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Double porosity modeling in elastic wave propagation for reservoir characterization

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ABSTRACT

To account for large-volume low-permeability storage porosity and low-volume high-permeability fracture/crack porosity in oil and gas reservoirs, phenomenological equations for elastic wave propagation in a double porosity medium have been formulated and the coefficients in these linear equations identified. The generalization from single porosity to double porosity modeling increases the number of independent inertial coefficients from three to six, the number of independent drag coefficients from three to six, and the number of independent stress-strain coefficients from three to six for an isotropic applied stress and assumed isotropy of the medium. The analysis leading to physical interpretations of the inertial and drag coefficients is relatively straightforward, whereas that for the stress-strain coefficients is more tedious. In a quasistatic analysis of stress-strain, the physical interpretations are based upon considerations of extremes in both spatial and temporal scales. The limit of very short times is the one most relevant for wave propagation, and in this case both matrix porosity and fractures are expected to behave in an undrained fashion, although our analysis makes no assumptions in this regard. For the very long times more relevant for reservoir drawdown, the double porosity medium behaves as an equivalent single porosity medium. At the macroscopic spatial level, the pertinent parameters (such as the total compressibility) may be determined by appropriate field tests. At the mesoscopic scale pertinent parameters of the rock matrix can be determined directly through laboratory measurements on core, and the compressiblity can be measured for a single fracture. We show explicitly how to generalize the quasistatic results to incorporate wave propagation effects and how effects that are usually attributed to squirt flow under partially saturated conditions can be explained alternatively in terms of the double-porosity model. The result is therefore a theory that generalizes, but is completely consistent with, Biot's theory of poroelasticity and is valid for analysis of elastic wave data from highly fractured reservoirs.

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INTRODUCTION

It is well-known in the phenomenology of earth materials that rocks are generally heterogeneous, porous, and often fractured or cracked. In situ, rock pores and cracks/fractures can contain oil, gas, or water. These fluids are all of great practical interest to us. Distinguishing these fluids by their seismic signatures is a key issue in seismic exploration and reservoir monitoring. Understanding their flow characteristics is typically the responsibility of the reservoir engineer.

Traditional approaches to seismic exploration have often made use of Biot's theory of poroelasticity [Biot, 1941; 1956; 1962; Gassmann, 1951]. This theory has always been limited by an explicit assumption that the porosity itself is homogeneous. Although this assumption is known to be adequate for acoustic studies of many rock core samples in a laboratory setting, it is probably not a very good assumption for applications to realistic heterogeneous reservoirs. One approach to dealing with the heterogeneity is to construct a model that is locally homogeneous, *i.e.*, a sort of finite element approach in which each block of the model satisfies Biot-Gassmann equations. This approach may be adequate in some applications, and is certainly amenable to study with large computers. However, such models avoid the question of how we are to deal with heterogeneity on the local scale, *i.e.*, much smaller than the size of blocks typically used in the codes.

Although it is clear that porosity in the earth can and does come in virtually all shapes and sizes, it is also clear that two types of porosity are most important: (1) Matrix porosity that occupies a finite and substantial fraction of the volume of the reservoir. This porosity is often called the storage porosity, because this is the volume that stores the fluids of interest to us. (2) Fracture or crack porosity that may occupy very little volume, but nevertheless has two very important effects on the reservoir properties. The first effect is that fractures/cracks drastically weaken the rock elastically, and at very low effective stress levels introduce nonlinear behavior since very small changes in stress can lead to large changes in the fracture/crack apertures (and at the same time change the fracture strength for future changes). The second effect is that the fractures/cracks often introduce a high permeability pathway for the fluid to escape from the reservoir. This effect is obviously key to reservoir analysis and the economics of fluid withdrawal.

It is therefore not surprising that there have been many attempts to incorporate fractures into rock models, and especially models that try to account for partial saturation effects and the possibility that fluid moves (or squirts) during the passage of seismic waves [Budiansky and O'Connell, 1975; O'Connell and Budiansky, 1977; Mavko and Nur, 1979; Mavko and Jizba, 1991; Dvorkin and Nur, 1993]. Previous attempts to incorporate have generally been rather ad hoc in their approach to the introduction of the fractures into Biot's theory, if Biot's theory is used at all. The present authors have recently started an effort to make a rigorous extension of Biot's poroelasticity to include fractures/cracks by making a generalization to double-porosity/dual-permeability modeling [Berryman and Wang, 1995]. The previ-

ously published work concentrated on the fluid flow aspects of this problem in order to deal with the interactions between fluid withdrawal and the elastic response (closure) of fractures during reservoir drawdown.

It is the purpose of the present work to point out that a similar analysis applies to the wave propagation problem. We expect it will be possible to incorporate all of the important physical effects in a very natural way into this double-porosity extension of poroelasticity for seismic wave propagation. The price we pay for this rigor is that we must solve coupled equations of motion locally. Within traditional poroelasticity, there are two types of equations that are coupled. These are the equations for the elastic behavior of the solid rock and the equations for elastic and fluid flow behavior of the pore fluid. In the double-porosity extension of poroelasticity, we will have not two types of equations but three. The equations for the elastic behavior of the solid rock will be unchanged except for a new coupling term, while there will be two types of pore-fluid equations (even if there is only one fluid present) depending on the environment of the fluid. Pore fluid in the matrix (storage) porosity will have one set of equations with coupling to fracture fluid and solid; while fluid in the fractures/cracks will have another set of equations with coupling to pore fluid and solid. Although solution of these equations is no doubt more difficult than for simple acoustics/elasticity, it is probably not significantly more difficult than traditional single-porosity poroelasticity. We are not going to solve these equations in the present paper. We will instead derive them and then show that the various coefficients in these equations can be readily identified with measurable quantities.

EQUATIONS OF MOTION

The seismic equations of motion for a double-porosity medium have been derived recently by Tuncay and Corapcioglu [1996] using a volume averaging approach. (These authors also provide a thorough review of the prior literature on this topic.) We will present instead a quick derivation based on ideas similar to those of Biot's original papers [Biot, 1956; 1962], wherein a Lagrangian formulation is presented and the phenomenological equations derived.

Physically what we need is quite simple — just equations embodying the concepts of *force = mass×acceleration* and dissipation due to viscous loss mechanisms. The forces are determined by taking a derivative of an energy storage functional. The appropriate energies are discussed at length later in this paper, so for our purposes in this section it will suffice to assume that the constitutive laws relating stress and strain are known, and so the pertinent forces are the divergence of the solid stress field $\tau_{ij,j}$ and the gradients of the two fluid pressures $p^{(1)}_{,i}$ and $p^{(2)}_{,i}$ for the matrix and fracture fluids, respectively. (In this notation, i, j index the three Cartesian coordinates x_1, x_2, x_3 and a comma preceding a subscript indicates a derivative with respect to the specified coordinate direction.) Then, the only work we need to do to establish the equations of motion for dynamical double-porosity systems concerns the inertial terms arising from the kinetic energy of the system, and drag terms arising

from dissipation due to relative motion of the constituents.

Generalizing Biot's approach [Biot, 1956] to the formulation of the kinetic energy terms, we find that, for a system with two fluids, the kinetic energy T is determined by

$$2T = \rho_{11}\dot{\mathbf{u}}\cdot\dot{\mathbf{u}} + \rho_{22}\dot{\mathbf{U}}^{(1)}\cdot\dot{\mathbf{U}}^{(1)} + \rho_{33}\dot{\mathbf{U}}^{(2)}\cdot\dot{\mathbf{U}}^{(2)} + 2\rho_{12}\dot{\mathbf{u}}\cdot\dot{\mathbf{U}}^{(1)} + 2\rho_{13}\dot{\mathbf{u}}\cdot\dot{\mathbf{U}}^{(2)} + 2\rho_{23}\dot{\mathbf{U}}^{(1)}\cdot\dot{\mathbf{U}}^{(2)}, \tag{1}$$

where $\mathbf{u}$ is the displacement of the solid, $\mathbf{U}^{(k)}$ is the displacement of the kth fluid which occupies volume fraction $\phi^{(k)}$, and the various coefficients ρ_{11}, ρ_{12}, etc., are inertial mass coefficients that take into account the fact that the relative flow of fluid through the pores is not uniform, and that oscillations of solid mass in the presence of fluid leads to induced mass effects. Clarifying the precise meaning of these displacements is beyond our current scope, but recent publications help with these interpretations [Pride and Berryman, 1998].

Dissipation plays a crucial role in the motion of the fluids and so cannot be neglected in this context. The appropriate dissipation functional will take the form

$$2D = b_{12}(\dot{\mathbf{u}} - \dot{\mathbf{U}}^{(1)})\cdot(\dot{\mathbf{u}} - \dot{\mathbf{U}}^{(1)}) + b_{13}(\dot{\mathbf{u}} - \dot{\mathbf{U}}^{(2)})\cdot(\dot{\mathbf{u}} - \dot{\mathbf{U}}^{(2)}) + b_{23}(\dot{\mathbf{U}}^{(1)} - \dot{\mathbf{U}}^{(2)})\cdot(\dot{\mathbf{U}}^{(1)} - \dot{\mathbf{U}}^{(2)}). \tag{2}$$

This formula assumes that all dissipation is caused by motion of the fluids either relative to the solid, or relative to each other. (Other potential sources of attenuation, especially for partially saturated porous media [Miksis, 1988], should also be considered, but will not be discussed here.) We expect the coefficient b_{23} will generally be small and probably be negligible, whenever the double-porosity model is appropriate for the system under study.

Lagrange's equations then show easily that

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{u}_i}\right) + \frac{\partial D}{\partial \dot{u}_i} = \tau_{ij,j}, \quad \text{for} \quad i = 1,2,3, \tag{3}$$

and that

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{U}_i^{(k)}}\right) + \frac{\partial D}{\partial \dot{U}_i^{(k)}} = -p_i^{(k)}, \quad \text{for} \quad i = 1,2,3; k = 1,2. \tag{4}$$

These equations now account properly for inertia and elastic energy, strain, and stress, as well as for the specified types of dissipation mechanisms, and are in complete agreement with those developed by Tuncay and Corapcioglu [1996] using a different approach. In (4), the parts of the equation not involving the kinetic energy can be shown to be equivalent to a two-fluid Darcy's law in this context, so b_{12} and b_{13} are related to Darcy's constants for two single phase flow and b_{23} is the small coupling coefficient. Explicit relations between the b's and the appropriate permeabilities (see Eqs. (53) and (54) of Berryman and Wang [1995]) are not difficult to establish (see

the next section). The harder part of the analysis concerns the constitutive equations required for the right hand side of (3). After the section on inertia and drag, the remainder of the paper will necessarily be devoted to addressing some of these issues concerning stress-strain relations.

In summary, equations (3) and (4) can be combined into

$$\begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12} & \rho_{22} & \rho_{23} \\ \rho_{13} & \rho_{23} & \rho_{33} \end{pmatrix} \begin{pmatrix} \ddot{u}_i \\ \ddot{U}_i^{(1)} \\ \ddot{U}_i^{(2)} \end{pmatrix} + \begin{pmatrix} b_{12}+b_{13} & -b_{12} & -b_{13} \\ -b_{12} & b_{12}+b_{23} & -b_{23} \\ -b_{13} & -b_{23} & b_{13}+b_{23} \end{pmatrix} \begin{pmatrix} \dot{u}_i \\ \dot{U}_i^{(1)} \\ \dot{U}_i^{(2)} \end{pmatrix} = \begin{pmatrix} \tau_{ij,j} \\ -p_{,i}^{(1)} \\ -p_{,i}^{(2)} \end{pmatrix}, \quad (5)$$

showing the coupling between the solid and both types of fluid components.

In the next section we show how to identify the inertial and drag coefficients with physically measureable quantities.

INERTIAL AND DRAG COEFFICIENTS

Inertial coefficients

It is easy to understand that the inertial coefficients appearing in the kinetic energy T must depend on the densities of solid and fluid constituents ρ_s and ρ_f, and also on the volume fractions $\phi^{(1)}$ and $\phi^{(2)}$ of the matrix and fracture porosities. The total porosity is given by $\phi = \phi^{(1)} + \phi^{(2)}$ and the volume fraction occupied by the solid material is therefore $1 - \phi$.

For a single porosity material, there are only three inertial coefficients and the kinetic energy can be written as

$$2T = \begin{pmatrix} \dot{\mathbf{u}} & \dot{\mathbf{U}} \end{pmatrix} \begin{pmatrix} \bar{\rho}_{11} & \bar{\rho}_{12} \\ \bar{\rho}_{12} & \bar{\rho}_{22} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{U}} \end{pmatrix}, \quad (6)$$

where $\dot{\mathbf{U}}$ is the velocity of the only fluid present. Then, it is easy to see that, if $\dot{\mathbf{u}} = \dot{\mathbf{U}}$, the total inertia $\bar{\rho}_{11} + 2\bar{\rho}_{12} + \bar{\rho}_{22}$ must equal the total inertia present in the system $(1-\phi)\rho_s + \phi\rho_f$. Furthermore, Biot [1956] has shown that $\bar{\rho}_{11} + \bar{\rho}_{12} = (1-\phi)\rho_s$, and that $\bar{\rho}_{22} + \bar{\rho}_{12} = \phi\rho_f$. These three equations are not linearly independent and therefore do not determine the three coefficients. So we make the additional assumption that $\bar{\rho}_{22} = \alpha\phi\rho_f$, where α was termed the structure factor by Biot [1956] but has more recently been termed the tortuosity [Brown, 1980; Johnson *et al.*, 1982]. Berryman [1980] has shown that

$$\alpha = 1 + r\left(\frac{1}{\phi} - 1\right), \quad (7)$$

follows from interpreting the coefficient $\bar{\rho}_{11}$ as resulting from the solid density plus the induced mass due to the oscillation of the solid in the surrounding fluid. Then, $\bar{\rho}_{11} = (1-\phi)(\rho_s + r\rho_f)$, where r is a factor dependent on microgeometry that is expected to lie in the range $0 \le r \le 1$, with $r = \frac{1}{2}$ for spherical grains. For example, if $\phi = 0.2$ and $r = 0.5$, equation (7) implies $\alpha = 3.0$, which is a typical value for tortuosity of sandstones.

For double porosity, the kinetic energy may be written as

$$2T = (\,\dot{\mathbf{u}} \quad \dot{\mathbf{U}}^{(1)} \quad \dot{\mathbf{U}}^{(2)}\,) \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12} & \rho_{22} & \rho_{23} \\ \rho_{13} & \rho_{23} & \rho_{33} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{U}}^{(1)} \\ \dot{\mathbf{U}}^{(2)} \end{pmatrix}. \tag{8}$$

We now consider some limiting cases. First, suppose that all the solid and fluid material moves in unison. Then, we again have the result $\rho_{11} + \rho_{22} + \rho_{33} + 2\rho_{12} + 2\rho_{13} + 2\rho_{23}$ must equal the total inertia of the system $(1-\phi)\rho_s + \phi\rho_f$. Next, if we suppose that the two fluids can be made to move in unison, but independently of the solid, then we can take $\dot{\mathbf{U}} = \dot{\mathbf{U}}^{(1)} = \dot{\mathbf{U}}^{(2)}$, and telescope the expression for the kinetic energy to

$$2T = (\,\dot{\mathbf{u}} \quad \dot{\mathbf{U}}\,) \begin{pmatrix} \rho_{11} & (\rho_{12}+\rho_{13}) \\ (\rho_{12}+\rho_{13}) & (\rho_{22}+2\rho_{23}+\rho_{33}) \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{U}} \end{pmatrix}. \tag{9}$$

We can now relate the matrix elements in (9) directly to the barred matrix elements appearing in (6), which then gives us three equations for our six unknowns. Again these equations are not linearly independent, so we still need four more equations.

Next we consider the possibility that the fracture fluid can oscillate independently of the solid and the matrix fluid, and furthermore that the matrix fluid velocity is locked to that of the solid so that $\dot{\mathbf{u}} = \dot{\mathbf{U}}^{(1)}$. For this case, the kinetic energy telescopes in a different way to

$$2T = (\,\dot{\mathbf{u}} \quad \dot{\mathbf{U}}^{(2)}\,) \begin{pmatrix} (\rho_{11}+2\rho_{12}+\rho_{22}) & (\rho_{13}+\rho_{23}) \\ (\rho_{13}+\rho_{23}) & \rho_{33} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{U}}^{(2)} \end{pmatrix}. \tag{10}$$

This equation is also of the form (6), but we must be careful to account properly for the parts of the system included in the matrix elements. Now we treat the solid and matrix fluid as a single unit, so

$$\rho_{11} + 2\rho_{12} + \rho_{22} = (1-\phi)\rho_s + \phi^{(1)}\rho_f + (\alpha^{(2)} - 1)\phi^{(2)}\rho_f, \tag{11}$$

$$\rho_{13} + \rho_{23} = -(\alpha^{(2)} - 1)\phi^{(2)}\rho_f, \tag{12}$$

and

$$\rho_{33} = \alpha^{(2)}\phi^{(2)}\rho_f, \tag{13}$$

where $\alpha^{(2)}$ is the tortuosity of fracture porosity alone.

Finally, we consider the possibility that the matrix fluid can oscillate independently of the solid and the fracture fluid, and furthermore that the fracture fluid velocity is locked to that of the solid so that $\dot{\mathbf{u}} = \dot{\mathbf{U}}^{(2)}$. The kinetic energy telescopes in a very similar way to the previous case with the result

$$2T = \begin{pmatrix} \dot{\mathbf{u}} & \dot{\mathbf{U}}^{(1)} \end{pmatrix} \begin{pmatrix} (\rho_{11} + 2\rho_{13} + \rho_{33}) & (\rho_{12} + \rho_{23}) \\ (\rho_{12} + \rho_{23}) & \rho_{22} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{U}}^{(1)} \end{pmatrix}. \tag{14}$$

We imagine that this thought experiment amounts to analyzing the matrix material alone without fractures being present. The equations resulting from this identification are completely analogous to those in (11)-(13), so we will not show them explicitly here.

We now have nine equations in the six unknowns and six of these are linearly independent, so the system can be solved. The result of this analysis is that the off-diagonal terms are given by

$$2\rho_{12}/\rho_f = (\alpha^{(2)} - 1)\phi^{(2)} - (\alpha^{(1)} - 1)\phi^{(1)} - (\alpha - 1)\phi, \tag{15}$$

$$2\rho_{13}/\rho_f = (\alpha^{(1)} - 1)\phi^{(1)} - (\alpha^{(2)} - 1)\phi^{(2)} - (\alpha - 1)\phi, \tag{16}$$

and

$$2\rho_{23}/\rho_f = \alpha\phi - \alpha^{(1)}\phi^{(1)} - \alpha^{(2)}\phi^{(2)}. \tag{17}$$

The diagonal terms are given by

$$\rho_{11} = (1 - \phi)\rho_s + (\alpha - 1)\phi\rho_f, \tag{18}$$

$$\rho_{22} = \alpha^{(1)}\phi^{(1)}\rho_f, \tag{19}$$

and ρ_{33} is given by (13).

Estimates of the three tortuosities α, $\alpha^{(1)}$, and $\alpha^{(2)}$ may be obtained using (7), or direct measurements may be made using electrical methods as advocated by Brown [1980] and Johnson *et al.* [1982].

Drag coefficients

The drag coefficients may be determined by first noting that the equations presented here reduce to those of Berryman and Wang [1995] in the low frequency limit by merely neglecting the inertial terms. What is required to make the direct identification of the coefficients is a pair of coupled equations for the two increments of fluid content $\zeta^{(1)}$ and $\zeta^{(2)}$. These quantities are related to the displacements by $\zeta^{(1)} = -\phi^{(1)}\nabla \cdot (\mathbf{U}^{(1)} - \mathbf{u})$ and $\zeta^{(2)} = -\phi^{(2)}\nabla \cdot (\mathbf{U}^{(2)} - \mathbf{u})$.

The pertinent equations from Berryman and Wang [1995] are

$$\eta \begin{pmatrix} \dot{\zeta}^{(1)} \\ \dot{\zeta}^{(2)} \end{pmatrix} = \begin{pmatrix} k^{(11)} & k^{(12)} \\ k^{(21)} & k^{(22)} \end{pmatrix} \begin{pmatrix} p^{(1)}_{,ii} \\ p^{(2)}_{,ii} \end{pmatrix}, \tag{20}$$

where η is the shear viscosity of the fluid, and the ks are permeabilities including possible cross-coupling terms. We can extract the terms we need from (5), and then take the divergence to obtain

$$\begin{pmatrix} b_{12}+b_{23} & -b_{23} \\ -b_{23} & b_{13}+b_{23} \end{pmatrix} \begin{pmatrix} \nabla\cdot(\dot{\mathbf{U}}^{(1)}-\dot{\mathbf{u}}) \\ \nabla\cdot(\dot{\mathbf{U}}^{(2)}-\dot{\mathbf{u}}) \end{pmatrix} = -\begin{pmatrix} p^{(1)}_{,ii} \\ p^{(2)}_{,ii} \end{pmatrix}. \tag{21}$$

Comparing these two sets of equations and solving for the b coefficients, we find

$$b_{12} = \frac{\eta\phi^{(1)}(k^{(22)}-k^{(21)})}{k^{(11)}k^{(22)}-k^{(12)}k^{(21)}}, \tag{22}$$

$$b_{12} = \frac{\eta\phi^{(2)}(k^{(11)}-k^{(12)})}{k^{(11)}k^{(22)}-k^{(12)}k^{(21)}}, \tag{23}$$

and

$$b_{23} = \frac{\eta\phi^{(1)}k^{(21)}}{k^{(11)}k^{(22)}-k^{(12)}k^{(21)}} = \frac{\eta\phi^{(2)}k^{(12)}}{k^{(11)}k^{(22)}-k^{(12)}k^{(21)}}. \tag{24}$$

For many applications it will be adequate to assume that the cross-coupling vanishes. In this situation, $b_{23}=0$,

$$b_{12} = \frac{\eta\phi^{(1)}}{k^{(11)}} \tag{25}$$

and

$$b_{13} = \frac{\eta\phi^{(2)}}{k^{(22)}}, \tag{26}$$

which also provides a simple interpretation of these coefficients in terms of the porosities and diagonal permeabilities.

This completes the identification of the inertial and drag coefficients introduced in the previous section.

STRESS-STRAIN FOR SINGLE POROSITY

In the absence of driving forces that can maintain pressure differentials over long time periods, double porosity models must reduce to single porosity models in the long time limit when the matrix pore pressure and crack pore pressure become equal. It is therefore necessary to remind ourselves of the basic results for single porosity models

in poroelasticity. One important role these results play is to provide constraints for the long time behavior in the problems of interest. A second significant use of these results (see Berryman and Wang [1995]) arises when we make laboratory measurements on core samples having properties characteristic of the matrix material. Then the results presented in this section apply specifically to the matrix stiffnesses, porosity, etc.

For isotropic materials and hydrostatic pressure variations, the two independent variables in linear mechanics of porous media are the confining (external) pressure p_c and the fluid (pore) pressure p_f. The differential pressure $p_d \equiv p_c - p_f$ is often used to eliminate the confining pressure. The equations of the fundamental dilatations are then

$$-\frac{\delta V}{V} = \frac{\delta p_d}{K} + \frac{\delta p_f}{K_s} \tag{27}$$

for the total volume V,

$$-\frac{\delta V_\phi}{V_\phi} = \frac{\delta p_d}{K_p} + \frac{\delta p_f}{K_\phi} \tag{28}$$

for the pore volume $V_\phi = \phi V$, and

$$-\frac{\delta V_f}{V_f} = \frac{\delta p_f}{K_f} \tag{29}$$

for the fluid volume V_f. Equation (27) serves to define the various constants of the porous solid, such as the drained frame bulk modulus K and the unjacketed bulk modulus K_s for the composite frame. Equation (28) defines the jacketed pore modulus K_p and the unjacketed pore modulus K_ϕ. Similarly, (29) defines the bulk modulus K_f of the pore fluid.

Treating δp_c and δp_f as the independent variables in our poroelastic theory, we define the dependent variables $\delta e \equiv \delta V/V$ and $\delta\zeta \equiv (\delta V_\phi - \delta V_f)/V$, both of which are positive on expansion, and which are respectively the total volume dilatation and the increment of fluid content. Then, it follows directly from the definitions and from (27), (28), and (29) that

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \begin{pmatrix} 1/K & 1/K_s - 1/K \\ -\phi/K_p & \phi(1/K_p + 1/K_f - 1/K_\phi) \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}. \tag{30}$$

Now we consider two well-known thought experiments: the drained test and the undrained test [Gassmann, 1951; Biot and Willis, 1957; Geertsma, 1957]. (For a single porosity system, these two experiments are sometimes considered equivalent to the "slow loading" and "fast loading" limits respectively. However, these terms are relative since, for example, the fast loading — equivalent to undrained — limit is still assumed to be slow enough that the average fluid and confining pressures are assumed to have reached equilibrium.) The drained test assumes that the porous material is surrounded by an impermeable jacket and the fluid is allowed to escape through

a tube that penetrates the jacket. Then, in a long duration experiment, the fluid pressure remains in equilibrium with the external fluid pressure (*e.g.*, atmospheric) and so $\delta p_f = 0$ and hence $\delta p_c = \delta p_d$; so the changes of total volume and pore volume are given exactly by the drained constants $1/K$ and $1/K_p$ as defined in (27) and (28). In contrast, the undrained test assumes that the jacketed sample has no passages to the outside world, so pore pressure responds only to in confining pressure changes. With no means of escape, the increment of fluid content cannot change, so $\delta\zeta = 0$. Then, the second equation in (30) shows that

$$0 = -\phi/K_p(\delta p_c - \delta p_f/B), \tag{31}$$

where Skempton's pore-pressure buildup coefficient B [Skempton, 1954] is defined by

$$B \equiv \left.\frac{\delta p_f}{\delta p_c}\right|_{\delta\zeta=0} \tag{32}$$

and is therefore given by

$$B = \frac{1}{1 + K_p(1/K_f - 1/K_\phi)}. \tag{33}$$

It follows immediately from this definition that the undrained modulus K_u is determined by (also see Carroll [1980])

$$K_u = \frac{K}{1 - \alpha B}, \tag{34}$$

where we introduced the combination of moduli known as the Biot-Willis parameter $\alpha = 1 - K/K_s$. This result was apparently first obtained by Gassmann [1951] for the case of microhomogeneous porous media (*i.e.*, $K_s = K_\phi = K_m$, the bulk modulus of the single mineral present) and by Brown and Korringa [1975] and Rice [1975] for general porous media with multiple minerals as constituents.

Finally, we condense the general relations from (30) together with the reciprocity relations [Brown and Korringa, 1975] into symmetric form as

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \frac{1}{K}\begin{pmatrix} 1 & -\alpha \\ -\alpha & \alpha/B \end{pmatrix}\begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}. \tag{35}$$

The storage compressibility, which is a central concept in describing poroelastic aquifer behavior in hydrogeology, is related inversely to one defined in Biot's original 1941 paper by

$$S \equiv \left.\frac{\delta\zeta}{\delta p_f}\right|_{\delta p_c=0} = \frac{\alpha}{BK}. \tag{36}$$

This storage compressibility is the change in increment of fluid content per unit change in the fluid pressure, *defined for a condition of no change in external pressure*. It has also been called the three-dimensional storage compressibility by Kümpel [1991].

We may equivalently eliminate the Biot-Willis parameter α and write (35) in terms of the undrained modulus so that

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \frac{1}{K} \begin{pmatrix} 1 & -(1-K/K_u)/B \\ -(1-K/K_u)/B & (1-K/K_u)/B^2 \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}. \tag{37}$$

Equation (37) has the advantage that all the parameters have very well defined physical interpretations, and are also easily generalized for a double porosity model. Finally, note that (35) shows that $K_p = \phi K/\alpha$, which we generally refer to as the reciprocity relation.

The total strain energy functional (including shear) for this problem may be written in the form

$$2E = \delta\tau_{ij}\delta e_{ij} + \delta p_f \delta\zeta, \tag{38}$$

where δe_{ij} is the change in the average strain with $\delta e_{ii} \equiv \delta e$ being the dilatation, $\delta\tau_{ij}$ being the change in the average stress tensor for the saturated porous medium with $\frac{1}{3}\delta\tau_{ii} = -\delta p_c$. It follows that

$$\delta p_c = -\frac{\partial E}{\partial(\delta e)} \tag{39}$$

and

$$\delta p_f = \frac{\partial E}{\partial(\delta\zeta)}, \tag{40}$$

both of which are also consistent with Betti's reciprocal theorem [Love, 1927] since the matrices in (35) and (37) are symmetric. The shear modulus μ is related to the bulk modulus and Poisson's ratio by $\mu = 3(1-2\nu)K/2(1+\nu)$. Then, it follows that the stress equilibrium equation is

$$\tau_{ij,j} = (K_u + \tfrac{1}{3}\mu)e_{,i} + \mu u_{i,jj} - BK_u\zeta_{,i} = 0 \tag{41}$$

and Darcy's law takes the form

$$\frac{k}{\eta} p_{,ii} = \dot{\zeta}, \tag{42}$$

where η is the single-fluid shear viscosity.

STRESS-STRAIN FOR DOUBLE POROSITY

We now assume two distinct phases at the macroscopic level: a porous matrix phase with the effective properties $K^{(1)}$, $\mu^{(1)}$, $K_m^{(1)}$, $\phi^{(1)}$ occupying volume fraction $V^{(1)}/V = v^{(1)}$ of the total volume and a macroscopic crack or joint phase occupying the remaining fraction of the volume $V^{(2)}/V = v^{(2)} = 1 - v^{(1)}$. The key feature

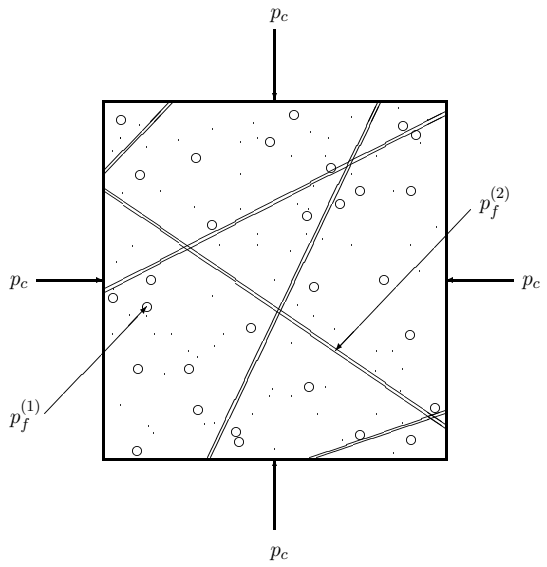

Figure 1: The double porosity model features a porous rock matrix intersected by fractures. Three types of macroscopic pressure are pertinent in such a model: external confining pressure p_c, internal pressure of the matrix pore fluid $p_f^{(1)}$, and internal pressure of the fracture pore fluid $p_f^{(2)}$. A single porosity medium is one in which either matrix or fracture porosity are present, but not both. jim-pic [NR]

distinguishing the two phases — and therefore requiring this analysis — is the very high fluid permeability $k^{(22)}$ of the crack or joint phase and the relatively lower permeability $k^{(11)}$ of the matrix phase. We could also introduce a third independent permeability $k^{(12)} = k^{(21)}$ for fluid flow at the interface between the matrix and crack phases, but for simplicity we assume here that this third permeability is essentially the same as that of the matrix phase, so $k^{(12)} = k^{(11)}$.

We have three distinct pressures: confining pressure δp_c, matrix-fluid pressure $\delta p_f^{(1)}$, and joint-fluid pressure $\delta p_f^{(2)}$. Treating $\delta p_c, \delta p_f^{(1)}$, and $\delta p_f^{(2)}$ as the independent variables in our double porosity theory, we define the dependent variables $\delta e \equiv \delta V/V$ (as before), $\delta\zeta^{(1)} = (\delta V_\phi^{(1)} - \delta V_f^{(1)})/V$, and $\delta\zeta^{(2)} = (\delta V_\phi^{(2)} - \delta V_f^{(2)})/V$, which are respectively the total volume dilatation, the increment of fluid content in the matrix phase, and the increment of fluid content in the joints. We assume that the fluid in the matrix is the same kind of fluid as that in the cracks or joints, but that the two fluid regions may be in different states of average stress and therefore need to be distinguished by their respective superscripts.

Linear relations among strain, fluid content, and pressure then take the general form

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}. \tag{43}$$

By analogy with (35) and (37), it is easy to see that $a_{12} = a_{21}$ and $a_{13} = a_{31}$. The symmetry of the new off-diagonal coefficients may be demonstrated by using Betti's reciprocal theorem in the form

$$\begin{pmatrix} \delta e & -\delta\zeta^{(1)} & -\delta\zeta^{(2)} \end{pmatrix} \begin{pmatrix} 0 \\ -\delta\overline{p}_f^{(1)} \\ 0 \end{pmatrix} = \begin{pmatrix} \delta\overline{e} & -\delta\overline{\zeta}^{(1)} & -\delta\overline{\zeta}^{(2)} \; cr \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -\delta p_f^{(2)} \end{pmatrix}, \tag{44}$$

where unbarred quantities refer to one experiment and barred to another experiment to show that

$$\delta\zeta^{(1)}\delta\overline{p}_f^{(1)} = a_{23}\delta p_f^{(2)}\delta\overline{p}_f^{(1)} = a_{32}\delta\overline{p}_f^{(1)}\delta p_f^{(2)} = \delta\overline{\zeta}^{(2)}\delta p_f^{(2)}. \tag{45}$$

Hence, $a_{23} = a_{32}$. Similar arguments have often been used to establish the symmetry of the other off-diagonal components. Thus, we have established that the matrix in (43) is completely symmetric, so we need to determine only six independent coefficients. To do so, we consider a series of thought experiments, including tests in both the short time and long time limits. The key idea here is that at long times the two pore pressures must come to equilibrium ($p_f^{(1)} = p_f^{(2)} = p_f$ as $t \to \infty$) as long as the cross permeability $k^{(12)}$ is finite. However, at very short times, we may assume that the process of pressure equilibration has not yet begun, or equivalently that $k^{(12)} = 0$ at $t = 0$. We nevertheless assume that the pressure in each of the two components have individually equilibrated on the average, even at short times.

Undrained joints, undrained matrix, short time

There are several different, but equally valid, choices of time scale on which to define Skempton-like coefficients for the matrix/fracture system under consideration. Elsworth and Bai [1992] use a definition based on the idea that for very short time both fluid systems will independently act undrained after the addition of a sudden change of confining pressure. This idea implies that $\delta\zeta^{(1)} = 0 = \delta\zeta^{(2)}$ which, when substituted into (43), gives

$$\begin{aligned} -\delta e &= a_{11}\delta p_c + a_{12}\delta p_f^{(1)} + a_{13}\delta p_f^{(2)} \\ 0 &= a_{12}\delta p_c + a_{22}\delta p_f^{(1)} + a_{23}\delta p_f^{(2)} \\ 0 &= a_{13}\delta p_c + a_{23}\delta p_f^{(1)} + a_{33}\delta p_f^{(2)}. \end{aligned} \tag{46}$$

Defining

$$B_{EB}^{(1)} \equiv \left.\frac{\delta p_f^{(1)}}{\delta p_c}\right|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} \quad \text{and} \quad B_{EB}^{(2)} \equiv \left.\frac{\delta p_f^{(2)}}{\delta p_c}\right|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} \tag{47}$$

we can solve (46) for the two Skempton's coefficients and find the results

$$B_{EB}^{(1)} = \frac{a_{23}a_{13} - a_{12}a_{33}}{a_{22}a_{33} - a_{23}^2} \tag{48}$$

and

$$B_{EB}^{(2)} = \frac{a_{23}a_{12} - a_{13}a_{22}}{a_{22}a_{33} - a_{23}^2}. \tag{49}$$

The effective undrained modulus is found to be given by

$$\frac{1}{K_{u^{EB}}} \equiv -\frac{\delta e}{\delta p_c}|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} = a_{11} + a_{12}B_{EB}^{(1)} + a_{13}B_{EB}^{(2)}. \tag{50}$$

These definitions will be compared to others.

Drained joints, undrained matrix, intermediate time

Now consider a sudden change of confining pressure on a jacketed sample, but this time with tubes inserted in the joint (fracture) porosity so $\delta p_f^{(2)} = 0$, while $\delta\zeta^{(1)} = 0$. We will call this the drained joint, undrained matrix limit. The resulting equations are

$$\begin{aligned} \delta e &= -a_{11}\delta p_c - a_{12}p_f^{(1)} \\ -\delta\zeta^{(2)} &= -a_{31}\delta p_c - a_{32}\delta p_f^{(1)}, \end{aligned} \tag{51}$$

showing that the pore-pressure buildup in the matrix is

$$B[u^{(1)}] \equiv \left.\frac{\delta p_f^{(1)}}{\delta p_c}\right|_{\delta\zeta^{(1)}=\delta p_f^{(2)}=0} = -\frac{a_{21}}{a_{22}}. \tag{52}$$

Similarly, the effective undrained modulus for the matrix phase is found from (51) to be determined by

$$\frac{1}{K[u^{(1)}]} \equiv -\left.\frac{\delta e}{\delta p_c}\right|_{\delta\zeta^{(1)}=\delta p_f^{(2)}=0} = a_{11} + a_{12}B[u^{(1)}]. \tag{53}$$

Notice that if $a_{23} = 0$ then (48) and (52) are the same.

Drained matrix, undrained joints, intermediate time

Next consider another sudden change of confining pressure on a jacketed sample, but this time the tubes are inserted in the matrix porosity so $\delta p_f^{(1)} = 0$, while $\delta\zeta^{(2)} = 0$. We will call this the drained matrix, undrained joint limit. The equations are

$$\begin{aligned} \delta e &= -a_{11}\delta p_c - a_{13}p_f^{(2)} \\ -\delta\zeta^{(1)} &= -a_{21}\delta p_c - a_{23}\delta p_f^{(2)} \\ 0 &= -a_{31}\delta p_c - a_{33}\delta p_f^{(2)}, \end{aligned} \tag{54}$$

showing that the pore-pressure buildup in the cracks is

$$B[u^{(2)}] \equiv \left.\frac{\delta p_f^{(2)}}{\delta p_c}\right|_{\delta\zeta^{(2)}=\delta p_f^{(1)}=0} = -\frac{a_{31}}{a_{33}}. \tag{55}$$

Similarly, the effective undrained modulus for the joint phase is found

$$\frac{1}{K[u^{(2)}]} \equiv -\left.\frac{\delta e}{\delta p_c}\right|_{\delta\zeta^{(2)}=\delta p_f^{(1)}=0} = a_{11} + a_{13}B[u^{(2)}]. \tag{56}$$

We may properly view Eqs. (52), (53), (55), and (56) as "defining" relations among these parameters.

Notice that if $a_{23} = 0$ then (49) and (55) are the same.

Drained test, long time

The long duration drained (or "jacketed") test for a double porosity system should reduce to the same results as in the single porosity limit. The conditions on the pore pressures are $\delta p_f^{(1)} = \delta p_f^{(2)} = 0$, and the total volume obeys $\delta e = -a_{11}\delta p_c$. It follows therefore that

$$a_{11} \equiv \frac{1}{K}, \tag{57}$$

where K is the overall drained bulk modulus of the system including the fractures.

Undrained test, long time

The long duration undrained test for a double porosity system should also produce the same physical results as a single porosity system (assuming only that it makes sense at some appropriate larger scale to view the medium as homogeneous). The basic equations are

$$\begin{aligned} \delta p_f^{(1)} &= \delta p_f^{(2)} = \delta p_f, \\ \delta\zeta &\equiv \delta\zeta^{(1)} + \delta\zeta^{(2)} = 0, \end{aligned} \tag{58}$$

assuming the total mass of fluid is confined. Then, it follows that

$$\begin{aligned} \delta e &= -a_{11}\delta p_c - (a_{12} + a_{13})\delta p_f, \\ 0 &= -(a_{21} + a_{31})\delta p_c - (a_{22} + a_{23} + a_{32} + a_{33})\delta p_f, \end{aligned} \tag{59}$$

showing that the overall pore-pressure buildup coefficient is given by

$$B \equiv \left.\frac{\partial p_f}{\partial p_c}\right|_{\delta\zeta=0} \tag{60}$$

Similarly, the undrained bulk modulus is found to be given by

$$\frac{1}{K_u} \equiv -\left.\frac{\delta e}{\delta p_c}\right|_{\delta\zeta=0} = a_{11} + (a_{12} + a_{13})B. \tag{61}$$

Fluid injection test, long time

The conditions on the pore pressures for the long duration, single porosity limit for the three-dimensional storage compressibility S are $\delta p_f^{(1)} = \delta p_f^{(2)} = \delta p_f$, while the confining pressure remains constant. It follows therefore that

$$S \equiv \left.\frac{\partial\zeta}{\partial p_f}\right|_{\delta p_c=0} = a_{22} + a_{23} + a_{32} + a_{33}. \tag{62}$$

This completes the main analysis of the elastic coefficients for double porosity. Further details may be found in Berryman and Wang [1995].

TABLE 1. Material Properties

Parameter	Barre Granite	Bedford Limestone
K (GPa)	13.5[a]	23.0[a]
K_s (GPa)	54.5[a]	66.0[a]
α	0.75[a]	0.65[a]
$K^{(1)}$ (GPa)	21.5[a]	27.0[a]
$\nu^{(1)}$	0.15	0.15
$K_s^{(1)}$ (GPa)	55.5[a]	66.0[a]
$\alpha^{(1)}$	0.61[a]	0.59[a]
K_f (GPa)	3.3	3.3
$\phi^{(1)}$	0.0007	0.119[a]
$B^{(1)}$	0.996	0.389
$S^{(1)}$ (GPa^{-1})	0.0558	0.0561
$v^{(2)}$	0.007[a]	0.0119

[a]From Coyner [1984]

EXAMPLES

Table 1 presents data for Barre granite and Bedford limestone taken from laboratory measurements by Coyner [1984]. Coyner's experiments included a series of tests on several types of laboratory scale rock samples at different confining pressures. The values quoted for K and K_s are those for a moderate confining pressure of 10 MPa (values at lower confining pressures were also measured but we avoid using these values because the rocks generally exhibit nonlinear behavior in that region of the parameter space), while the values quoted for $K^{(1)}$ and $K_s^{(1)}$ are at 25 MPa, which is close to the value beyond which the constants cease depending on pressure — and therefore for which we assume all the cracks were closed. Thus, based on the idea that the pressure behavior is associated with two kinds of porosity in the laboratory samples — a crack porosity, which is being closed between 10 and 25 MPa, and a residual matrix porosity above 25 MPa, we assume the available data are K, K_s, $K^{(1)}$, $K_s^{(1)}$, K_f, $\phi^{(1)}$, and $v^{(2)}$. We find that these data are sufficient to compute all the coefficients. In Table 2, we find in both types of rock that the coefficient a_{23} is positive and small — about an order of magnitude smaller than the other matrix elements. Another unusual feature of the results computed using these laboratory data is the occurrence of values larger than unity for B and $B[u^{(1)}]$ in Barre granite; similar results were observed by Berryman and Wang [1995] for Chelmsford granite, and in subsequent work for Westerly granite. Also note that $\alpha^{(2)}$ for both rocks is close to unity for these calculations. In this example, seven measurements (together with Poisson's ratio) are sufficient to determine completely the mechanical behavior of the double-porosity model. Having a direct measurement of K eliminates the necessity of assuming $a_{23} = 0$, which we have found is sometimes necessary when dealing with field data [Berryman and Wang, 1995].

TABLE 2. Double porosity parameters computed from material properties in Table 1.

Parameter	Formula	Barre Granite	Bedford Limestone
a_{11} (GPa^{-1})	$1/K$	0.0741	0.0435
a_{12} (GPa^{-1})	$-\alpha^{(1)}K_s^{(1)}/K^{(1)}K_s$	-0.0289	-0.0219
a_{13} (GPa^{-1})	$-\alpha/K - a_{12}$	-0.0267	-0.0064
a_{22} (GPa^{-1})	$v^{(1)}\alpha^{(1)}/B^{(1)}K^{(1)}$	0.0283	0.0555
a_{23} (GPa^{-1})	$-v^{(1)}\alpha^{(1)}/K^{(1)} - a_{12}$	0.00072	0.00026
a_{33} (GPa^{-1})	$v^{(2)}/K_f + v^{(1)}/K^{(1)} - (1-2\alpha)/K + 2a_{12}$	0.0275	0.00954
$\overline{a}_{33}$ (GPa^{-1})	$a_{33} - v^{(2)}/K_f$	0.0254	0.00594
$\alpha^{(2)}$	$(\overline{a}_{33}a_{12} - a_{13}a_{23})/(a_{11}a_{23} - a_{13}a_{12})$	0.995	0.994
B	$-(a_{12}+a_{13})/(a_{22}+2a_{23}+a_{33})$	1.023	0.373
$B^{(1)}$	$-(a_{12}+a_{23})/a_{22}$	0.996	0.389
$B[u^{(1)}]$	$-a_{12}/a_{22}$	1.022	0.394
$B_{EB}^{(1)}$	$(a_{23}a_{13} - a_{12}a_{33})/(a_{22}a_{33} - a_{23}^2)$	0.998	0.391
$B^{(2)}$	$(\overline{a}_{33}a_{12} - a_{13}a_{23})/[a_{33}(a_{12}+a_{23}) - a_{23}(a_{13} - \overline{a}_{33})]$	0.969	0.631
$B[u^{(2)}]$	$-a_{13}/a_{33}$	0.969	0.672
$B_{EB}^{(2)}$	$(a_{23}a_{12} - a_{13}a_{22})/(a_{22}a_{33} - a_{23}^2)$	0.943	0.661
K_u (GPa)	$[a_{11} - (a_{12}+a_{13})^2/(a_{22}+2a_{23}+a_{33})]^{-1}$	58.0	33.7
$K^{(2)}$ (GPa)	$v^{(2)}(a_{12}+a_{13})/(a_{11}a_{23} - a_{13}a_{12})$	0.542	2.61
S (GPa^{-1})	α/BK	0.0543	0.0757
$S^{(2)}$ (GPa^{-1})	$\alpha^{(2)}/B^{(2)}K^{(2)}$	1.895	0.603

CONCLUSIONS

We now have a complete set of equations for the double-porosity/dual-permeability model. The next step in the analysis will be to solve these equations and find their wave propagation properties. This work is in progress and will be reported at a later date.

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