Separate Downward Continuation of Shots and Receivers

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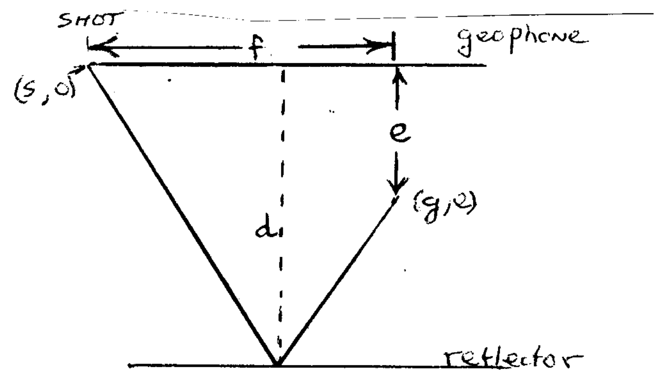
The basic idea of wave equation migration is that wave fields get simpler as they are projected back into the earth. A wave field resulting from a smooth reflector at depth may become very complex, due to diffraction effects and buried foci, by the time it is recorded at the earth's surface. However, if the wave field is projected back into the the earth it simplifies until at the depth of the reflector it essentially mirrors the reflector's shape. Thus, migration can be thought of as a downward continuation operation on the data.

Since projecting the data down to the reflector is essentially the same as projecting the receivers down to the reflectors and then recording the data, migration can also be thought of as downward continuing the shots and the receivers. If only shots are downward continued, diffraction effects occurring on the downgoing path are removed while diffraction occurring after reflection remain. If the receivers are downward continued, upgoing path diffraction is removed. If both are downward continued all diffraction is removed.

In some instances it is desirable to treat diffraction effects occurring on downgoing paths separately from those occurring on upgoing paths. This is easily accomplished by continuing the data on two separate grids, one downward continuing the shots and the other the receivers.

Migration equations for downward continuing shots or receivers can be derived in the following coordinate system.



$$y = (s + g)/2$$

$$f = (g - s) \qquad (0)$$

$$z = e$$

$$d = [(\bar{c}^2 t^2 - (g - s)^2)^{1/2} + e] \, 1/\bar{c}$$

Thus we have 'z' as the depth of the geophone, 'd' as the two way vertical travel time from the earth's surface to the reflector, 'f' as the horizontal shot receiver offset. 'y' is like the midpoint coordinate (it is the midpoint if the shot and receiver are at the surface). Examination of the definition of 'd' shows that this coordinate transformation performs normal moveout correction. In other words, for a flat reflector, ideal transformed data $Q(y,d)$ will be independent of 'f' . The transformation maps travel time hyperbolas in 't' to horizontal lines in 'd' .

Since we are dealing with waves our migration equation will have to be consistent with the wave equation. Let's express the wave equation in this system (0) . First, since the disturbance in the cartesian coordinates $P(g,s,e,t)$ must be the same as that in the new coordinates $Q(y,f,z,d)$ we have

$$P(g,s,e,t) = Q(y,f,z,d)$$

The wave equation is

$$P_{gg} + P_{ee} = \frac{1}{\tilde{c}^2} P_{tt} \qquad (1)$$

To transform (1) into the new system we need to evaluate some partial derivatives. Using the chain rule we have

$$P_g = Q_y\, y_g + Q_f\, f_g + Q_d\, d_g + Q_z\, z_g = 1/2\, Q_y + Q_f + d_g\, Q_d$$

$$P_e = Q_y\, y_e + Q_f\, f_e + Q_d\, d_e + Q_z\, z_e = \frac{1}{\bar{c}}\, Q_d + Q_z$$

$$P_t = d_t\, Q_d$$

Forming second derivatives, being careful to keep all derivatives of the coordinate frame, and substituting to (1) we have

$$\frac{1}{4} Q_{yy} + Q_{zz} + Q_{ff} + 2d_g\, Q_{fd} + d_g\, Q_{yd} + \frac{2}{\bar{c}} Q_{dz} \qquad (2)$$

$$[\, d_g^2 + \frac{1}{\bar{c}^2} - \frac{d_t^2}{\tilde{c}^2}\,]\, Q_{dd} + (d_g\, d_{gd} + d_{gf} - \frac{1}{\tilde{c}^2}\, d_t\, d_{td})\, Q_d = 0$$

The Q_{zz} term can be deleted if we restrict ourselves to small offsets (near trace sections) and moderate reflector dips. The assumption that Q_{zz} is small will be called the 'parabolic approximation'.

Note that we have used two velocities, $\bar{c}$, the velocity used to generate the coordinate system and, $\tilde{c}$, the velocity in the wave equation. These two need not be the same but for simplicity we will assume they are. If $\tilde{c} = \bar{c}$, the coefficient of Q_{dd} is later shown to be identically zero and the coefficient of Q_d greatly simplifies and (2) becomes

$$\frac{1}{4} Q_{yy} \quad \frac{1}{c^2(d-z/c)} Q_d + \frac{2}{c} Q_{dz} + \underline{Q_{ff} + 2d_g\, Q_{fd}} + \overline{d_g\, Q_{yd} + Q_{yf}} = 0$$

where $d_g = \dfrac{-f}{c^2(d-z/c)}$

The Q_d term is simply a geometrical spreading term so if we include a geometrical spreading correction before migration we can neglect it. The underlined terms are at least second order in offset, so we will delete them using the small offset assumption. The overlined terms are proportional to the product of offset and dip, both of which were assumed small. Neglecting both sets of terms we get

$$Q_{dz} = -\frac{c}{8} Q_{yy} \qquad (3)$$

Equation (3) can be used to downward continue data in the z-direction. Since 'z' is the receiver depth (3) really downward continues the receiver, and thus (3) can be used to remove diffraction effects occurring on the path from the reflector to the earth's surface. If it is run backwards (by reversing the direction of time) this equation can also be used to perform the diffraction necessary to propagate a wave from the reflector to the surface.

The equation governing downward continuation of the shots can be found from (2) and (3) almost by inspection. We could have defined a transformation which kept the receivers in place and downward continued the shots. The resulting equation, (2), would have been the same except that 'z' would now be the shot depth and d_g would be replaced by d_s. Since $d_s = -d_g$ the same terms cancel and the same terms can be neglected. Thus (3) is also the migration equation for downward continuing the shots.

Appendix

Proof that $\frac{1}{c^2} - \frac{1}{c^2} d_t^2 + d_g^2 = 0$

We know $d = ((c^2 t^2 - (g-s)^2)^{1/2} + e) \ 1/c$

$$d_t = c\, t(c^2t^2 - (g - s)^2)^{-1/2}$$

$$d_g = - \frac{(g-s)}{c} (c^2t^2 - (g - s)^2)^{-1/2} \qquad \text{first order in 'f'}$$

thus

$$\frac{1}{c^2} - \frac{1}{c^2} d_t^2 + d_g^2 = \frac{1}{c^2} - \frac{1}{c^2} c^2t^2(c^2 t^2 (g - s)^2)^{-1} + \frac{(g-s)^2}{c^2} (c^2t^2-(g-s)^2)^{-1}$$

$$= \frac{1}{c^2} ((c^2t - (g-s)^2) - (c^2t^2 + (g-s)^2))/ (c^2t^2-(g-s)^2)$$

$$= 0$$

Simplification of Q_d coefficient when $\tilde{c} = \bar{c}$

In terms of (f,d,z,y) we have

$$d_g = \frac{-f}{c^2(d-z/c)} \qquad \text{thus} \qquad d_{gd} = \frac{+f}{c^2\ell^2}$$

$$d_{gf} = \frac{-1}{c^2\ell} \qquad \ell = (d-z/c)$$

We also know

$$d_t = (1 + f^2/c^2(d-z/c)^2)^{1/2}$$

$$d_{td} = \frac{-f^2}{c^2\ell^3} / (1+f^2/c^2\ell^2)^{1/2} \qquad d_t\, d_{td} = \frac{-f^2}{c^2\ell^3}$$

Substituting these expressions into the Q_d coefficient, K, gives

$$K = (d_g\, d_{gd} + d_{gf} - \frac{1}{c^2} d_t\, d_{td}) = \frac{-f^2}{c^4\ell^3} + \frac{-1}{c^2\ell} + \frac{1}{c^2} \frac{f^2}{c^2\ell^3} = \frac{-1}{c^2\ell}$$