Reconciliation of Various Downward Continuation Equations

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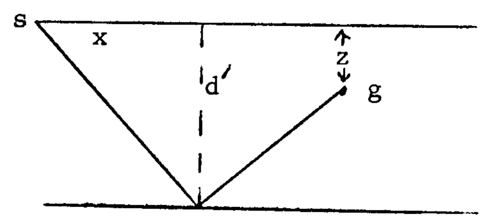
We have three downward continuation equations which are valid in three different coordinate systems. From our October '72 Geophysics paper.

$$Q_{d'z} = -\left(\frac{d'}{2d'-z}\right)^2 Q_{xx} \tag{1}$$

for the coordinate system:

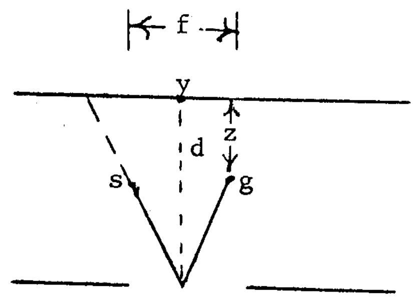


Note that d' has units of feet and is the depth to the reflector.

The second equation is

$$Q_{dz} = -\frac{c}{4} Q_{yy} \tag{2}$$

for the coordinate system:



Now d has units of seconds and is the moveout corrected two way travel time.

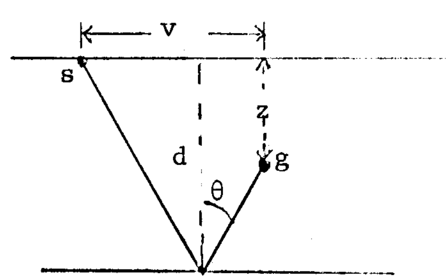
In getting (2) we have assumed all the terms in the transformed wave equation depending if Q_f were zero.

The third equation (the migration version of equation (8) in the '72 paper) is

$$Q_{dz} = -\frac{c}{2} Q_{vv} \tag{3}$$

Equation (3) operates in either of two translating coordinate systems (one in time, one in space) given by

space	time
$v = y$	$v = y$
$z = z - ct$	$z = z$
$d = t$	$d = t + z/c$



The time translating system can be interpreted in terms of the figure on the bottom of the previous page. Here d is two way travel time, v is the coordinate of the geophone.

Let us relate equation (3) to (2). First we see that d and z have the same meaning for each equation. Notice that y is defined as the coordinate of the reflection point in the system for equation (2). If we apply equation (3) to zero offset sections we see that v is coordinate of the geophone relative to some arbitrary origin. Thus we have

$$v = 2y$$

$$\frac{\partial}{\partial v} = \frac{1}{2}\frac{\partial}{\partial y} \qquad (4)$$

Substituting (4) into (3) gives

$$Q_{dz} = -\frac{c}{8} Q_{yy} \qquad (5)$$

Equation (2) downward continues both shots and receivers. Equation (4) downward continues receivers only. Thus we must double the right side of (5) if we wish to compare it to (2). With this correction we see both equations are the same.

Let's try and relate equations (1) and (2). To do this we will transform the variables of (1) to those of (2). First we see that z has the same meaning for both (1) and (2). Also we have

$$d' = \frac{c}{2} d$$

thus

$$\frac{\partial}{\partial d'} = \frac{\partial d}{\partial d'} \frac{\partial}{\partial d} = \frac{2}{c} \frac{\partial}{\partial d} \qquad (6)$$

Finally we need to relate x and y. If we apply (1) to zero offset sections we can see that x is the coordinate of the reflection point relative to some arbitrary origin. In this case the definition of y and x seem to be the same, so we have

$$x = y \qquad \frac{\partial}{\partial x} = \frac{\partial}{\partial y} \tag{7}$$

Substituting (6) and (7) to (1) we get

$$Q_{dz} = - \left(\frac{\frac{c}{2} d}{cd-z} \right)^2 \frac{c}{2} Q_{yy} \tag{8}$$

Suppose we use (8) to integrate a wavefield from $z = 0$ to $z = \frac{cd}{2}$ (from the surface to the reflector). The average value of $\left(\frac{cd/2}{(cd-z)}\right)^2$ is then

$$\text{Average} = \left[\int_0^{\frac{cd}{2}} \left(\frac{cd/2}{(cd-z)}\right)^2 dz \right] \Big/ \frac{c'd}{2}$$

$$= \frac{2}{cd} \frac{c^2 d^2}{4} \left(cd-z \right)^{-1} \Big|_0^{\frac{cd}{2}} = \frac{1}{2}$$

Substituting this average into (8) we get

$$Q_{dz} = - \frac{c}{4} Q_{yy}$$

If substitution of the average is valid (I think this is a $\sin\theta = \theta$, θ actual angle of propagation, assumption), then both equations give the same result when integrated to the reflector. However intermediate results (1/2 way to the reflector) will be different.