and do not monkey around with predictable translation may be derived from the coordinate transformations

$$x' = x - ct \sin\theta \qquad (4a)$$

$$z' = z \qquad (4b)$$

$$t' = t \pm z/c \cos\theta - x/c \sin\theta \qquad (4c)$$

As usual we define

$$P( x,z,t ) = Q( x',z',t' ) \qquad (5)$$

and form the partial derivatives by the chain rule

$$P_x = Q_{x'} x'_x + Q_{z'} z'_x + Q_{t'} t'_x = Q_{x'} - c^{-1}\sin\theta\, Q_{t'} \tag{6a}$$

$$P_z = Q_{x'} x'_z + Q_{z'} z'_z + Q_{t'} t'_z = Q_{z'} \pm c^{-1}\cos\theta\, Q_{t'} \tag{6b}$$

$$P_t = Q_{x'} x'_t + Q_{z'} z'_t + Q_{t'} t'_t = -c\sin\theta\, Q_{x'} + Q_{t'} \tag{6c}$$

Forming second derivatives of (6) and inserting into the wave equation $P_{xx} + P_{zz} = c^{-2} P_{tt}$ we get an equation for $Q$. To simplify the sequel we omit primes on the coordinates of $Q$. We have

$$\begin{aligned} & Q_{xx} - 2\, c^{-1} \sin\theta\, Q_{xt} + c^{-2} \sin^2\theta\, Q_{tt} + \\ & + Q_{zz} \pm 2\, c^{-1} \cos\theta\, Q_{zt} + c^{-2} \cos^2\theta\, Q_{tt} = \\ & = \sin^2\theta\, Q_{xx} - 2\, c^{-1} \sin\theta\, Q_{xt} + c^{-2}\, Q_{tt} \end{aligned} \tag{7}$$

which with the parabolic approximation $Q_{zz} = 0$ collapses all the way down to

$$Q_{zt} = \pm .5\, c^{-1} \cos\theta\, Q_{xx} \tag{8}$$

Equation (8) is a familiar equation indeed, and there are no convection terms. To use (8) in the problems of figures (1) and (2) the downgoing waves $D$ are extrapolated with the minus sign in (4c) (the plus in (8)). The up-coming waves are extrapolated with the other signs. The angle $\theta$ is taken positive in all cases except for the up-coming wave in figure 2.