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The effects of lateral velocity variations and ambient noise source location on seismic imaging by cross-correlation

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ABSTRACT

Finite-difference modeling is used to investigate the conjecture that, by cross-correlating noise traces recorded at the surface, we can construct what would be recorded at one of locations if there was a source at the other. Synthetic experiments are conducted on a variety of Earth models with lateral variations in velocity. For each model, conventional seismograms are compared with seismograms constructed by cross-correlating noise traces. Sources of the ambient noise are first taken to be at infinity (planar wavefronts), and then within the zone of interest (curved wavefronts). The kinematics of the reflection events in all the cross-correlation seismograms are consistent with the conventional seismograms, suggesting the conjecture is robust to moderate lateral velocity variations and the location of the source of the background noise. In cases where most of the incoming energy has one angle of incidence, it is shown that by applying a weighting function in the slant-stack domain, the coherency of hyperbolic reflection events is increased.

INTRODUCTION

The idea that reflection seismograms could be constructed by cross-correlating ambient background noise was first discussed by Claerbout (1979). He found that for a one-dimensional Earth the autocorrelation function of an earthquake seismogram should equal what would be recorded in a reflection seismic experiment. This 1-D theory can simply be extended into plane layered Earth by considering slant stacks, leading to the conjecture that:

> By cross-correlating noise traces recorded at the surface we can construct the wavefield that would be recorded at one of the locations if there

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was an impulsive source at the other.

Cole (1995) tested this conjecture on synthetic data generated by a phase shift method, modeling the background noise as many random plane waves coming from all directions in the subsurface. However, his results were not conclusive, and his tests on field data were unsuccessful.

Rickett and Claerbout (1996) used phase shift modeling of random incident plane waves to show that cross-correlating noise traces gives the correct kinematics for horizontally layered and point diffractor models. The length of the time series before cross-correlation was shown to be the critical parameter in determining the signal to noise ratio; whereas the number and spatial distribution of plane waves determined the coherency of events. They also suggested that a half-differential filter, $\sqrt{i\omega}$, be applied to the output (in the 2-D case) to boost higher frequencies that may have been suppressed by the stacking of many plane waves.

Plane layers versus lateral velocity variations

The correct kinematics for layered models were predicted by the theory. An illustration of the raypaths for a single horizontal layer is shown in Figure 1. The cross-correlation picks out the times $T_{QB} - T_{PA}$ and $T_{PAB} - T_{PA} = T_{AB}$. These two times correspond to a 'direct wave' traveling directly between the geophones with apparent slowness equal to that of the incoming wave, and the reflection received at B if there was a source at A.

The theory has not yet been extended to Earth models with lateral variations in velocity. It is important to establish if the conjecture still holds for $v(x, z)$ models; and if it does not, then the limits of its applicability should be found and understood.

Incorporating a simple lateral velocity variation into Figure 1 gives Figure 2. Again the cross-correlation picks out two events. The velocity anomaly affects T_{PA} and T_{PAB} equally so the kinematics of the reflection event remain correct. However T_{QB} is not effect by the velocity anomaly, so the kinematics of the 'direct wave' are altered. Although this figure does not disprove the conjecture for a $v(x, z)$ models, it does show that lateral variations may affect the direct wave but not the reflection event.

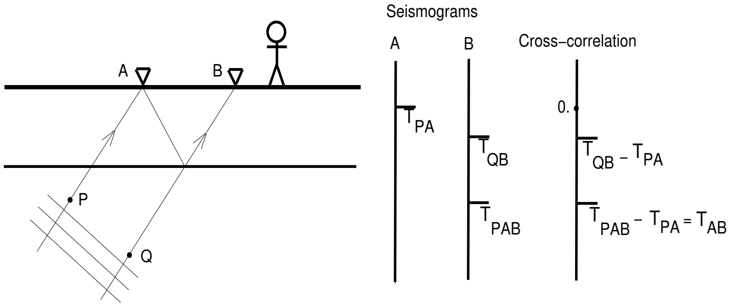


Figure 1: Ray paths for a single plane wave incident from below a horizontal layered model. Cross-correlation lag between events recorded at A and B corresponds to reflection event in conventional seismogram. james-rays1 [NR]

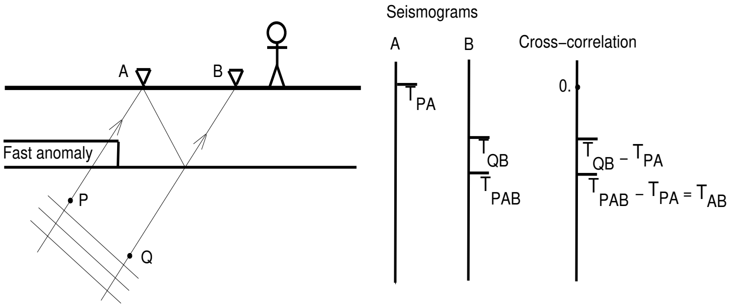


Figure 2: Ray paths for a single plane wave incident from below a model with lateral velocity anomaly. Time of reflection event is not affected by the anomaly, but the 'direct wave' is. james-rays2 [NR]

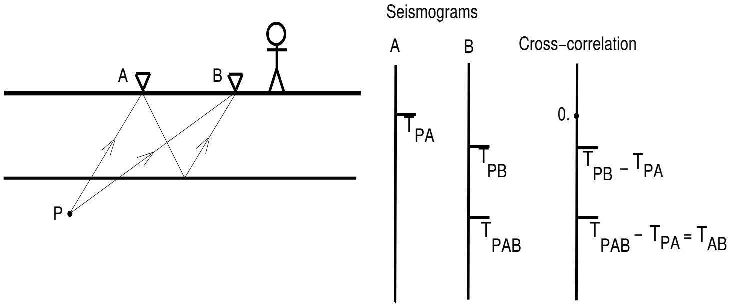


Figure 3: Ray paths for a single point source below a horizontal layered model. Time of reflection event is not affected by the location of the source, but the 'direct wave' is. james-rays3 [NR]

Sources within the zone of interest

So far the studies have been based on the assumption that the ambient background noise consists of 'random plane waves incident from below'. Whether or not this is a valid assumption is an open question; however, people have observed relatively strong seismic events coming from the reservoir interval due to micro-fracturing during hydrocarbon production. It is also likely that the surface noise will propagate into the subsurface, and in turn be reflected and recorded. So if the dominant source of ambient noise is not plane waves incident from below, but rather point sources within the zone of interest, is the conjecture still valid?

Figure 3 shown a ray diagram for a point source. As with the case with the velocity anomaly, the kinematics of the reflection event are not affected by the location of the source; although the kinematics of the direct wave are. Again this suggests that although there may be problems with the kinematics of the direct wave, the reflection event may be correct.

Slowness spectra

Rickett and Claerbout (1996) observed that for complete reflection seismograms to be constructed, waves must be incident with all incident angles, or slowness values. They suggested that a filter could be constructed and applied that 'whitened' the slowness spectrum of the incoming waves, producing coherent events.

The effect of lateral velocity variations, the effect of point sources as opposed to plane waves, and the whitening of the slowness-spectra are investigated in this paper.

The phase shift modeling was abandoned in favor of the flexibility of finite-difference simulation.

FINITE-DIFFERENCE MODELING

Finite difference modeling offers the flexibility that make it suitable for testing more complicated Earth models, illuminated by a variety of types of background noise.

Earth models

Four different Earth models were constructed (Figure 4):

1. A simple horizontal plane layer
2. A dipping plane layer
3. Two plane layers with opposite dips
4. A faulted block model

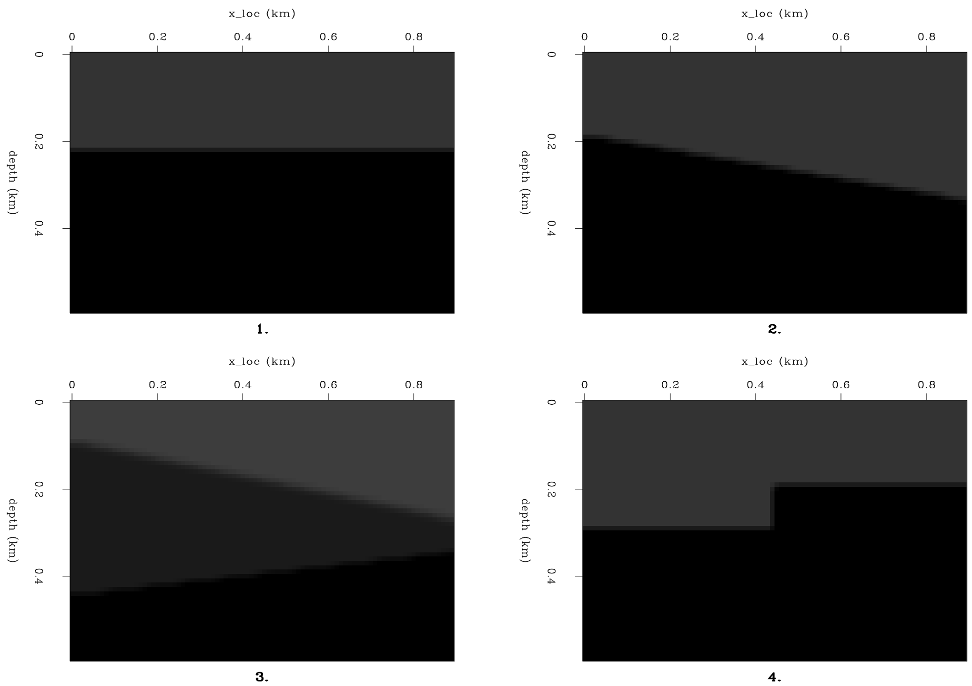


Figure 4: Models for finite-difference experiments: 1. Simple plane layer, 2. Single dipping layer, 3. Two dipping layers, and 4. Fault model. All axes show km's. Velocities vary from 1500 m/s in the uppermost layer to 2000 m/s in the half-space below. james-models [ER]

The velocities in each model varied from 1500 m/s in the uppermost layer to 2000 m/s in the half-space below. The density values were engineered artificially to give large impedance contrasts at the boundaries and hence strong reflections. In all cases the top boundary was a free surface.

The simulations were done using a versatile finite-difference modeling program written by Martin Karrenbach (1995). A cell size of 5 m by 5 m were used, with time-steps of 2 ms. The input grids were surrounded by an exponential damping boundary that was 20 cells wide, and a one-way boundary, 4 cells wide. To reduce artifacts due to the discrete nature of the mesh, the models were slightly smoothed. This can be seen in Figure 4.

Experiments

Three synthetic experiments were then carried out on each Earth model:

1. A conventional reflection seismic experiment. A common shot gather was created by simulating a point source at the surface, and recording the reflected waves.
2. An imaging by cross-correlation experiment, where the ambient noise was a family of plane waves incident from all angles below.
3. An imaging by cross-correlation experiment, where the ambient noise was a family of point sources located within the zone of interest.

The modeling for the cross-correlation experiments was done independently for each incoming wave; these results were then cross-correlated and then added together (Figure 5). This approach can be justified by considering the following (Claerbout, 1979):

If $B_1 =$ the impulse response at 1,
$B_2 =$ the impulse response at 2, and
$N =$ white noise.

Therefore $B_1 N =$ the white response at 1, and
$B_2 N =$ the white response at 2.

Since white noise is defined so that the expectation of $\bar{N}N = I$, the cross-correlation of the two white responses equals the cross-correlation of the impulse responses.

$$\bar{B}_1 \bar{N} N B_2 = \bar{B}_1 I B_2 = \bar{B}_1 B_2 \tag{1}$$

The advantage of modeling each incident wave separately is that the seismograms are free from random noise: the results are the same as would be expected from

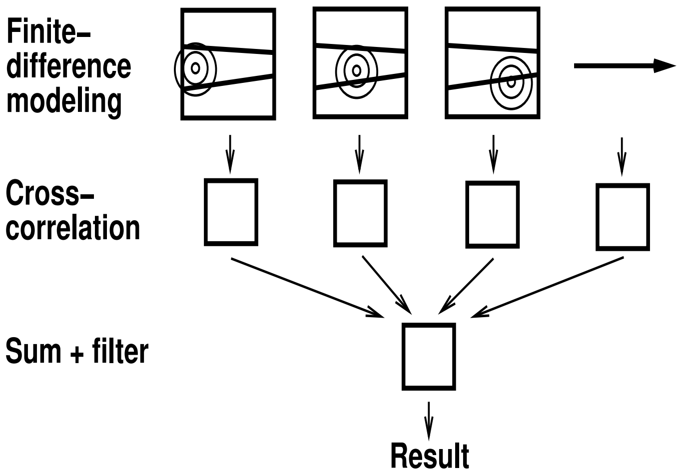


Figure 5: Flow used to generate common shot gathers by cross-correlating finite-difference noise seismograms. james-flow [NR]

cross-correlating infinite time series. The effects of the finite length time series are therefore removed from the experiment, allowing the issues of signal to noise to be separated from issues of resolution.

RESULTS

Common shot gathers constructed using the synthetic experiments described above are shown in Figures 6 to 9.

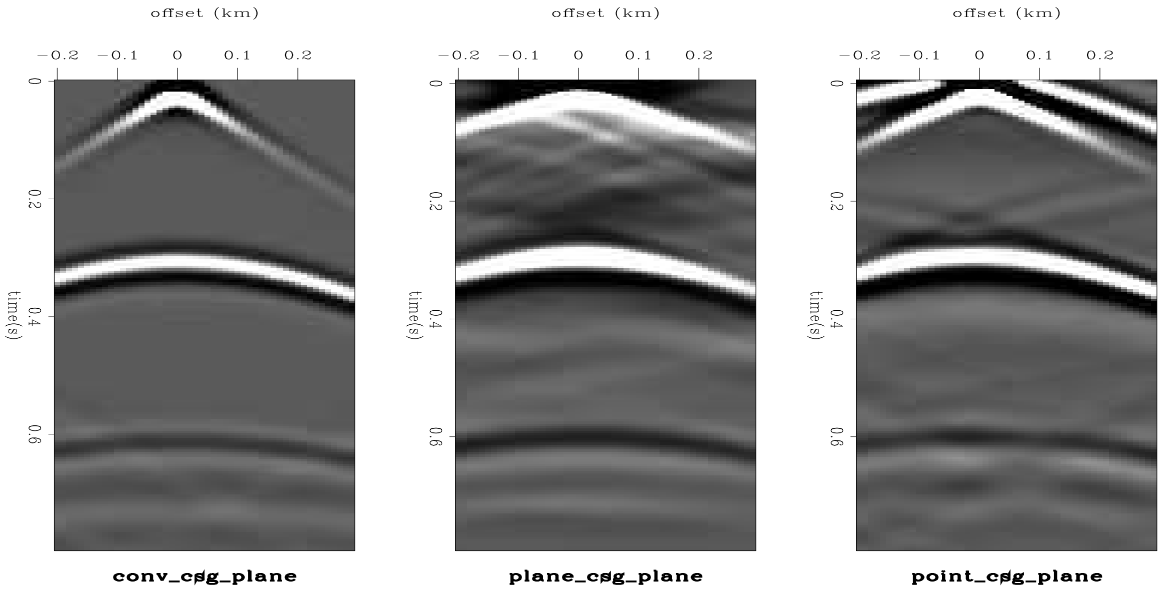


Figure 6: Common midpoint gathers over a single horizontal reflector. Left panel: conventional finite-difference experiment. Center panel: cross-correlation method with planar background noise. Right panel: cross-correlation method with sources of the background noise within the zone of interest. james-csg.plane [CR]

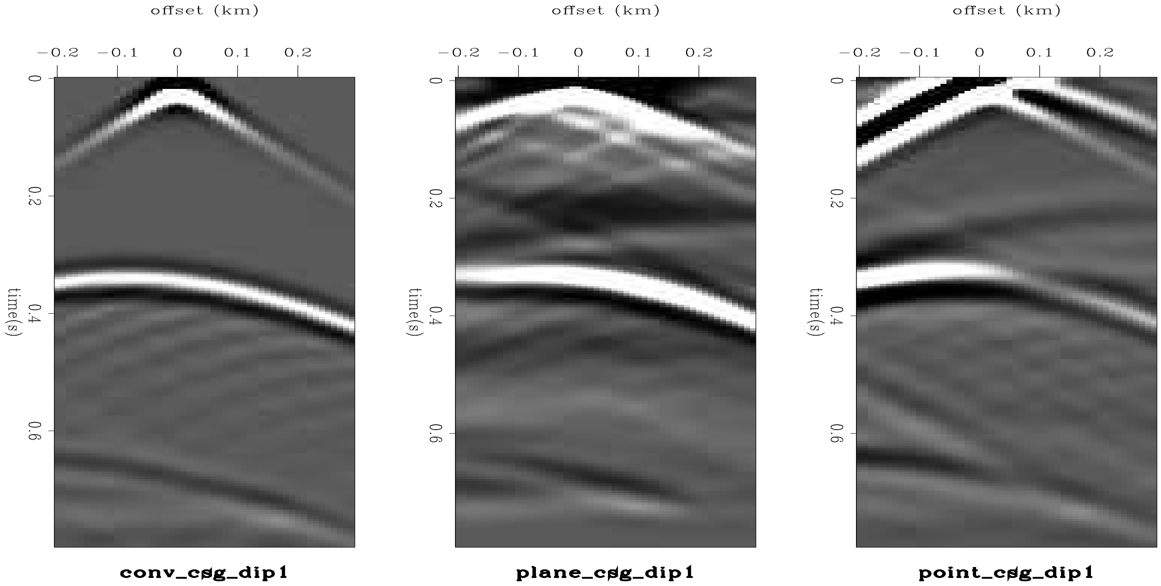


Figure 7: Common midpoint gathers over a single dipping reflector. Left panel - conventional finite-difference experiment. Center panel - cross-correlation method with planar background noise. Right panel - cross-correlation method with sources of the background noise within the zone of interest. james-csg.dip1 [CR]

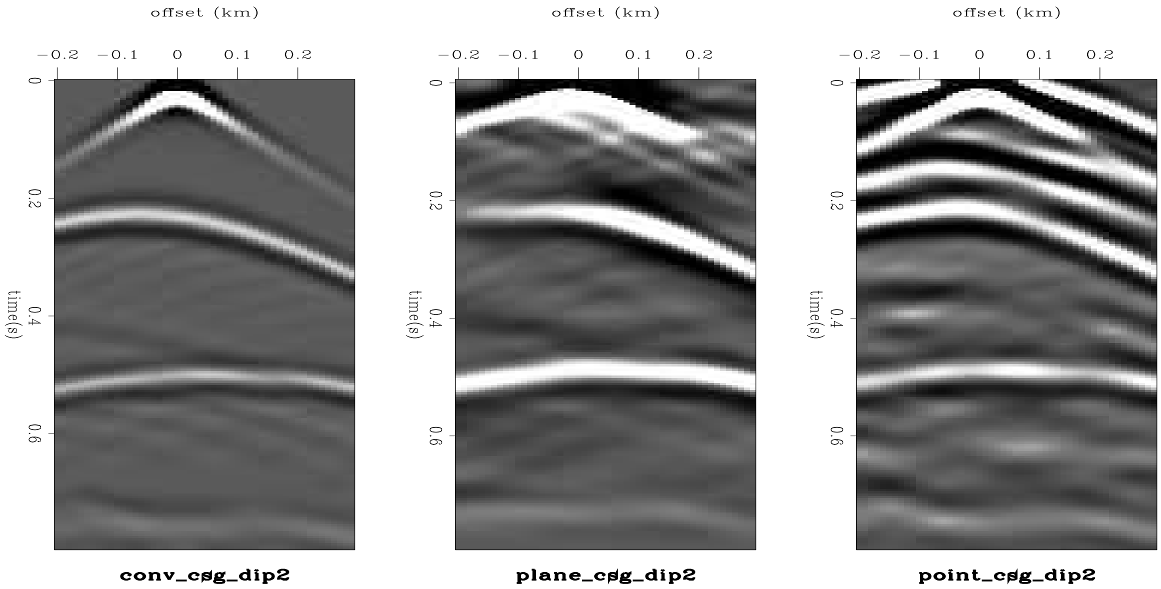


Figure 8: Common midpoint gathers over two dipping reflectors. Left panel - conventional finite-difference experiment. Center panel - cross-correlation method with planar background noise. Right panel - cross-correlation method with sources of the background noise within the zone of interest. james-csg.dip2 [CR]

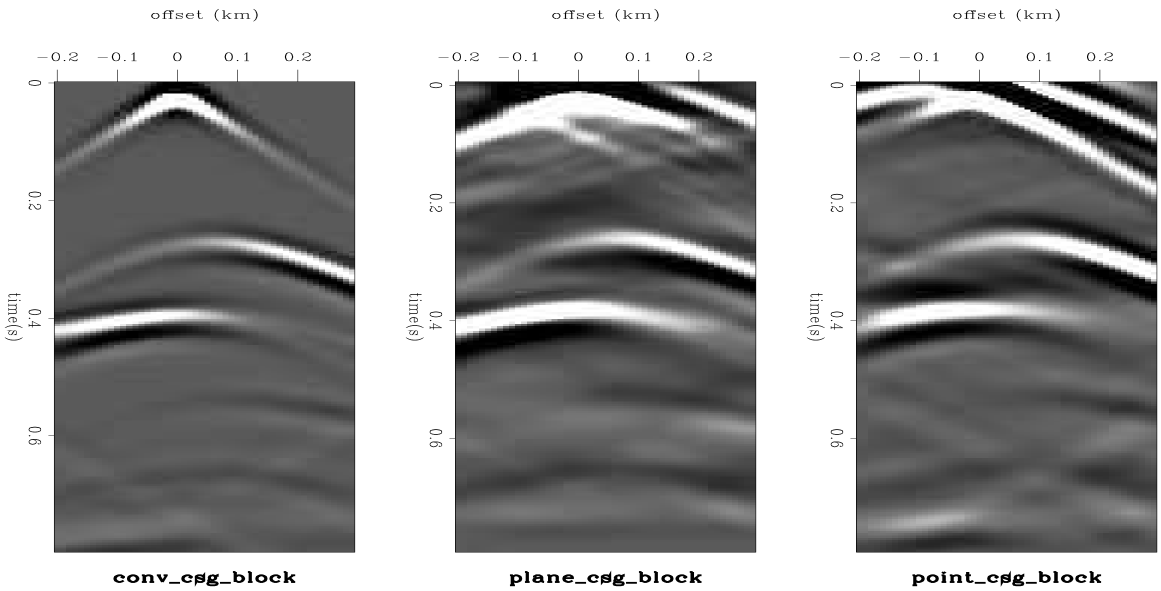


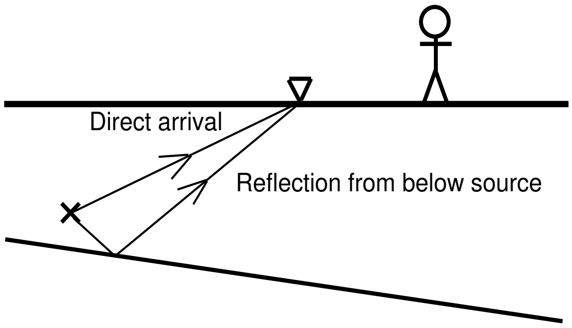
Figure 9: Common midpoint gathers over a fault block model. Left panel - conventional finite-difference experiment. Center panel - cross-correlation method with planar background noise. Right panel - cross-correlation method with sources of the background noise within the zone of interest. james-csg.block [CR]

Figures 6 and 7 shows common shot gathers over Models 1 and 2, a single horizontal reflector and a single dipping reflector. The left panels were generated by a conventional finite difference experiment, while the middle and right panels were generated by cross-correlating waves from below. The difference between the middle and left panel is that the sources of the incoming waves were at infinity for the middle panel, giving planar waves; and within the zone of interest for the right panel, giving curved wavefronts.

The reflection events visible in the cross-correlation seismograms seem to be consistent with the conventional shot gathers to within the resolution of the modeling; this true for both sources located within the zone of interest as well as at infinity. The direct waves, however, are not consistent with the conventional seismograms. This is due to the factors discussed in the introduction and the lack of incident waves with an apparent slowness equal to the slowness of the near surface layer.

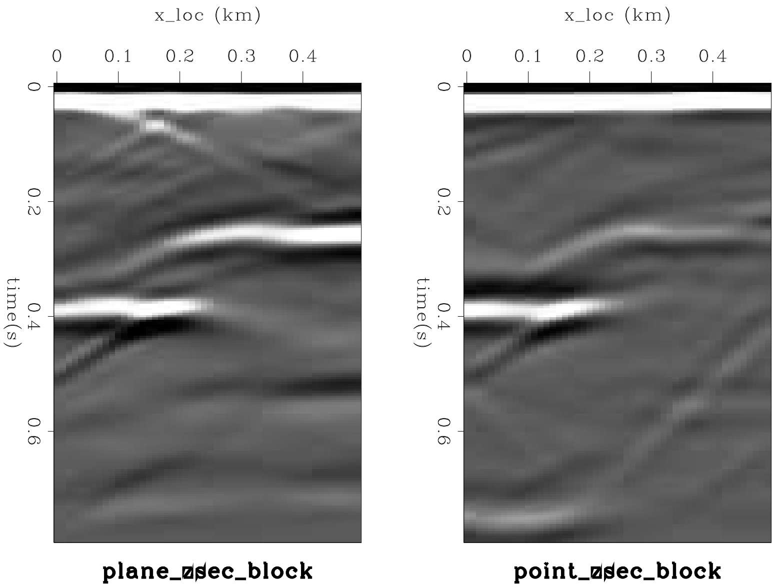
Common shot gathers modeled over Model 3 are shown in Figure 8. Again the kinematics of the two reflections appear correct to within the resolution of the modeling, for both plane wave and point sources of ambient noise. However, in the point source case (right panel), an extra event appears at 0.15 s travel-time. This is a corresponds to the primary reflection from the event *below* the sources (see Figure 10). This event is coherent in this example because the point source were all situated at the same depth of 0.3 km. If the point source had been located at random depths then this event would not have been coherent.

Figure 10: Reflection from below source that causes extra event in common shot gather created by cross-correlation. james-belowref [NR]



The shot gathers for the fault block model are shown in Figure 9; and zero-offset sections of this model, created by auto-correlating noise traces, are shown in Figure 11. Again the kinematics are consistent to within the resolution of the experiment. Diffractions can even be seen from the edge of the fault.

Figure 11: Zero-offset sections of fault block model generated by cross-correlation method. Left panel: sources of ambient noise at infinity. Right panel: sources of ambient noise within zone of interest. james-zsec.block [CR]



Another point that can be seen in the Figures is that the bandwidth of the cross-correlation seismograms is comparable to the bandwidth of the conventional seismograms. This implies that the resolution we may expect from field data is governed by the bandwidth of the ambient noise, not by the length of the time series that are cross-correlated.

WHITENING THE SLOWNESS SPECTRA

For the case of a plane layered Earth, for a particular slowness to be present in the cross-correlation seismograms, then there must be ambient energy incident with that slowness. Similarly, if the incident energy has one predominant slowness then this will overwhelm the rest. In this case, a filter should be applied so the resultant seismograms contain information with all slowness values.

In this paper, we have applied a filter to cross-correlation common shot gathers in the slant-stack (linear $\tau - p$) domain. The transformation to the slant-stack domain is easily invertible (Thorson, 1984). It also represents a decomposition into plane waves of given slowness. Since this is the energy which we would like to balance, it is a natural domain to work in both practically and theoretically.

The filter that was applied in this domain tapered the r.m.s. values of each p trace by a Gaussian weighting function. The width of the Gaussian was determined by the range of slowness values desired in the output.

For a $v(z)$ model illuminated by plane waves from below, each plane wave will contribute to one slowness value, and so the slant-stack decomposition well separates the energy from each incident angle. The left panel of Figure 12 shows a cross-correlation seismogram generated for single horizontal layer illuminated by plane waves from all directions in the subsurface. However some of the plane waves were stronger than others, so the seismogram is dominated by energy with positive p-values. This can be seen in the slant-stack domain shown in the right panel.

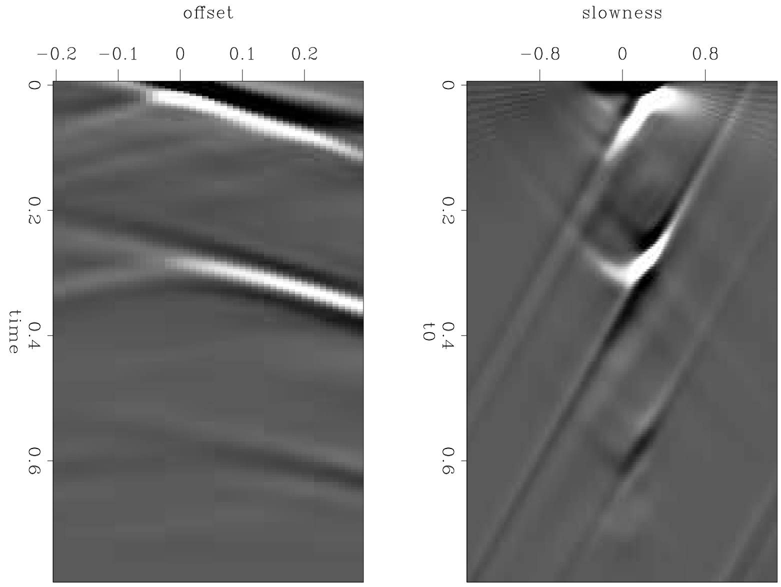


Figure 12: Cross-correlation seismograms over a single horizontal layer with dominant energy from one incident angle. Right panel: common shot gather. Left panel: slant-stack. james-pw.plane.b4 [CR]

The filtered slant-stack of the same data is shown in Figure 13, along with the resultant seismogram after inversion back to the $t-x$ domain. The primary and first order multiple reflection events are now more coherent.

For a model with lateral velocity variations, we expect less successful results. Plane waves in the subsurface are not necessarily plane waves when they reach the surface, so the slant-stack transformation does not separate the energy as cleanly. Similarly if the sources of the ambient noise are with in the zone of interest, the corresponding wavefronts will be diverging when they reach the surface, contributing to a range of

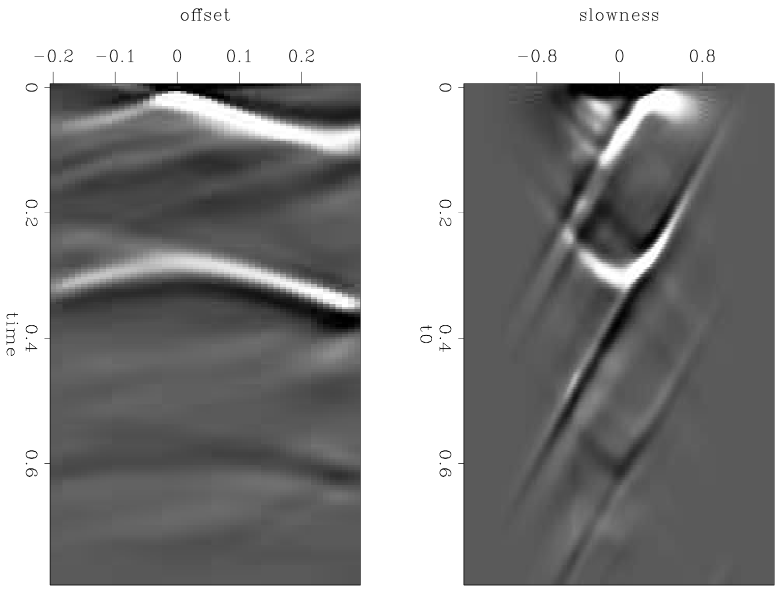


Figure 13: Cross-correlation seismograms over a single horizontal layer after slowness filtering. Right panel: common shot gather. Left panel: slant-stack. james-pw.plane.after [CR]

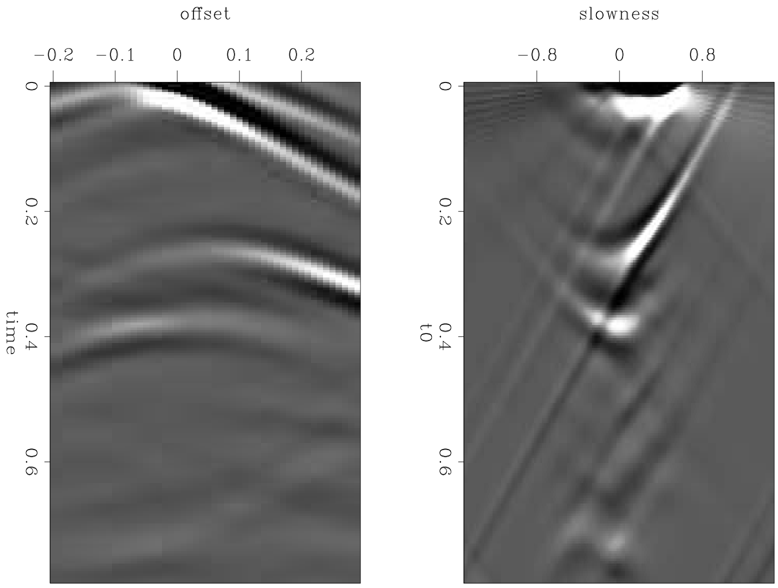


Figure 14: Cross-correlation seismograms over fault block model with dominant energy from a single source in the subsurface. Right panel: common shot gather. Left panel: slant-stack. james-pw.block.b4 [CR]

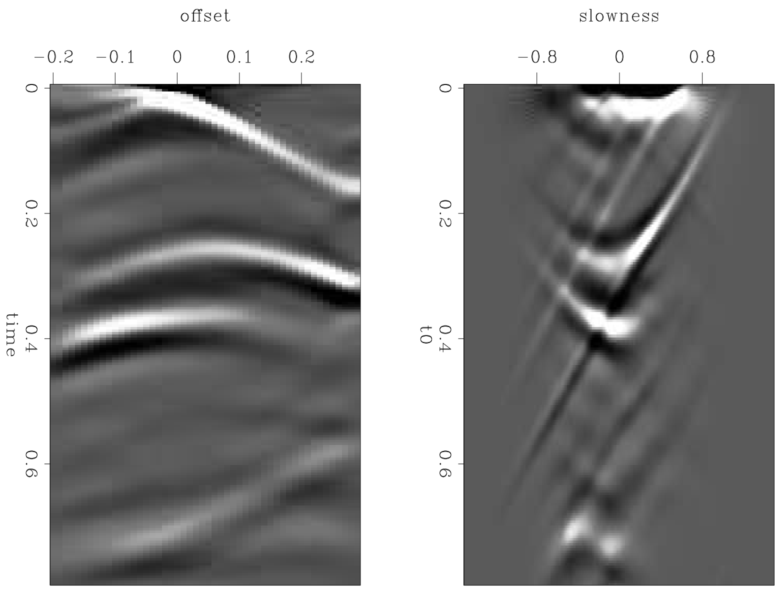


Figure 15: Cross-correlation seismograms over fault block model after slowness filtering. Right panel: common shot gather. Left panel: slant-stack. james-pw.block.after [CR]

p-values. These problems can be seen in Figures 14 and 15, where there is an extra dipping event towards the bottom of the gather.

It should be noted that this method of filtering will boost under-represented slownesses even if they contain no signal, in which case noise will be boosted. It would be better if a filtering scheme was designed that only boosted slownesses containing useful information.

CONCLUSIONS

Seismograms have been generated by cross-correlating noise traces for a variety of Earth models with moderate lateral variations in velocity. Sources of the ambient energy were modeled to be both at infinity, leading to plane waves at the Earth's surface, and within the zone of interest, leading to diverging wavefronts at the surface. In all cases the kinematic of the reflections agree with either the results of a conventional seismic reflection experiment (or with simple ray theory) to within the resolution of the finite-difference modeling. This suggests that the kinematics of the cross-correlation method are robust to moderate lateral velocity variations, and the location of the sources of the ambient noise.

However, both lateral variations in velocity and source locations at finite distance mean the wavefronts arriving at the surface are not planar. Although this does not affect the kinematics of the reflections, it does affect the kinematics of the first arrival, the event that resembles the direct wave.

In cases where most of the incoming energy has one angle of incidence it is recognized that it may be beneficial to apply a filter which whitens the slowness spectra of the incoming energy. In this paper we have applied a weighting function in the slant-stacked shot domain. This approach works best for a plane-layered Earth

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