Stanford Exploration Project, Report 94, November 11, 1997, pages 1–??

Two-step equalization of irregular data

Nizar Chemingui and Biondo Biondi[1]

keywords: *AMO, irregular geometry, normalization, least squares, data space inverse*

ABSTRACT

Sampling irregularities in seismic data may introduce noise, cause amplitude distortions and even structural distortions when wave equation processes such as dip moveout, azimuth moveout, and prestack migration are applied. Data regularization before imaging becomes a processing requirement to preserve amplitude information and produce a good quality final image. We propose a new technique to invert for reflectivity models while correcting for the effects of irregular sampling. The final reflectivity model is a two-step solution where the data is equalized in a first stage with an inverse filter and an imaging operator is then applied to the equalized data to invert for a model. Based on least-squares theory, the solution estimates an equalization filter that corrects the imaging operator for the interdependencies between data elements. Each element of the filter is a mapping between two data elements. It reconstructs a data trace with given input geometry at the geometry of the other data element. This mapping represents an AMO transformation. The filter is therefore a symmetric AMO matrix with diagonal elements being the identity and the off-diagonal elements being the trace-to-trace AMO transforms. We explore the effectivness of the method in the 2D case for the application of partial stacking by offset continuation. The equalization step followed by imaging has proved to correct and equalize the processing for the effects of fold variations.

INTRODUCTION

Processing seismic data for amplitude inversion has many applications in AVO analysis, reservoir monitoring, detection of anisotropy, estimation of fracture density and orientation and other related applications. Such detailed exploration objectives have led to the careful design of seismic surveys. Unfortunately, during the acquisition stage, obstructions, cable feathering, environmental objectives, economic constraints and many other factors cause seismic data to be sampled in sparse and irregular fashion. These irregularities are often observed in the form of variations in fold cover-

[1]**email:** nizar@sep.stanford.edu, biondo@sep.stanford.edu

age, which can manifest itself as an acquisition foot-imprint on prestack data or even the stacked image. If not accounted for, irregular sampling can affect prestack data analyses and may introduce noise, cause amplitude distortions, and even structural distortions in the final image. (Black and Schleicher, 1989; Gardner and Canning, 1994; Beasley, 1994; Chemingui and Biondi, 1996a). Many techniques with varying accuracy and cost have been proposed for processing irregularly-sampled data; among them equalized DMO (Beasley and Klotz, 1992), geometrically calibrated DMO (Ronen et al., 1995), spatial dealiasing (Ronen and Liner, 1987) and fold normalization (Chemingui and Biondi, 1996b). The goal has been always to avoid aliasing, interpolate missing data and normalize the imaging process for the effects of fold variations.

Chemingui and Biondi (1996) have demonstrated that the effects of irregularly-sampled data on seismic amplitudes can be substantial and have proposed a method for processing wide-azimuth 3D surveys that can largely overcome these problems. The technique is based on applying the AMO transformation (Biondi et al., 1996) in order to organize the data into common-azimuth common-offset cubes and, therefore, to allow interpolation to a regular grid before imaging. In a subsequent SEP report, Chemingui and Biondi (1996) proposed an additional development in their technique to compensate for the effects of irregular fold distributions. The method extends the multiplicity concept to wave equation processes and uses a normalization procedure to correct the imaging operator for the effects of irregular coverage. The normalization presents an approximate solution to the problem of fold variation by normalizing each input trace in the prestack process according to the local fold of its corresponding bin.

In this report we present a new technique to invert for reflectivity models while properly handling the irregularities in spatial sampling. The technique is based on the method of least squares and consists of a two-step solution to the imaging problem. In the first stage an inverse AMO filter is computed to account for the interdependencies between data traces, then an imaging operator is applied to the filtered data to invert for the final reflectivity model. In the next section we show the relationship between irregular sampling and inverse theory and present a formalism for the normalization filter. The computation of each element of the filter requires the evaluation of an inner product in the model space. We show that each inner product corresponds to an AMO transformation between two data elements. We explore the effectiveness of the method in the 2D case for the application of offset continuation and partial stacking and compare our results to conventional processing techniques.

THEORY

Similarly to Ronen (1987), we pose processing as the inverse of modeling irregular data from regularly sampled model. In mathematical notation we formulate the problem with the following system of equations:

$$d = Lm \tag{1}$$

where the vector d represents the irregular input data, L represents the modeling operator, and the vector m is the model.

Given the nature of multi-channel recording, the design of 3D surveys and the acquisition problems mentioned earlier, it must be expected that the number of data traces is different from the number of model traces, most likely the number of observations is larger than the number of model parameters. One way to solve such a system of inconsistent equations is to look for a solution that minimizes the average error in the set of equations. This minimization can be done in a least-squares sense where the norm $\|Lm - d\|_2$ is minimized. The choice of m that makes this error a minimum gives the least-squares solution which can be expressed for the overdetermined case as

$$m = (L^T L)^{-1} L^T d \tag{2}$$

When solving the underdetermined problem, this solution takes a different expression:

$$m = L^T (L L^T)^{-1} d \tag{3}$$

where m is the minimum energy model that satisfies the linear equations.

These solutions define a least square inverse or pseudo-inverse to the operator L. From equation (2), we write this inverse in terms of L and its adjoint L^T as:

$$L_m^\dagger = (L^T L)^{-1} L^T \tag{4}$$

whereas in (3) the inverse for the underdetermined problem is:

$$L_d^\dagger = L^T (L L^T)^{-1} \tag{5}$$

Applying the pseudo-inverse of (4) is equivalent to applying the adjoint operator L^T followed by a spatial filtering of the model space by the inverse of $L^T L$. Therefore, we will refer to this inverse as model-space inverse.

In equation (5) the adjoint operator is applied after the data have been filtered with the inverse of LL^T and, consequently, we will refer to this inverse as data-space inverse.

In the next section we discuss the connection between the data-space inverse solution and the inversion problem in seismology using irregularly sampled data.

Data-space inverse for irregular geometry

Multichannel recording results in an abundance of seismic traces at every CMP bin. Imaging aims at inverting for a reflectivity model using the entire prestack volume. The model is regularly sampled at the nominal CMP spacing. Therefore, considering multiple records at every CMP bin to present redundant information, the inversion for a reflectivity model from multi-offset seismic input is generally an overdetermined problem.

The reality of seismic acquisition results into variations in CMP locations within the nominal bin spacing and, therefore, could introduce extra degrees of freedom if the model space were allowed to be sampled at the resolution of these spatial variations. Moreover, whenever gaps in seismic coverage occur, the inversion problem becomes locally underdetermined. Therefore, the problem is never genuinely overdetermined as often perceived.

In the context of preserving amplitudes by adapting the data to fit the imaging operator, we seek a solution in which the input is first regularized to correct for the interdependencies between data elements, then an imaging operator (the adjoint of L) is applied to solve for the final model. This type of solution lends itself readily to the definition of a data-space inverse. The problem then reduces to estimating an inverse for the cross-product matrix LL^T, which we shall first define, and explain its properties.

The cross-product filter

The inverse of the cross product operator LL^T acts as a filter for the data space. Each element A_{ij} of $(LL^T)^{-1}$ measures the correlation between a data element d_i and another data element d_j. The computation of each element A_{ij} requires the evaluation of an inner product in the model space. Since the model space is regularly sampled, the inner products for several imaging operators can be computed analytically which leads to a fast and affordable evaluation of the elements of LL^T. This is the case for the solution we present in this paper.

Let's consider an irregularly sampled input of n seismic traces and let L_{m,d_i} be the operator that maps trace d_i into the model space m. The operator that performs the inverse mapping is therefore L^T_{m,d_i}. In matrix notation, we write the cross-product matrix LL^T as

$$
LL^T = \begin{bmatrix}
\left[L_{(m,d_1)} L^T_{(m,d_1)}\right] & \left[L_{(m,d_1)} L^T_{(m,d_2)}\right] & \cdots\cdots & \left[L_{(m,d_1)} L^T_{(m,d_n)}\right] \\
\left[L_{(m,d_2)} L^T_{(m,d_1)}\right] & \left[L_{(m,d_2)} L^T_{(m,d_2)}\right] & \cdots\cdots & \left[L_{(m,d_2)} L^T_{(m,d_n)}\right] \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\left[L_{(m,d_n)} L^T_{(m,d_1)}\right] & \left[L_{(m,d_n)} L^T_{(m,d_2)}\right] & \cdots\cdots & \left[L_{(m,d_n)} L^T_{(m,d_n)}\right]
\end{bmatrix} \tag{6}
$$

Each inner product $\left[L_{(m,d_i)} L^T_{(m,d_j)}\right]$ is therefore a reconstruction of a data trace with input offset h_i to a new trace with offset h_j. We recognize this mapping as our previously-defined AMO transformation. Therefore, we call this cross product filter **A**, and we write it in terms of its AMO elements as

$$
\mathbf{A} = \begin{bmatrix}
I & A_{(h_1,h_2)} & A_{(h_1,h_3)} & \cdots\cdots & A_{(h_1,h_n)} \\
A_{(h_2,h_1)} & I & A_{(h_2,h_3)} & \cdots\cdots & A_{(h_2,h_n)} \\
A_{(h_3,h_1)} & A_{(h_3,h_2)} & I & \cdots\cdots & A_{(h_3,h_n)} \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
A_{(h_n,h_1)} & A_{(h_n,h_2)} & A_{(h_n,h_3)} & \cdots\cdots & I
\end{bmatrix} \tag{7}
$$

where $A_{(h_i,h_j)}$ is AMO from input offset h_i to output offset h_j and, I is the identity operator (mapping from h_i to h_i). Comforming to the definition of AMO (Biondi et al., 1996), $A_{(h_i,h_j)}$ is the adjoint of $A_{(h_j,h_i)}$; therefore, the filter **A** is Hermitian with diagonal elements being the identity and off-diagonal elements being AMO transforms.

This is the fundamental definition of **A** that will allow a fast and efficient numerical approximation of its inverse, and thus of the whole prestack imaging inverse problem. Being a narrow operator, the cost of applying AMO to prestack data is almost negligible compared to other imaging operators such as prestack migration. When the azimuth rotation or the offset continuation is small, as is the case for geometry regularization problems, the size of the operator is very small, but also has a strong curvature. Biondi et al (1996) discussed the design of an efficient implementation of the AMO operator that avoids aliasing and simultaneously takes advantage of the opportunity for saving computation by properly limiting the spatial extent of the numerical integration.

DATA EQUALIZATION

For the data-space inverse solution, the input is first filtered with the inverse of the operator $\mathbf{A}$. The main challenge is then to solve for this inverse and for that we start by writing the solution for m from equation (3) in terms of $\mathbf{A}$ as

$$m = L^T \mathbf{A}^{-1} d \tag{8}$$

Then we change the problem formulation variable d to a new variable $\hat{d}$ and recast the problem as

$$m = L^T \hat{d} \tag{9}$$

where $\hat{d}$ is the filtered input given by the substitution:

$$\hat{d} = \mathbf{A}^{-1} d \tag{10}$$

Solving for $\hat{d}$, we then need to compute the inverse of $\mathbf{A}$ from the system of equations:

$$d = \mathbf{A}\hat{d} \tag{11}$$

Once the inverse of $\mathbf{A}$ is estimated to yield the filtered data $\hat{d}$, we merely evaluate $m = L^T \hat{d}$ to get the solution for the original problem.

Note that after filtering, one can use any imaging operator L^T to invert for m. The new input is well suited to prestack imaging based on any wave equation operator. The role of the equalization filter was to correct the imaging operator for the interdependencies between data elements.

Comparing the fold-normalization technique proposed by Chemingui and Biondi (1996) to the least-squares solution in (9), the inverse of the cross product matrix (LL^T) was approximated by a diagonal matrix in the normalization solution. The diagonal elements in the normalization solution were heuristically derived to be proportional to the inverse of the fold coverage at the output bin that corresponds to the data trace. The fold-normalization procedure, therefore, provides an approximate solution to the problem of varying fold coverage. Nevertheless, experience has proved it can provide good results.

Iterative solution for the inverse filter

The inversion of the cross-product matrix $\mathbf{A}$ is a computationally challenging task. We use an iterative solution to estimate the inverse of $\mathbf{A}$. This solves a huge set of simultaneous equations without the need to write down the matrix of coefficients. The iterative technique is based on the conjugate gradient method, which produces a good result at a reasonable cost. Experience has shown that a satisfactory solution for equation (11) can be achieved in less than 10 iterations, where each iteration involves the application of the adjoint followed by the forward operator. In every iteration, a total of $2n$ AMO operations are performed in order to project each trace to all the other traces in the selected input. Note that both the forward and transpose operations are AMO transformations. The cost of each operation is very cheap giving the narrow aperture of AMO.

EXAMPLE OF TWO-STEP SOLUTION

We now apply the concepts developed in the previous sections for equalizing a seismic dataset before imaging. An application that is of interest to us is the reduction of the size of prestack data while correcting for the effects of its irregular geometry. The example presented in this section was designed to illustrate the two steps in our method and demonstrate the effects of fold variations on prestack imaging.

We consider the application of partial stacking by offset continuation of a 2D synthetic data example. The model consists of a horizontal bed, a dipping reflector, and a point diffractor in a constant velocity medium of 2.5/sec. The input data represents eight constant-offset sections between 500 and 1200 meter offsets with a 100 meter offset spacing. To simulate the fold variations between CMP bins, we randomly deleted about half the total input traces. An area of missing coverage (zero fold) is generated around CMP bin 80, as indicated on the fold chart (Figure 5). Figures 1 and 2 show four out of the eight irregularly sampled input sections.

To illustrate the effects of the irregular geometry on the imaging operator, we show in Figure 3 the output of an offset continuation from 500 to 850 meter offset, whereas in Figure 4 we show the results of AMO from 1000 meters offset to an effective offset of 850 meters. Both results suffer poor resolution and severe amplitude and phase distortions. The aliasing effects strongly contaminated the dipping events and destroyed their phase. The non-aliased implementation of the offset continuation operator following (Bevc and Claerbout, 1992) eliminated that noise around the flat reflector even in presence of irregular sampling.

For the sake of consistency, all the results are displayed after NMO correction since the AMO transformation operates on moveout-corrected data.

Step 1: Equalization of irregular data

The first step in our method involves applying the inverse of the matrix **A** to correct for the interdependencies between the data traces. Each element of **A** is a 2D AMO transformation from one data trace with a given offset to another offset geometry. The offset-continuation operator is applied as an integral operator in the time-space domain. For small offset continuations, the operator is very compact and inexpensive to apply. The implementation defines a true amplitude transformation with amplitude weights following (Fomel, 1996). A phase factor is also taken into account and consists of applying a causal half differentiation for continuations to low offsets and anticausal half differentiation for continuations to large offsets. No antialiasing filter was implemented for the computations of the trace-to-trace AMO transforms.

A satisfactory solution based on a conjugate gradient scheme was obtained after 5 iterations. Given the affordable cost of these 2D transformations, we experimented with large numbers of iterations but without much improvement in the results. Judging the quality of the equalized data is not a straightforward procedure. The number of output traces is the same as in the input volume and the equalized data is still irregularly sampled. Only its amplitude and phase information were equalized for the varying fold coverage. We can only evaluate the equalization step through the quality of the final image after applying an imaging operator to the equalized data.

Step 2: Partial stacking by AMO

The second step in the solution involves imaging the equalized data. As mentioned earlier, after the equalization stage, one can use any imaging operator to invert for the model. We use the AMO operator to invert for a non-zero offset model from the multi-offset input data. From a processing point of view, this is equivalent to the partial stacking of the equalized data after transformation to a common offset of 850 meters.

Previously, Chemingui and Biondi (1994) investigated the application of 2D AMO for coherent partial stacking. They demonstrated on the Marmoussi dataset that stacking after AMO transformation to a common offset eliminates the dip filtering action of NMO stacking by preserving the dipping events and diffractions.

In the context of true-amplitude imaging, we will examine the effects of fold variations on partial stacking by offset continuation and discuss the improvements provided by our solution.

Synthetic results

In a first experiment, we stacked the traces after correcting for NMO and normalizing by the CMP fold in each bin. As expected, NMO produced accurate amplitudes at the horizontal reflector and the flat top of the diffraction hyperbola. The NMO action, however, failed to preserve the steep flanks of the hyperbola. The CMP stacking process acted as a dip filter which destroyed the steep slopes of the diffraction. Moreover, the NMO stack action could not interpolate for the zero coverage area.

Partial stacking after AMO transformtion to a common offset of 850 meters better preserved the steep flanks of the hyperbola. As result of the narrow range of offset continuations and the efficient non-aliased implementation of the 2D AMO operator, the quality of the amo-stack as shown on Figure 7 is better than expected. Nevertheless, data aliasing effects contaminated the dipping arrivals and introduced phase distortions. Most noticeable are the amplitude distortions along the flat reflector (Figure 11) and along the flanks of the diffraction and the dipping bed (Figure 9).

The results of partial stacking after equalizing the input are shown in Figure 8. The output is now smoother, it shows more continuity and better resolution than the unequalized result. The equalization step eliminated the phase distortions along the flanks of the dipping events (Figures 8 and 10) and helped restore the amplitude scales for the horizontal reflector (Figure 12) as well as the dipping arrivals (Figure 10).

This simple inversion was inexpensive. Has anything been gained over the AMO stacking action? First, we reduced the phase distortions due to aliasing of the data. Second, we better preserved the true amplitude scales without ever bothering to think about the number of contributing traces.

CONCLUSIONS

We have presented a new technique to invert for reflectivity models while properly handling the irregular sampling of seismic data. The technique is based on the method of least squares and consists of a two-step solution for imaging. The data is equalized in a first stage with an inverse filter and an imaging operator is then applied to the preconditioned data to invert for a model. The equalization filter corrects the imaging operator for the interdependencies between data parameters. Each element of the filter is a mapping between two data elements. It reconstructs a data trace with given input geometry at the geometry of the other data element. The filter is therefore a symmetric AMO matrix with diagonal elements being the identity and the off-diagonal elements being the trace to trace AMO transforms. We tested the effectiveness of the method in the 2D case for the application of partial stacking by offset continuation. The equalization step followed by imaging has proved to correct and equalize the processing for the effects of fold variations by properly handling the amplitude and phase of the data.

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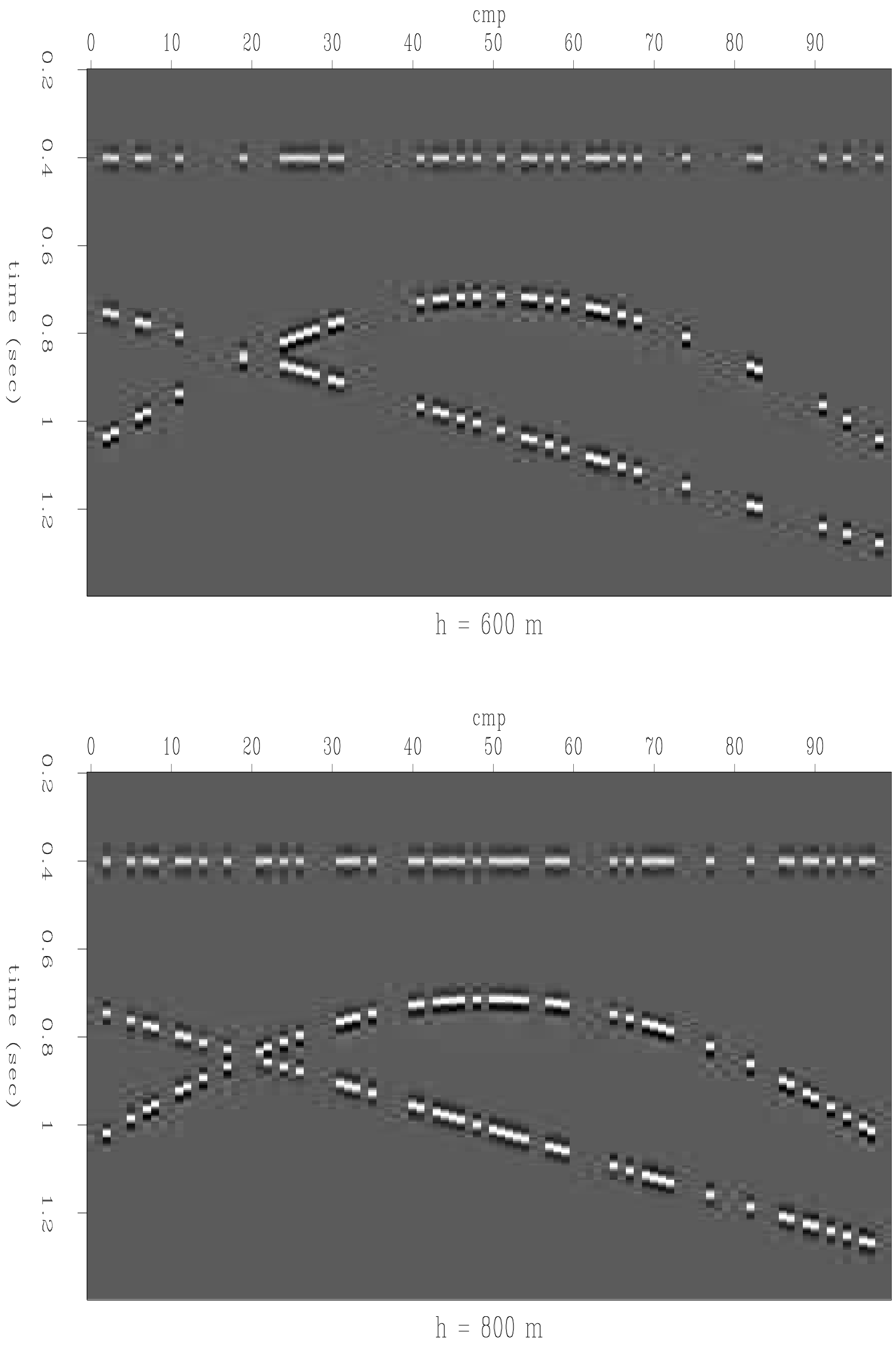

Figure 1: Irregularly sampled constant offset sections with 600 and 800m offsets. $\boxed{\text{nizar1-input24}}$ [CR]

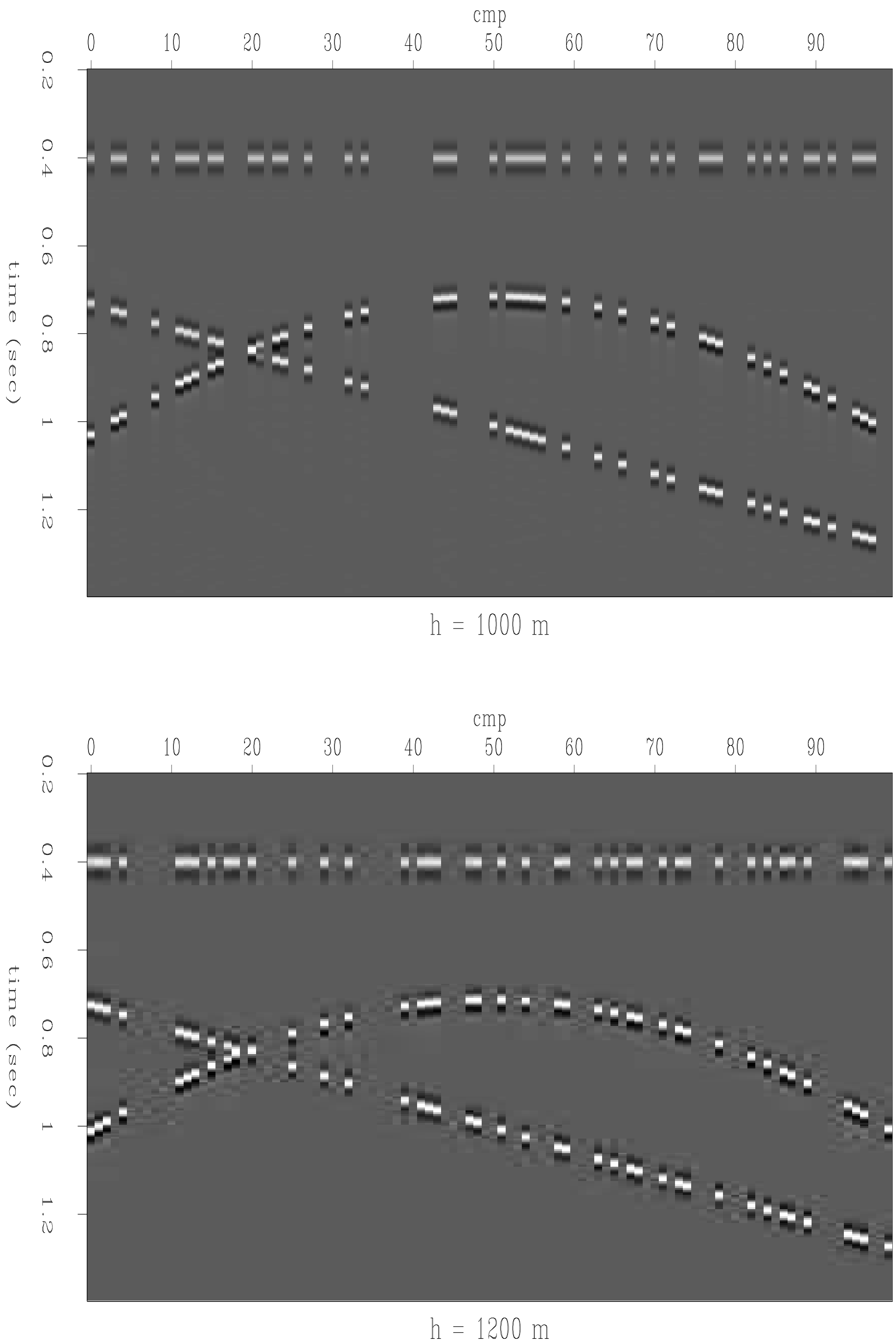

Figure 2: Irregularly sampled constant offset sections with 1000 and 1200m offsets $\boxed{\text{nizar1-input68}}$ [CR]

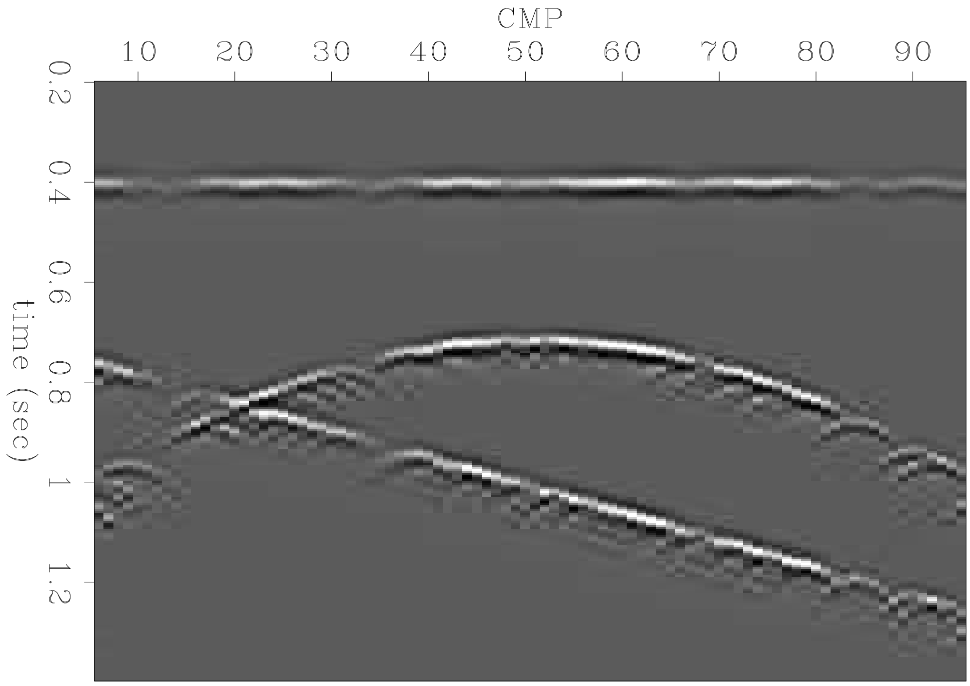

Figure 3: Offset continuation of irregularly sampled constant-offset section from 600m to 850m. nizar1-amo-600 [CR]

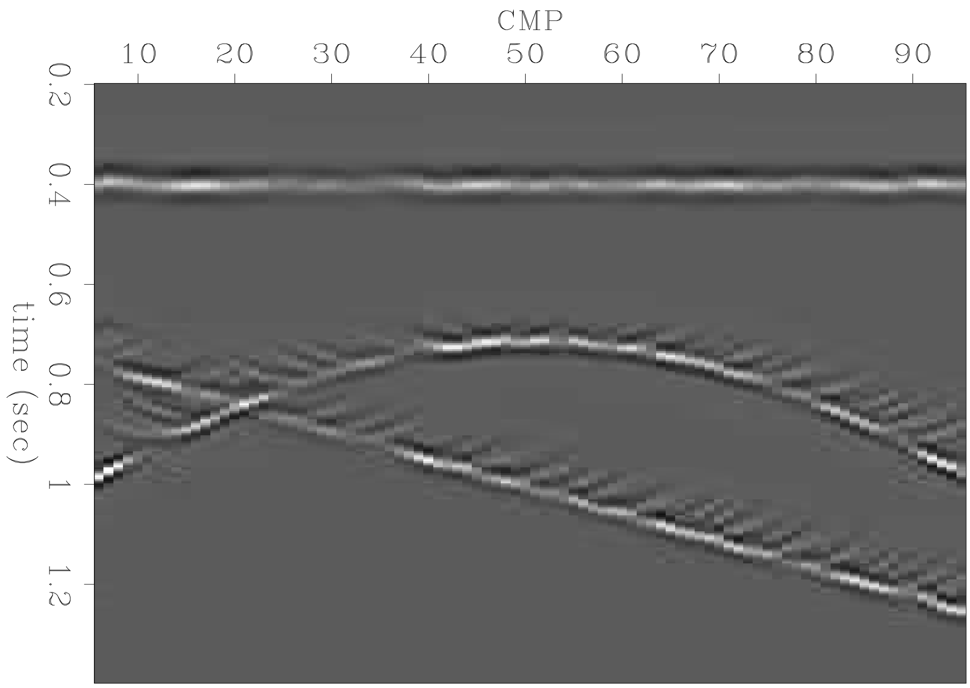

Figure 4: Offset continuation of irregularly sampled constant-offset section from 1200m to 850m. nizar1-amo-1200 [CR]

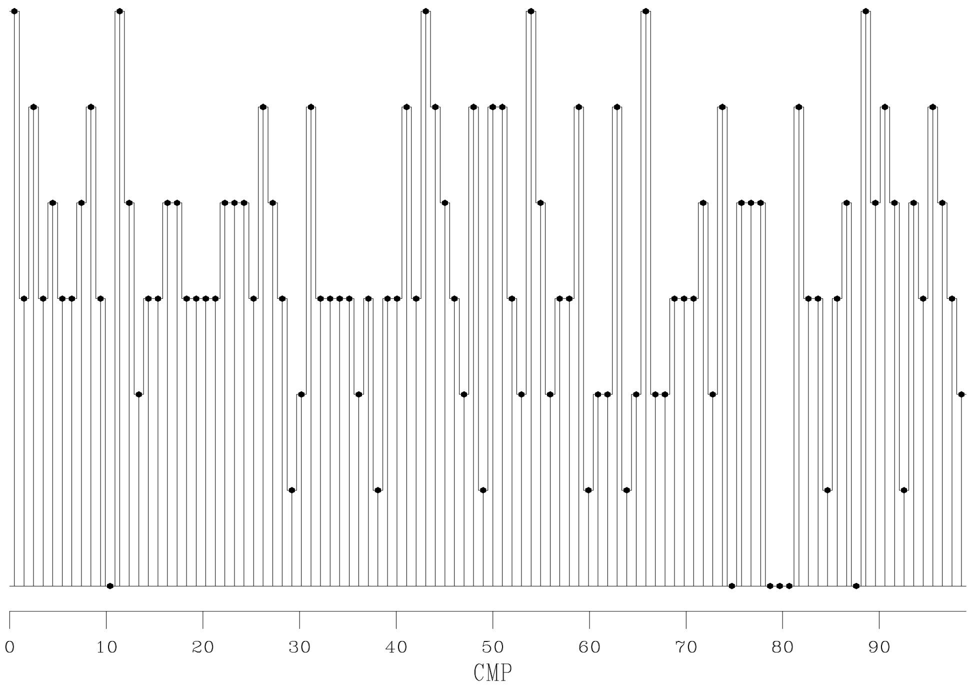

Figure 5: Fold distribution in the input data. The maximum fold is 7 and minimum is 0. nizar1-fold [CR]

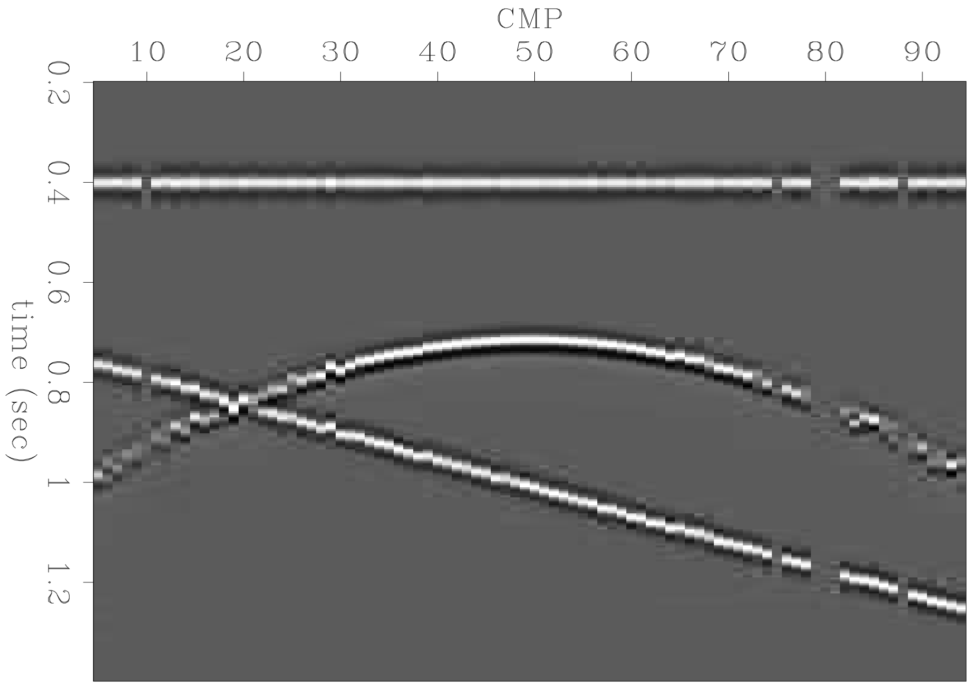

Figure 6: Partial stack after NMO and normalization by the CMP fold in each bin. nizar1-nmo-stack [CR]

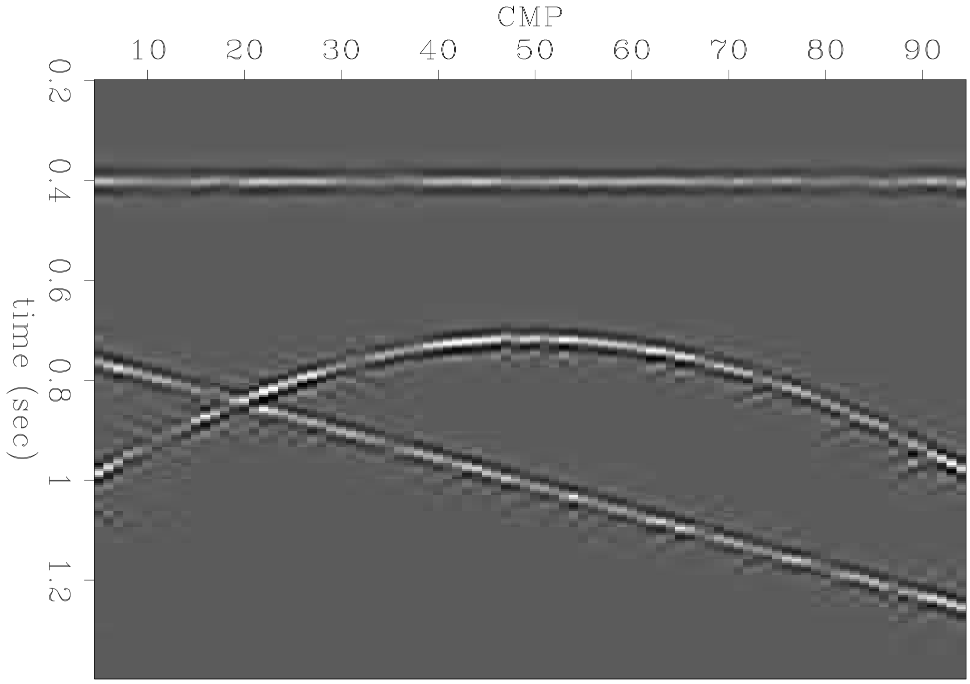

Figure 7: Partial stack after AMO to a common offset of 850m. nizar1-amo-stack [CR]

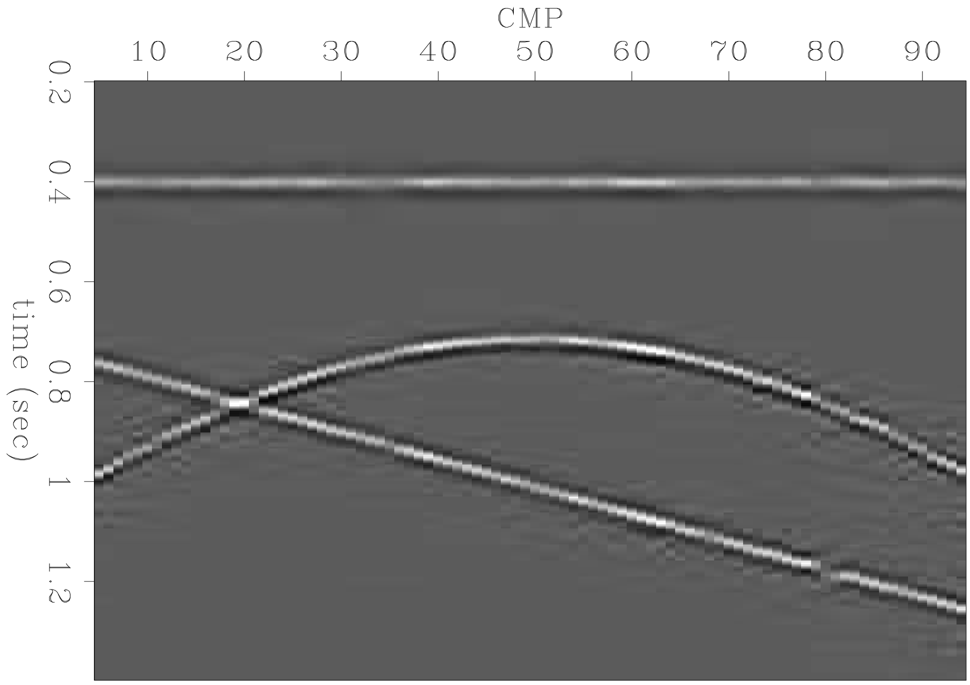

Figure 8: Output of the two-step inversion at an effective offset of 850 meters. nizar1-equal-stack [CR]

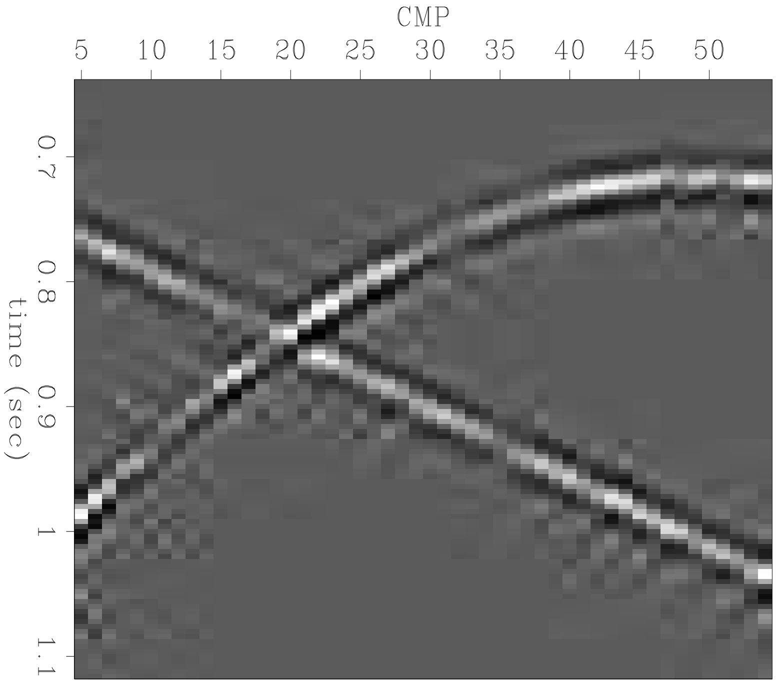

Figure 9: Window on the dipping events from the amo-stack result. nizar1-amo-stack-win [CR]

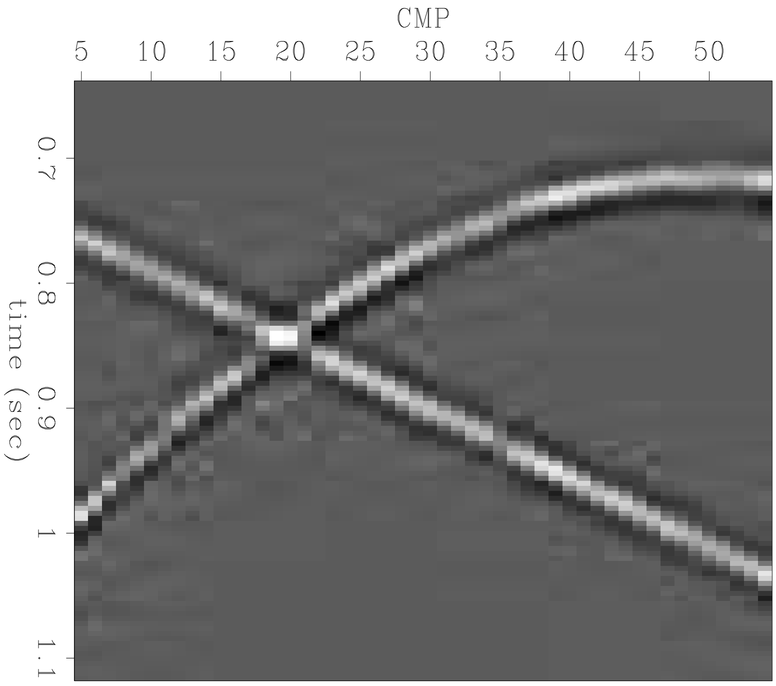

Figure 10: Window on the dipping events from the two-step inversion. nizar1-equal-stack-win [CR]

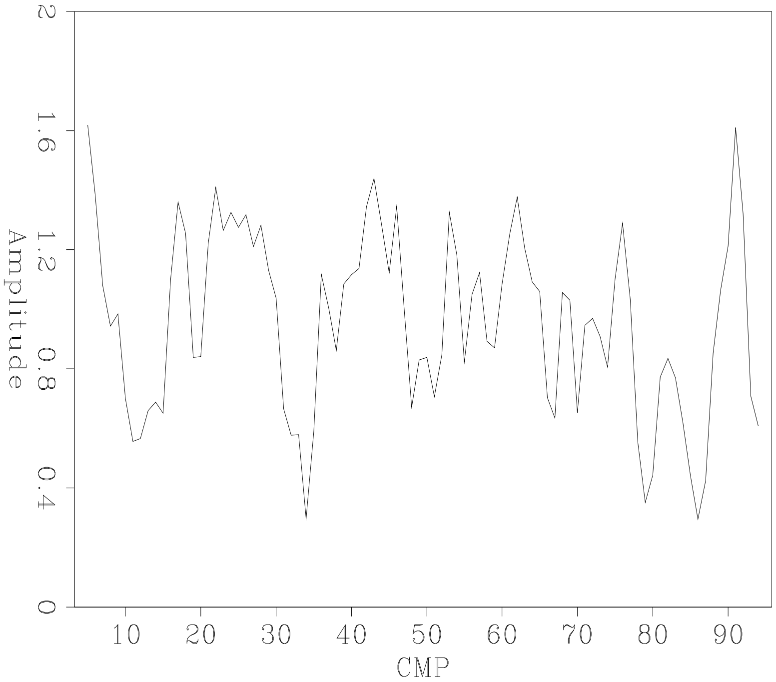

Figure 11: Peak amplitudes on the flat reflector from the amo-stack result. nizar1-amp-amo [CR]

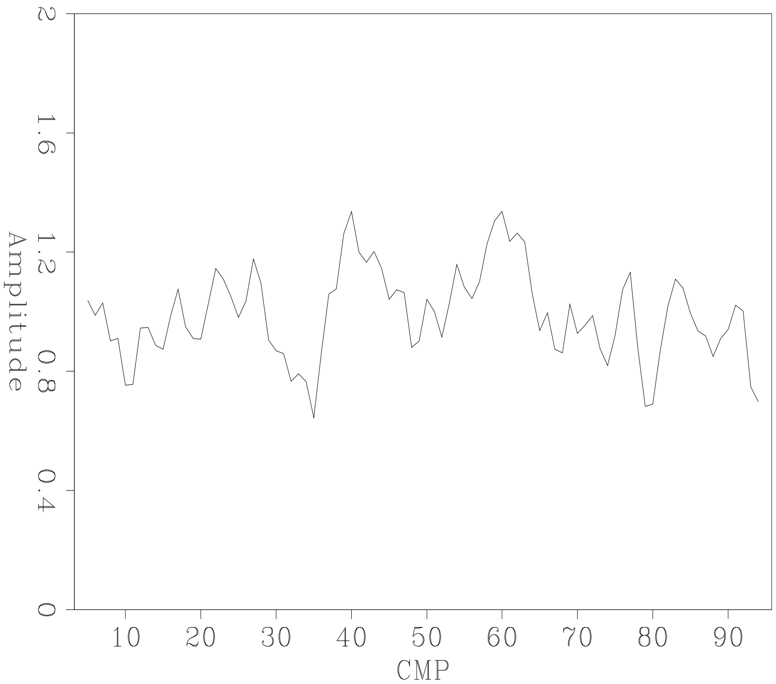

Figure 12: Peak amplitudes on the flat reflector from the two-step solution. nizar1-amp-norm [CR]

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