One Dimensional Noah's Deconvolution

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January 14, 1974

Noah was a geophysicist, but he used a submarine

An ancient geophysicist by the name of Noah had an unusual method of reflection profiling. Noah recognized that a large part of the problem (at normal sea level) with multiple reflections was due to the presence of the near-perfect reflector at the sea surface. Being the practical man that he was and having knowledge of the latest weather forecast, Noah proposed to collect his data in a submarine during the flood. The very good result was that Noah's seismograms were free of sea floor multiples and structure peglegs.

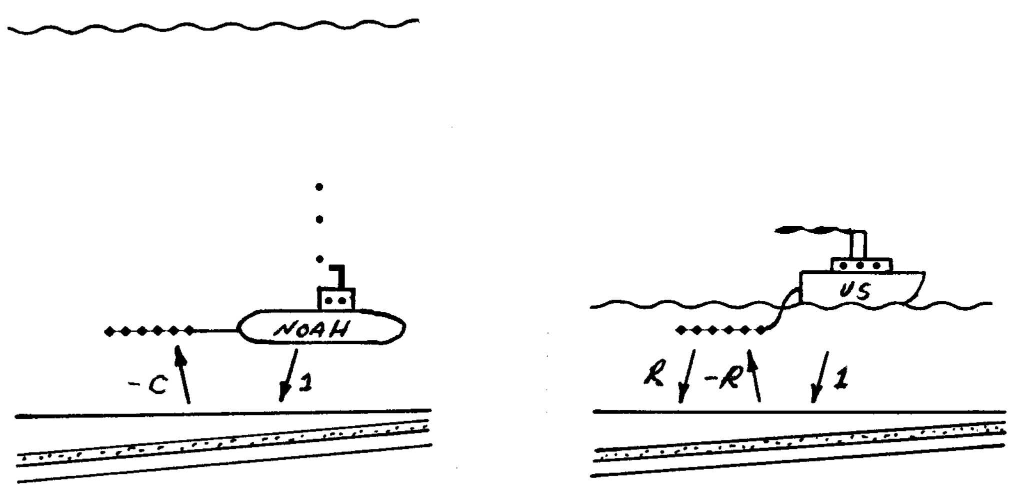


Fig. 1. Noah's recording geometry on the left, ours on the right.

While we cannot record data like Noah we can try to synthesize Noah's seismograms from ours in the computer. To begin we will need to make some simplyfying assumptions, and we'll start with the worst. It is easy to argue for a two dimensional model of the subsurface since most data are shot along lines, usually "dip" lines, and most often interpreted with zero cross-dips in mind. But a necessary step here is to next make the layered media approximation; that our earth model is a function of depth only. We don't believe in this assumption of course, but probably neither do single channel deconvolution folks or even layered media theorists. The idea is that it makes the problem immediately tractable and hopefully we will gain insight into how we may do the two-dimensional problem.

In addition, we'll also neglect material dispersion, and attenuation with<u>in</u> the layers. It is also convenient to define a depth sampling such that one sample interval in time is the two-way travel time within one layer.

First-Order Algorithm

To synthesize Noah's seismogram from ours we place all receivers at the same datum and equate the z-transfer function $G(z)$ of the earth beneath the free surface as deduced from our experiment and Noah's. This follows from the assumption that the earth behaves as a one-dimensional, time-invariant, linear system and as such is completely described by $G(z) = U(z)/D(z)$. For our geometry we have upcoming waves or $-R(z)$, the z-transform of the reflection seismogram, and downgoing waves of the ideal impulsive source 1 and $R(z)$ reflecting off the free surface.

In our definition of the reflection seismogram, being only the upcoming waves, we have excluded the possibility of recording the direct arrival of the shot. In practice we record a direct arrival, but, because of always finite offset, we receive the horizontally travelling source waveform. We will later see how we may estimate the desired <u>vertically</u> transmitted source waveform. Thus, we disregard as unmeasurable and unmodelled the early portion of the seismogram containing the shot waveform.

Noah's upcoming waves are $-c(z)$ and the downgoing wave is simply the shot since the free surface is absent.

$$G(z) \triangleq \left. \frac{U(z)}{D(z)} \right|_{\text{surface}}$$

where U, D are the up and downgoing waves at the receivers. For the same earth below and same datum above we may equate the transfer function for both geometries.

$$\left. \frac{U(z)}{D(z)} \right|_{\text{surface}} = \frac{-C(z)}{1} = \frac{-R(z)}{1+R(z)} \qquad (1)$$

$$C(z) = \frac{R(z)}{1+R(z)} \qquad (2)$$

In order to do the transformation (2) a necessary condition is that $1 + R(z)$ be physically realizable (since $c_t \equiv 0$, $t \leqslant 0$). In the ideal case before us of a horizontally layered linear medium, this is guaranteed since it may be shown that $1 \pm R(z)$ is positive-real. In considering transforming field data with effectively unknown shot waveforms $1 + R(z)$ is not measurable. This is due to the fact that even for very small offsets we record the horizontal path shot rather than the vertical path waveform transmitted into the earth.

If in (2) we identify and equate coefficients of like powers of z we have

$$(c_0 + c_1 z + c_2 z^2 + \cdots)(1 + r_0 + r_1 z + r_2 z^2 + \cdots) = r_0 + r_1 z + r_2 z^2 + \cdots$$

$$c_0 \equiv r_0 \equiv 0$$

$$c_1 \equiv r_1$$

$$c_2 = r_2 - c_1 r_1$$

$$c_3 = r_3 - c_1 r_2 - c_2 r_1$$

$$\vdots$$

which generalizes to

$$c_t = r_t - \sum_{k=1}^{t-1} c_k r_{t-k} \qquad (3a)$$

$$r_t = c_t + \sum_{k=1}^{t-1} r_k c_{t-k} \qquad (3b)$$

Thus the recursion (3a) shows how to develop a new point in Noah's seismogram from the convolution of the previously computed portion on to the original reflection seismogram. In examining (3b), which is the synthesis transformation, we note that the reflection seismogram is the sum of an innovative part c_t and a predicted, deterministic multiple part $r_t * c_t$. Thus, like statistical deconvolution we are, in (3a), attempting to subtract the predictable portion from the seismogram. Unlike deconvolution, however, the "filter" is the deconvolved seismogram itself.

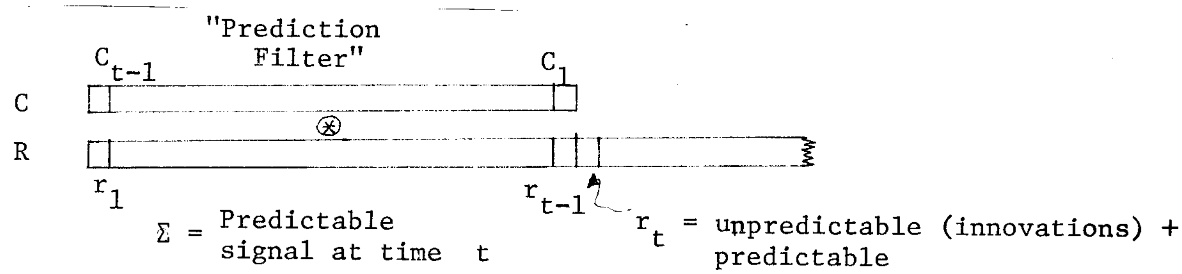


Fig. 2. Predictable energy is the convolution of the past reflection seismogram with the past unpredictable portion of the reflection seismogram.

Practical Considerations

Several problems arise in dealing with realistic cases, either synthetic or field data. One is that of computational efficiency since the number of operations for a trace n samples long is like n^2 MADS and quickly goes out of sight. The solution to this is careful gating of the algorithm and is discussed under classes of multiples treated.

The other is that so far we haven't taken into account a realistic source waveform of finite duration and bandwidth. We shall rewrite the transfer function equation, this time taking into account a shot pulse $S(z)$ of duration ℓ in which case our wave fields are now

$$U = -R'(z) = -S(z)\,R(z)$$

$$D = S(z) + R'(z) \quad ,$$

and Noah's

$$U = -C'(z) = -S(z)\,C(z)$$

$$D = S(z)$$

where the prime denotes the reflected wave with the source waveform included. Thus, we write again

$$\frac{C'(z)}{S(z)} = \frac{R'(z)}{S(z) + R'(z)} \tag{4}$$

Now define an inverse of S to be $U \triangleq S^{-1}$ and dividing through (4) by U we have

$$C'(z) = \frac{R'(z)}{1 + U(z)\, R'(z)} \tag{5}$$

And again equating coefficients of like powers of z results in a similar recurrance pair

$$c'_t = r'_t - u * \sum_{k=1}^{t-1} c'_k \, r'_{t-k} \tag{6a}$$

$$r'_t = c'_t + u * \sum_{k=1}^{t-1} r'_k \, c'_{t-k} \tag{6b}$$

We now need to choose the form of the source inverse U(z) . Since under the summation both c' and R' have the source waveform, in the process of convolving the two the source will not only spread out but also its center of gravity will be delayed.

Therefore, in designing the inverse operator U it should not only whiten S but advance it up to its original position as illustrated in fig. 3.

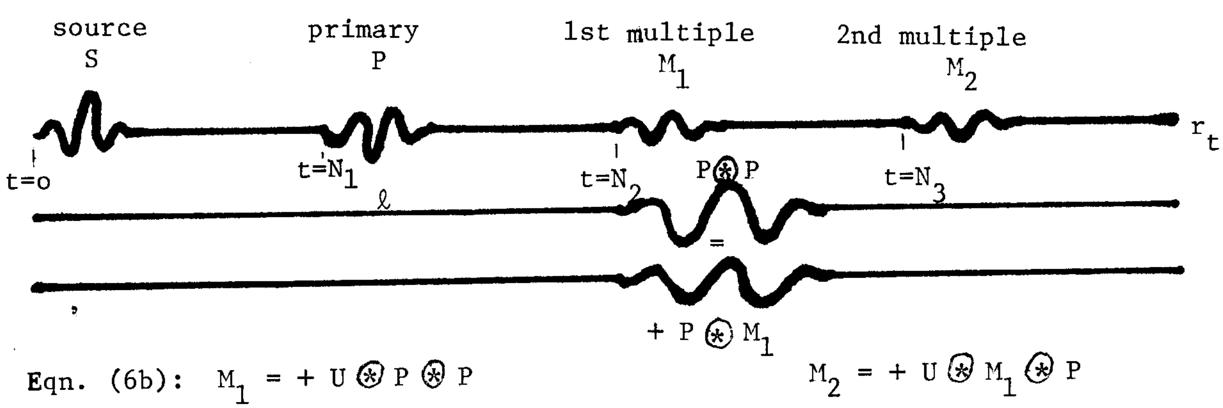


Fig. 3. U is taken to be that anticausal filter which is the inverse of S such that U pushes $P * P$ up to match M_1 .

Recall, though, that we have not allowed ourselves direct access to the vertical path source waveform. However, we may consider the case where the various multiples are separated in time as in fig. 3. Thus (6b) allows us to estimate U either from the primary convolved on itself and the 1st multiple or the primary convolved on the jth multiple and the j + 1st multiple.

$$U{*}P{*}P \quad = M_1 : \quad \underset{u}{\text{Min}} \sum_{t=N_2}^{N_2+\ell} \left(r'_t - u * \sum_{k=N_1}^{N_1+\ell} r'_k \, r'_{t-k} \right)^2$$

or (7)

$$U{*}P{*}M_1 \quad = M_2 : \quad \underset{u}{\text{Min}} \sum_{t=N_3}^{N_3+\ell} \left(r'_t - u * \sum_{k=N_1}^{N_1+\ell} r'_k \, r'_{t-k} \right)^2$$

Therefore having estimated U from the data for the inverse transformation, or directly for data synthesis for the known source waveform, U may be applied to R' and C', respectively. Thus moving U inside the summation we have

$$c'_t = r'_t - \sum_{k=1}^{t-1} c'_k \, r_{t-k} \qquad r'_t = c'_t + \sum_{k=1}^{t-1} r'_k \, c_{t-k} \tag{8}$$

Classes of Multiples Treated

By careful gating of the summation in the recursion, a large amount of control is possible over the type of multiple reflections we may synthesize or remove. Running the recursion without any gates, as in (8), will accommodate all types of multiples that were absent on Noah's seismograms. The only multiples Noah was left with were interbed multiples.

For our purposes, we choose to define interbed multiples as those rays which suffer a reflection on their upcoming path with the exception of reflections at the free surface. Since the one-dimensional Noah transformation above does not model interbed energy it remains in the inverted data. This we view as an acceptably small portion of the multiple energy distribution. The important thing is that the disturbing amount of seafloor and pegleg multiple energy is properly treated within the framework of our simple model.

In the above approximation we assumed the usual situation where structure reflection coefficients are small compared to the seafloor reflectivity. Depending on the strength of this condition we may choose to neglect another class of multiples, structure-structure multiples. Although this is really a data dependent question we are often led to consider it an economic one.

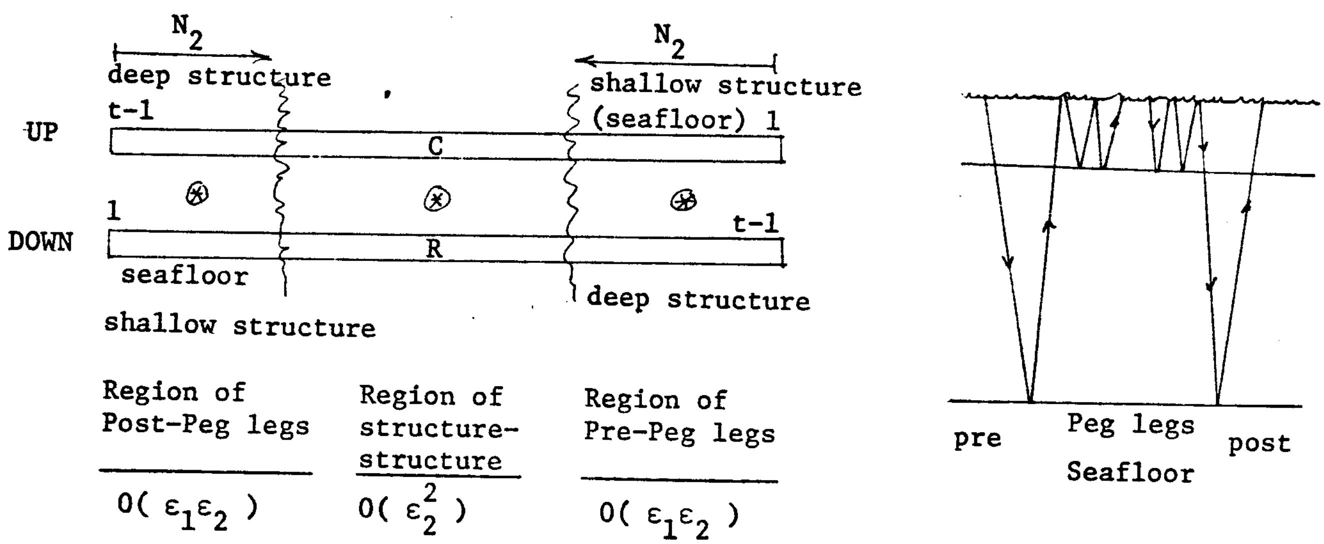


Fig. 4. Illustrated how the middle terms in the summation of equation (8) may be gated out if structure-structure multiples may be neglected. ε_1 is reflection coeff. of seafloor, ε_2 reflection coeff. of deep structure. For $\varepsilon_2^2 << \varepsilon_1\varepsilon_2$ we gate in the N_2 terms at each end.

If ε_1 , ε_2 are representative values for seafloor and structure reflection coefficients, respectively, then the structure peglegs are of the order of $\varepsilon_1\varepsilon_2$. Structure-structure and interbed multiples are $O(\varepsilon_2^2)$ and $O(\varepsilon_2^3)$, respectively. If we choose to neglect the former the computations may be largely reduced. Referring to fig. 4 we gate in the final N_2 terms off each end where the deep structure peglegs are generated. Thus we may split (8) into

$$c'_t = r'_t - \sum_{k=1}^{N_2} (c'_k r_{t-k} + c'_{t-k} r_k) \quad \text{for} \quad t > 2 N_2 \tag{9a}$$

$$r'_t = c'_t + \sum_{k=1}^{N_2} (r'_k c_{t-k} + r'_{t-k} c_k) \quad \text{for} \quad t > 2 N_2 \tag{9b}$$

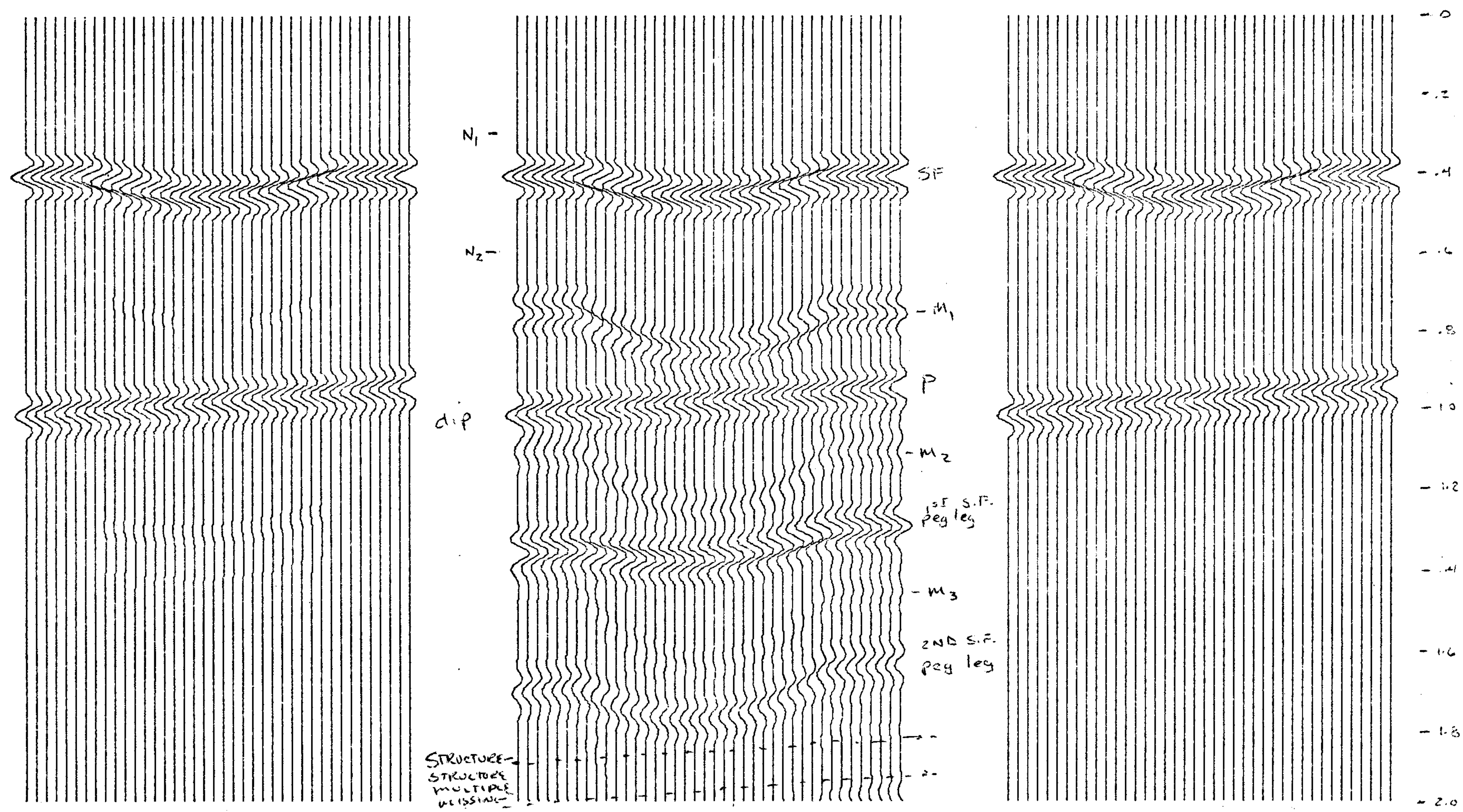
Thus the computations go as $2n*N_2$ MADS for a n length seismogram. Figure 5 is an example of a synthesis-inverse experiment using the split summation recursion (9). Note first the absence of the structure-structure multiple expected near the bottom of the section. Also note the quality of the reconstructed Noah on the left. This was surprising since only one inverse U was estimated from the first trace and subsequently used on all the others.

Possible Pitfalls

Perhaps the most obvious limitation with this, or for that matter any single channel-technique is one-dimensional violations in the data. Specifically, the presence of diffracted multiples is a likely source of trouble. Secondly, we rely heavily on reasonable estimates of the inverse shot waveform. Problems may be expected where the waveform changes rapidly from trace-to-trace, is of excessive duration, or simply cannot be reliably estimated. This will most often occur on very shallow water data.

Another source of difficulty may be due to selective path attenuation. Our model assumes that all possible multiple paths corresponding to a particular arrival time are identically attenuated. For physical reasons some paths may be unexpectedly lossy . Finally, the influence of typical field noise on this technique has not been evaluated.

SINGLE CHANNEL NOAH fig. 5



RECONSTRUCTED MODEL (NOAH) ← (REFLECTION) ← MODEL (NOAH)

Split Summation – t outer – Diffractionless – PRGAIN = 0

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