Stanford Exploration Project, Report 82, May 11, 2001, pages 1–136

Berryhill's "pure migration" = Gardner's PSI

Alexander M. Popovici[1]

ABSTRACT

Kinematically, Berryhill's "pure migration" or nonimaging shot-record migration is equivalent to dip moveout (DMO) before normal moveout (NMO) followed by Gardner's Prestack Imaging (PSI). The two processes have the same effect on a point diffractor in midpoint, offset, and time coordinates, compressing the Cheops pyramid surface to the same curve. The two methods produce the same kinematic result, although from a computational point of view, Berryhill's method is much faster.

INTRODUCTION

Prestack imaging (PSI) introduced by Gardner et al. (1986) is a prestack migration alternative. DMO followed by PSI migrates data without stacking. Gardner's claim to velocity independence is questionable, but it seems that by delaying the NMO step to the end of the processing flow, the data is better focused and it has a better signal to noise ratio. Recently, Berryhill (1994), presented a new focusing technique called nonimaging shot-record migration based on the the idea that for a given shot and receiver position, data can be spread along a line instead of the conventional prestack migration ellipse. The method and the examples were presented for constant velocity, although Berryhill does mention the possibility of extending the principle to variable velocity. The two methods are kinematically equivalent. I analyze the effect each method has on the reflection times from a point diffractor and show that both methods produce the same result. I start by reviewing Gardner's PSI and explain how the method reduces the Cheops-pyramid diffraction surface to a single curve. The resulting curve exists only in offset at the common-midpoint location above the diffractor. In other words the Cheops pyramid is compressed to a single curve in the common-midpoint section corresponding to the surface coordinate of the diffractor. Next, I analyze Berryhill's method and show that for the same diffractor, Berryhill's migration focuses the Cheops diffraction surface to the same curve generated by Gardner's method. Therefore, I conclude that the two methods are equivalent from a kinematic point of view.

[1]**email:** mihai@sep.stanford.edu

Gardner's prestack imaging

Gardner's Prestack Imaging (PSI) is a time migration process in common-midpoint, offset and time coordinates, based on focusing the Cheops-pyramid diffraction surface into a curve. To bring the Cheops surface to a suitable form for focusing, DMO before NMO is applied to the data. After DMO and PSI, the final image is obtained by NMO and stack. The equation for

../mihail/./Fig/DMOgeom.ps

Figure 1: DMO geometry for midpoint-offset coordinates.

the source-receiver traveltime t_h in the case of a single diffractor, schematically represented in Figure **??**, can be written as:

$$t_h = \sqrt{\frac{t_0^2}{4} + \frac{(m - A + h)^2}{v^2}} + \sqrt{\frac{t_0^2}{4} + \frac{(m - A - h)^2}{v^2}}, \tag{1}$$

where t_0 is the two way vertical time from the surface to the diffractor, m is the midpoint location, A is the surface location of the diffractor, and h is the half-offset. Equation (1) forms a diffraction surface in the system of coordinates formed by the common-midpoint m, the half-offset h and the traveltime t_h, commonly referred to as the Cheops-pyramid surface. A cut through the Cheops surface for a constant-offset, produces the flat-top hyperbola curve shown in Figure **??**. For the zero-offset case, the cut generates the customary zero-offset hyperbola pictured on the face of the cube. As the offset increases, the common-offset sections produced by setting the variable h to a constant in equation (1), become flat-top diffraction curves with apexes at increasing traveltimes. The standard processing sequence of NMO followed by DMO (or migration to zero-offset (MZO) in a single step) transforms the flat-top hyperbolas into identical zero-offset hyperbolas. Applying DMO before NMO will produce zero-offset hyperbolas shifted down in time with the NMO correction, as shown in the right cube of

../mihail/./Fig/DMOcubes2.ps

Figure 2: 3-D prestack cube data over a point diffractor.
The flat-top hyperbola in a common offset section is transformed into a zero offset hyperbola, shifted down by the NMO correction.

Figure **??**. After the NMO correction, the shifted zero-offset hyperbolas line up in offset, and therefore stacking greatly improves the signal to noise ratio. After DMO-before-NMO, the diffraction curve t_h is given by

$$t_d = 2\sqrt{\frac{t_0^2}{4} + \frac{(m-A)^2 + h^2}{v^2}}.$$

This definition of t_d creates a hyperboloid of revolution in the m, h, t_d space. Indeed for a constant t_d value we have

$$(m-A)^2 + h^2 = \frac{v^2}{4}(t_d^2 - t_0^2), \tag{2}$$

which is the equation of a circle of radius

$$\frac{v^2}{4}(t_d^2 - t_0^2),$$

and center of coordinates $(m = A, h = 0)$, as displayed in Figure **??**. Notice that in this formulation DMO is performed before NMO (or alternatively inverse NMO is performed after MZO). The hyperboloid of revolution still includes the NMO correction. This is made obvious by rewriting equation (2) as

$$\frac{4(m-A)^2}{v^2} + \frac{4h^2}{v^2} = t_d^2 - t_0^2.$$

The essence of PSI is to sum over circles in time-slices. The output of such a summation is represented on Figure **??**, where the circle on the left side is transformed into a single focused

../mihai1/./Fig/DMOcubes3.ps

Figure 3: In a time-slice, PSI sums data along a circle, and it focuses it at the apex of the semicircle.

point on the right side of the figure. The focused point is situated at the apex of the semicircle. The summation algorithm can be any choice of Kirchhoff or modified Stolt migration. The end result of applying DMO before NMO, and PSI to the Cheops surface, is a curve in the common-midpoint section passing through A, the coordinate of the diffractor. After NMO correction, the curve becomes a straight line and can be summed over offset to obtain the image of the point diffractor. The substance of this argument can be summarized in the fact that the result of applying DMO before NMO and PSI to the Cheops surface, reduces this surface to a hyperbola located in the common-midpoint gather containing the model diffractor. After NMO correction, the hyperbola becomes a line, positioned in time at the zero-offset traveltime t_0.

Berryhill's pure migration

Berryhill (1994) presented a new migration algorithm he calls "Nonimaging Shot-Record Migration" based on spreading the energy along a line in a common-shot section. The line represents the two-way shot traveltime in a constant velocity medium. The proof that for a source-receiver geometry the source traveltime is a line, is given in Appendix A. The equation of the line is shown to be:

$$t_s = t_h - \frac{4h}{v^2 t_h}(h - x_s), \tag{3}$$

where t_s is the two way traveltime from the shot, t_h is the source-receiver traveltime, h is the half-offset, v is the velocity, and x_s is the surface position corresponding to each value of t_s. The essence of the method is that spreading data over the time and surface coordinates in equation (3) focuses the hyperbola generated by a single diffractor in a common-shot record.

Figure **??** is an example of applying Berryhill's migration to a common-shot record over a point diffractor. Figure **??**a displays the input to Berryhill's migration and Figure **??**b displays the result of spreading data over the lines given by equation (3). In Appendix B I prove that the surface coordinate of the focusing point corresponds to the surface coordinate of the diffractor, and the time to the focusing point is twice the shot-time. The code used to produce Figure **??**b

Figure 4: Common-shot section in midpoint coordinates and the output of Berryhill's migration.
a. Common-shot section over a diffractor, with the shot located at surface location 0 m. The surface coordinates are common-midpoint locations. The physical position of the traces is at a distance 2h from the shot.
b. Output of Berryhill's migration. The diffraction curve was collapsed to the true diffractor location (-800 m), at a time equal to the source-receiver traveltime.

is very simple. The first two loops parse the data in space and time. The third loop parses the output in space and identifies for each input data point the time and space coordinates of the output line. The body of the Fortran kernel can be summarized as:

```
do ix=1,nx
  x=ox+(ix-1)*dx
  do it=1,nt
    t=it*dt
    do ixs=1,nx
      xs=ox+(ixs-1)*dx
      ts=t+4.*(x/v)*xv*(xs-x)/(t*v)
      its=int(ts/dt)+1
      if(its.ge.1.and.its.le.nt) then
        model(its,ixs)=model(its,ixs)+data(it,ix)
      endif
```

```
      enddo
    enddo
  enddo
```

../mihail/./Fig/BerryShots.ps

Figure 5: Two common-shot sections for Berryhill's nonimaging shot-record migration. The diffraction hyperbola is focused to a point at the correct surface location of the diffractor. For a different common-shot section, the diffractor is focused at the same surface location. In each case the NMO correction (different for each offset) will position the diffractor at the proper position. Note the fact that the receivers are actually positioned at half-offset distance from the source (at the common-midpoint location).

The data used for Berryhill's migration is taken from a single shot record, but the surface coordinates are the common-midpoints. In Figure **??**, where I tried to reproduce the same image as the one in the original Berryhill paper, the depth of the diffractor is 1406 meters, and the surface coordinate of the diffractor is -800 meters. The velocity used is 2800 meters per second. However, because data is represented in common-midpoint surface coordinates, in Figure **??** and Figure **??**, the top of the diffraction hyperbola is situated at -400 meters and not -800 meters, as it would be in a shot record. The focusing point after migration is located at the common-midpoint location of -800 meters, corresponding to a half offset of 800 meters. As a result, after resorting the prestack data in common-midpoint and offset coordinates, the shot record that constituted the input, contributes a single trace to the output dataset, with midpoint -800 and half offset -800. For a shot located at -800, after Berryhill's migration, the focusing point is situated at the same location but at half-offset zero. For more shot-record migrations, the surface coordinate of the focusing point remains unchanged but the time coordinate increases with offset. Figure **??** represents a sketch of the focusing geometry over a single point diffractor, after Berryhill's migration, for two different shot locations. Both migrations focus the diffraction hyperbola at the same surface location, but at different times.

After NMO correction, the time is identical for all the shot records. Therefore, in common-midpoint and offset coordinates, the output of Berryhill's migration is a curve passing through the surface coordinate of the diffractor. The curve is situated in the common-midpoint gather containing the diffractor, exactly like the output of PSI, and is shifted down in time with the NMO correction, again, precisely like the output of PSI.

CONCLUSIONS

Since the two methods, Berryhill's "pure migration" and DMO followed by Gardner's PSI, produce the same image given the same input, I conclude that the two methods are kinematically equivalent. Berryhill's method opens to research interesting questions like the velocity sensitivity of the method, the phase-shift equivalence of the algorithm, the full extraction of the operator amplitudes from the wave equation, and the formulation of an adjoint operator for modeling.

ACKNOWLEDGMENTS

I benefited from helpful discussions with Dimitri Bevc, Jon Claerbout, and John Berryhill. Andrei Popovici pointed out a flaw in my algebra, thus helping me to finish the mathematical demonstrations.

REFERENCES

Gardner, G. H. F., Wang, S. Y., Pan, N. D., and Zhang, Z., 1986, Dip Moveout and Prestack Imaging: paper presented at the 18th Annual Offshore Technology Conference, Houston, Texas.

Berryhill, J. R., 1994, Nonimaging Shot-Record Migration: Patent Application.

APPENDIX A

This appendix demonstrates that for a fixed shot-receiver geometry, the traveltime from the shot location to any point on the migration ellipse is a line. In other words an impulse which is spread over an ellipse in common-offset prestack migration can be spread over a straight line in shot-record migration. The time from the shot to the ellipse and back, t_s, is defined as

../mihai1/./Fig/Shotellipse.ps

Figure 6: Geometry for a single shot-receiver experiment.
The time from the source to any point on the ellipse is a line.

$$t_s = \frac{2}{v}\sqrt{x_s^2 + z_s^2}, \tag{4}$$

where v is the medium velocity, x_s is the surface coordinate of a point on the ellipse corresponding to the depth coordinate z_s. On the ellipse, the source-receiver traveltime t_h is equal to the sum of the time from the source to the reflection point on the ellipse (x_s, z_s) and the time from the point on the ellipse to the receiver, as follows

$$vt_h = \sqrt{x_s^2 + z_s^2} + \sqrt{(2h - x_s)^2 + z_s^2}.$$

Squaring the equation I have

$$v^2 t_h^2 = x_s^2 + z_s^2 + (2h - x_s)^2 + z_s^2 + 2\sqrt{x_s^2 + z_s^2}\sqrt{(2h - x_s)^2 + z_s^2},$$

and after reordering the free terms

$$v^2 t_h^2 - 2x_s^2 - 2z_s^2 - 4h^2 + 4hx_s = 2\sqrt{x_s^2 + z_s^2}\sqrt{(2h - x_s)^2 + z_s^2},$$

and squaring again

$$\left[v^2 t_h^2 - 2(x_s^2 + z_s^2) - (4h^2 - 4hx_s)\right]^2 = 4(x_s^2 + z_s^2)^2 + 4(x_s^2 + z_s^2)(4h^2 - 4hx_s),$$

I finally obtain:

$$v^4 t_h^4 + (4h^2 - 4hx_s)^2 - 2v^2 t_h^2 (4h^2 - 4hx_s) = 4v^2 t_h^2 (x_s^2 + z_s^2). \tag{5}$$

Introducing the right side of equation (5) in the equation for the double shot time (4) becomes:

$$\begin{aligned} \frac{v^2 t_s^2}{4} &= x_s^2 + z_s^2 \\ &= \frac{v^2 t_h^2}{4} + \frac{(4h^2-4hx_s)^2}{4v^2 t_h^2} - \frac{1}{2}(4h^2 - 4hx_s) \\ &= \left[\frac{vt_h}{2} - \frac{(4h^2-4hx_s)}{2vt_h}\right]^2. \end{aligned} \tag{6}$$

And finally for the shot time t_s I obtain Berryhill's migration line:

$$t_s = t_h - \frac{4h}{v^2 t_h}(h - x_s). \tag{7}$$

APPENDIX B

The purpose of this appendix is to demonstrate that using the Berryhill migration algorithm, a diffractor curve in a shot-record is focused to a point with the same surface coordinates as the diffractor. The time coordinate of the focusing point corresponds to twice the traveltime from the source to the diffractor. Suppose the diffractor is located at a distance x_d from the shot, at a depth z_d. For simplicity the shot is considered to be at the origin of coordinates. The traveltime t_h from the shot to a receiver situated at a distance $2h$ from the shot is:

$$t_h = \frac{1}{v}\left(\sqrt{x_d^2 + z_d^2} + \sqrt{(2h - x_d)^2 + z_d^2}\right).$$

Introducing the value of t_h into the Berryhill migration line expressed in equation (7)

$$vt_s = vt_h + \frac{y(x-y)}{vt_h},$$

and replacing $2h$ by a new variable y we obtain:

$$vt_s = \sqrt{x_d^2 + z_d^2} + \sqrt{(y - x_d)^2 + z_d^2} + \frac{y(x-y)}{\sqrt{x_d^2 + z_d^2} + \sqrt{(y - x_d)^2 + z_d^2}}. \tag{8}$$

Figure **??** shows an example of the family of lines described by equation (8). Each line corresponds to a fixed value of y and passes through the corresponding value of t_h. The lines have the remarkable property of passing through the same point x_s. We can easily find the value of x_s by solving equation (8) for two different values of y, for example $y_1 = 0, y_2 = 2x_d$. For

$x = 2x_d$ equation (8) becomes independent of y:

$$
\begin{aligned}
vt_s &= \sqrt{x_d^2+z_d^2}+\sqrt{(y-x_d)^2+z_d^2}+\frac{y(2x_d-y)}{\sqrt{x_d^2+z_d^2}+\sqrt{(y-x_d)^2+z_d^2}} \\
&= \sqrt{x_d^2+z_d^2}+\sqrt{(y-x_d)^2+z_d^2}+\frac{(x_d^2+z_d^2)-\left[(y-x_d)^2+z_d^2\right]}{\sqrt{x_d^2+z_d^2}+\sqrt{(y-x_d)^2+z_d^2}} \qquad (9) \\
&= \sqrt{x_d^2+z_d^2}+\sqrt{(y-x_d)^2+z_d^2}+\sqrt{x_d^2+z_d^2}-\sqrt{(y-x_d)^2+z_d^2} \\
&= 2\sqrt{x_d^2+z_d^2}.
\end{aligned}
$$

The shot-time for $x = 2x_d$ is constant for any value of the offset $y = 2h$. Since the surface coordinates in Figure **??** are the common midpoint positions, the diffractor is focused at the correct surface position x_d, for a half-offset equal to the surface distance from the shot to the top of the diffractor $x = x_d$. For this geometry, the NMO correction from t_s to zero-offset time t_0 gives us

$$t_0^2 = t_s^2 - \frac{4x_d^2}{v^2}.$$

Therefore, for each common-shot section, the NMO correction of the focused point is equal to the NMO correction of the equivalent common-offset section.

../mihai1/./Fig/Berrycurves.ps

Figure 7: Diffractor in a common-shot record.
The diffractor curve is displayed in common-midpoint surface coordinates (not the receiver surface position). The straight lines represent the Berryhill migration curves. The focus point is at the surface coordinate of the diffractor. The source is located at 0 m., the diffractor at -800 m. The apex of the hyperbola is at -400 m.