Chapter 1

Modeling, adjoint modeling, and migration in V(z) media, using mono-frequency Green's functions

Modeling and migration can be simply expressed in terms of the Green's functions of the wave equation. In the previous chapter I discussed calculating mono-frequency Green's functions using one way wave equations in polar coordinates. The complete outgoing Green's function is the superposition of all the mono-frequency Green's functions. Once Green's functions at all frequencies have been estimated they can be used for both modeling and migration. However, most practical implementations do not use the Green's function explicitly. The reason for this is that, using currently available computers, it is too expensive to routinely calculate and store these functions for all frequencies and all surface locations.

In this chapter I exploit the fact that when the velocity model is independent of the horizontal coordinate, only one set of Green's functions needs to be calculated. They can be used for prestack and poststack modeling and migration. Because my Green's functions are calculated in polar coordinates they are not limited to waves traveling at less that 90 degrees to the vertical. They can be used to image reflections due to overturned waves. I use a single set of Green's functions to demonstrate modeling and migration in a V(z) medium with overturned waves.

The true-amplitude nature of the wave-equation Green's functions reveals a problem with using adjoint of the modeling operator as an imaging operator. In the noise-free case, an inverse operator would be an ideal imaging operator because it would invert both the amplitude and phase information. Unfortunately, the true inverse operator is very expensive to compute. The adjoint operator is much cheaper, it inverts the phase correctly but it squares the amplitude. An alternative operator is one that correctly inverts the phase and attempts to approximately recover the original amplitudes. This process is sometimes referred to as migration/inversion (?).

1.1 Modeling in a V(Z) medium

In a general medium the scattered wavefield, $P(g,s,\omega)$, received by a receiver at position g on the surface, from a shot at surface position s, can be calculated by integrating the scattered field over all locations in the subsurface:

$$P(g,s,\omega) = \int\int G(s,x,z,\omega)G(x,g,z,\omega)S(x,z)\ dx\ dz$$

where $S(x,z)$ is a scalar/isotropic scattering function in the subsurface. This formula suggests that we need to calculate a Green's function for every surface position and frequency. However if the velocity field is only a function of depth and not of x, we only need one Green's function for every frequency. The integral can then be expressed as a convolution of the Green's function over x and an integral over depth:

$$P(g,s,\omega) = \int\int G(x-s,z,\omega)G(x-g,z,\omega)S(x,z)\ dx\ dz.$$

For a constant-offset panel in CMP coordinates this can be further simplified to a spatial convolution over a constant offset Green's function, and an integral over depth.

$$P(y,\omega;h) = \int\int \hat{G}(x-y,z,\omega;h)S(x,z)\ dx\ dz\ ,$$

where $\hat{G}(x-y,z,\omega;h) = G(x+h/2-y,z,\omega)G(x-h/2-y,z,\omega)$. The constant offset Green's function for one frequency is merely the one-way Green's function multiplied by a copy of itself shifted in x.

Figure 1.1 is the result of constant offset modeling for three different offsets. The first panel is the scattering function: two spikes in the center of the panel labeled "A" and "B". The second panel is the zero-offset modeling result. The third panel models an offset of 300m and the fourth an offset of 750m. In the zero offset panel the upward curvature of the limbs of the hyperbola due to overturned waves can clearly be seen. In the far-offset panel an interesting effect is visible, the table-top is concave. The traveltime to a point under the midpoint is greater than the traveltime to a point to one side.

1.2 Adjoint modeling and migration

Migration is often considered to be the adjoint of modeling; this would suggest the following adjoint operation as a reasonable constant offset imaging operation:

$$\hat{S}(x,z;h) = \int\int G^{\dagger}(x-y,z,\omega;h)P(y,z,\omega;h)\ dy\ d\omega\ .$$

When I apply this process to the output of constant offset modeling, the result is very disappointing. Figure 1.2 shows the result of adjoint modeling on the three constant-offset panels shown in Figure 1.1. Figure 1.3 shows the central trace of each

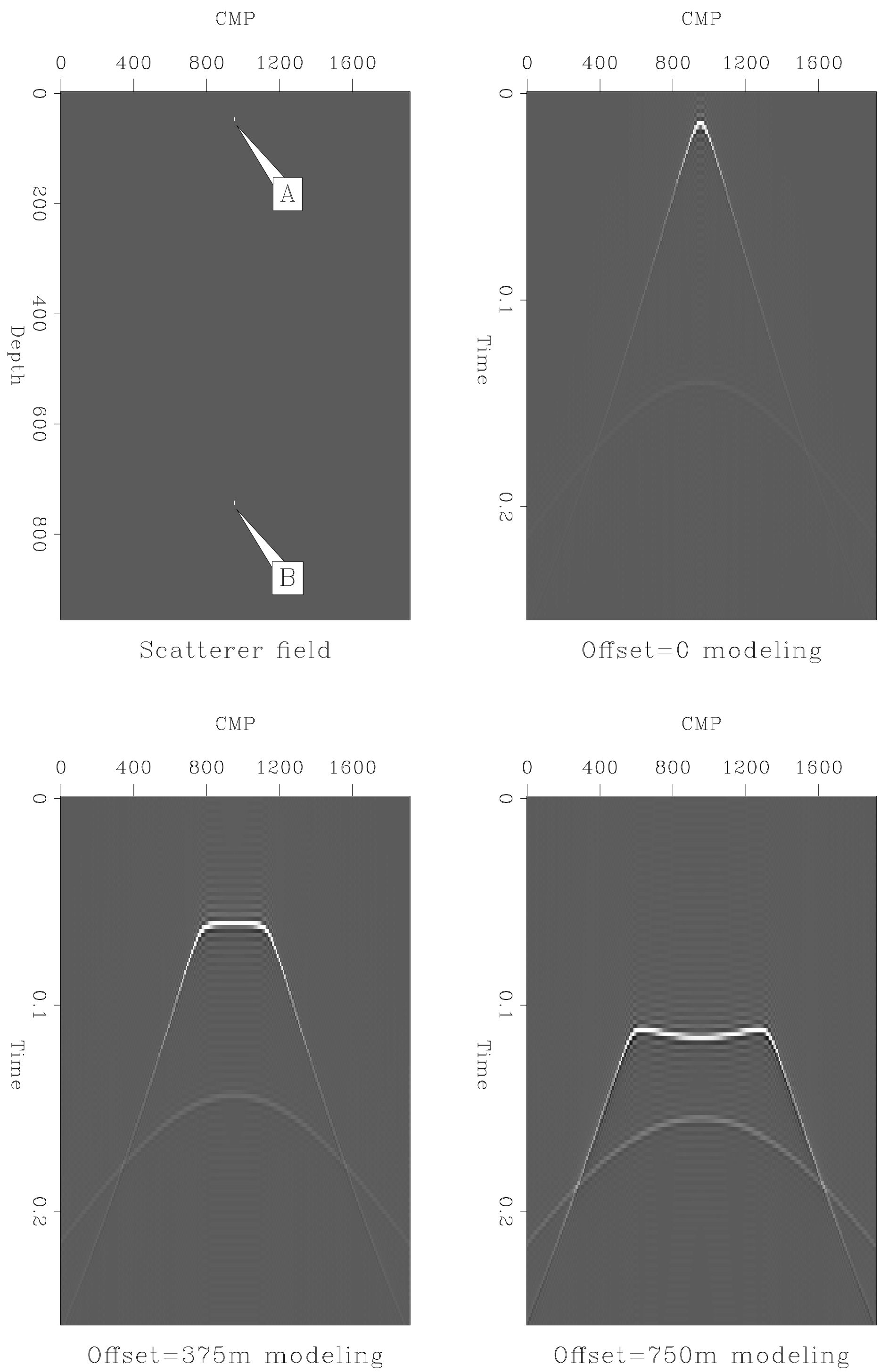

Figure 1.1: Constant offset modeling in a V(z) medium VZmodmig-const-mod [CR]

panel. The deeper event, B, has been imaged, but with such a small amplitude that it is barely visible on the panels in Figure 1.2. This is despite the fact that these plots are displaying the square root of the amplitude function in an attempt to make the weak events more visible. The reason for the weak amplitude is that the Green's functions I am using are not merely kinematically correct, they also have the correct amplitudes. The adjoint operation correctly inverts the phase but it squares the amplitude. This would not be a problem if the integration had been applied over infinite space and time. The truncation of the modeling operator and its adjoint results in a loss of energy from the image. This loss of energy is greatest at the bottom of the section where only a small piece of the hyperbola will be imaged.

One way to overcome this problem is to apply a time dependent gain, a "spherical spreading correction", to the data before the migration. By amplifying arrivals at later times the images of deeper events will be enhanced. However this will also increase the amplitude of the later parts of hyperbolas belonging to shallow events. Figure 1.4 shows the result of applying a spherical spreading correction to the modeled data. The events from the deeper scatterer are much more visible. Application of the adjoint modeling operator to this data produces the images shown in Figure 1.5. The correct amplitudes have still not been retrieved, and event B is still almost invisible. Figure 1.6 compares the amplitudes of the central traces on each panel. The result is a small improvement on Figure 1.3, but still a very poor image. The cause of this is the amplification of the limbs of the first event. In this strong gradient medium, the high angle arrivals from the shallow scatterer, A, arrive later than those from the deeper scatterer, B. The simple spherical spreading correction applied before imaging is unable to recover the true amplitudes.

If a true amplitude result is required this is not desirable. What should really be applied is a depth dependent gain *after* migration. Lumley and Beydoun (**?**), attempt to account for the effects of truncation and sampling by dividing by a "diagonal Hessian" operator, this is an approximation to the full inversion of the modeling operator. They call this process "migration/inversion." In a V(z) medium this becomes a depth and offset dependent scaling after migration.

1.2.1 Migration/inversion and preconditioning

Modeling can be regarded as a linear operator, where the domain of the operator is the scattering field and the range is a seismic dataset. When the scattering field and seismic data are discretized they can be represented as finite dimensional vectors. The modeling operator, A, maps from a scattering vector, s, to a data vector d.

$$As = d$$

When imaging we wish to obtain an estimate of s from the measured data d. One possible operation is to apply the adjoint operator,

$$\hat{s} = A'd$$

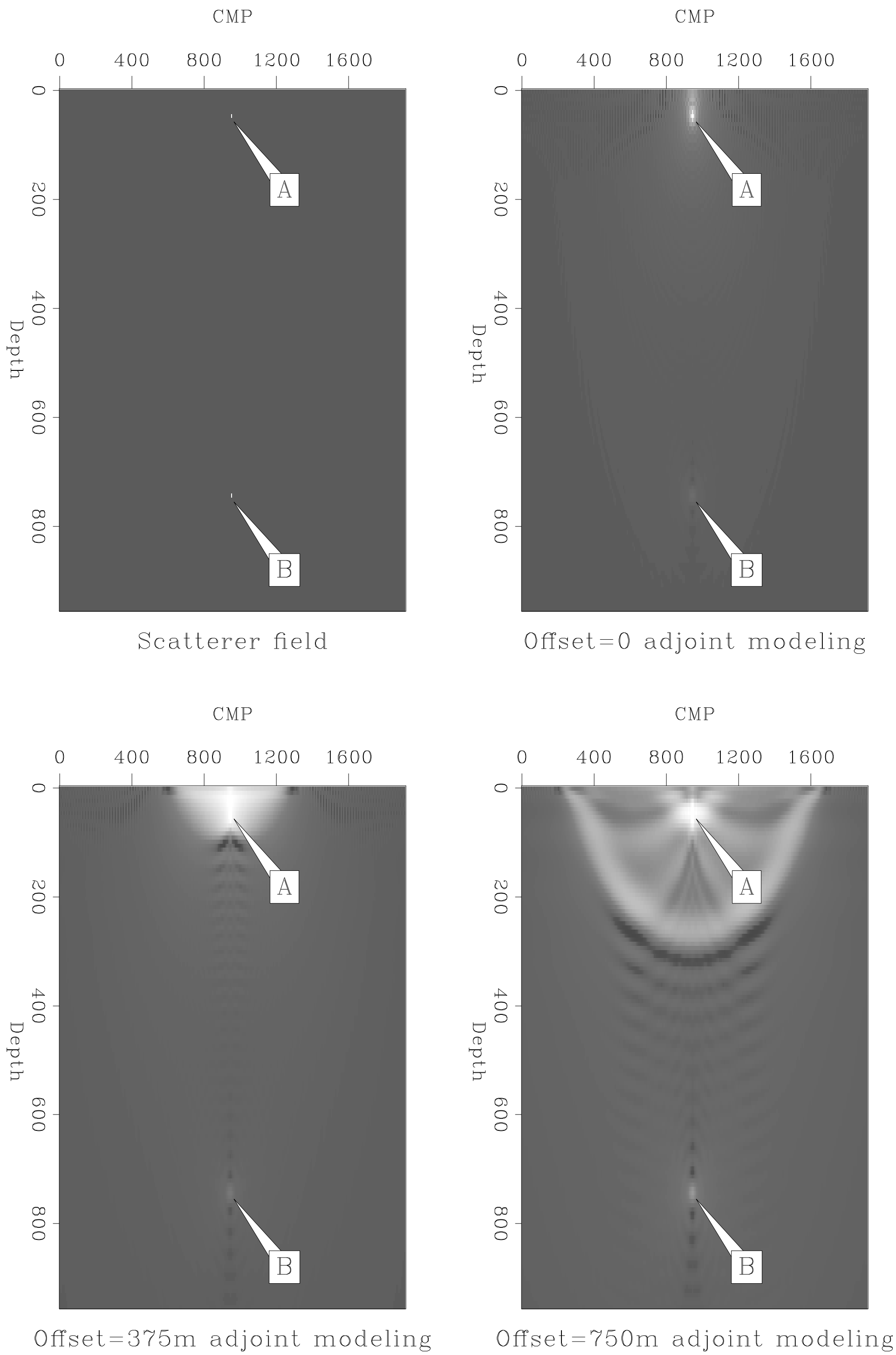

Figure 1.2: Adjoint modeling applied to the constant offset panels of the previous figure. The deeper events have not been imaged at the correct amplitude. A square-root gain function has been applied to enhance weak events. VZmodmig-adj-mig [CR]

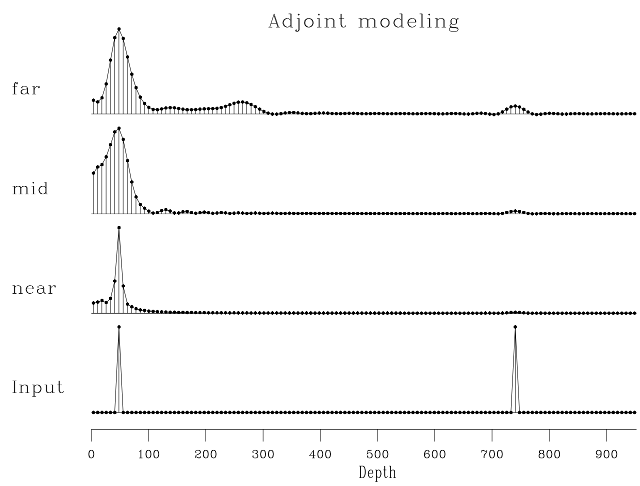

Figure 1.3: The central trace extracted from each panel of figure 1.2. The bottom trace is the true scattering function that was the input to the modeling. VZmodmig-adj-mig2 [CR]

If the operator is nearly unitary this will be a good estimate of the model. When the operator is not unitary, as when our modeling operator is truncated, this will not be a good estimate. The estimate can be improved by solving the least squares inverse problem

$$\min_{\hat{s}} ||A\hat{s} - d||^2 .$$

If a conjugate gradient algorithm is used, the application of the adjoint can be regarded as the first iteration of that algorithm.

Another method for obtaining an estimate is to note that the solution of the least squares problem is given by

$$\hat{s} = (A'A)^{-1}A'd$$

(As long as $(A'A)^{-1}$ exists). Calculation of $(A'A)^{-1}$ is very expensive. However we might obtain an acceptable result by approximating this inverse. If we assume $A'A$ is diagonally dominant we can approximate the inverse by $(\mathrm{Diag}(A'A))^{-1}$, where Diag() extracts the diagonal of $A'A$ and sets all other elements to zero. Here the estimate of the solution is given by

$$\hat{s} = (Diag(A'A))^{-1}A'd \,. \tag{1.1}$$

This procedure can also be interpreted as the first step of an iterative algorithm. In this case the problem being solved is a preconditioned version of the original problem. $M = (\mathrm{Diag}(A'A))^{-1/2}$ is a diagonal preconditioner that equalizes the Euclidean norm of the columns of the operator A. The preconditioned least squares problem is

$$\min_{\hat{t}} ||AM\hat{t} - d||^2 \tag{1.2}$$

$$\hat{s} = M\hat{t}. \tag{1.3}$$

One step of the preconditioned iterative algorithm followed by application of the preconditioner to solve for $\hat{s}$ is exactly the same as equation (1.1).

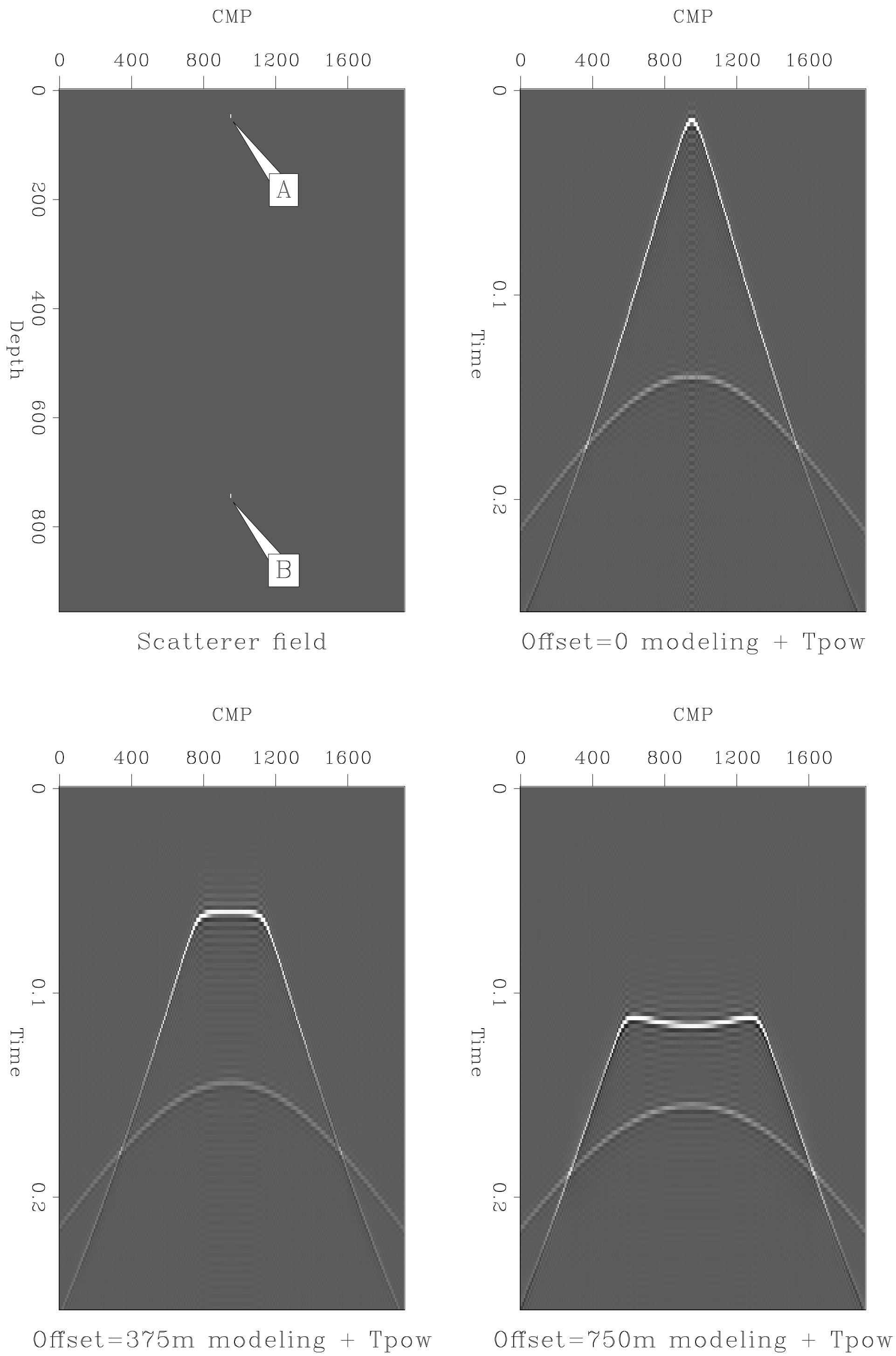

Figure 1.4: Constant offset modeling followed by spherical spreading correction. The later arrivals have increased amplitude but so do the limbs of the shallower event. VZmodmig-const-modt [CR]

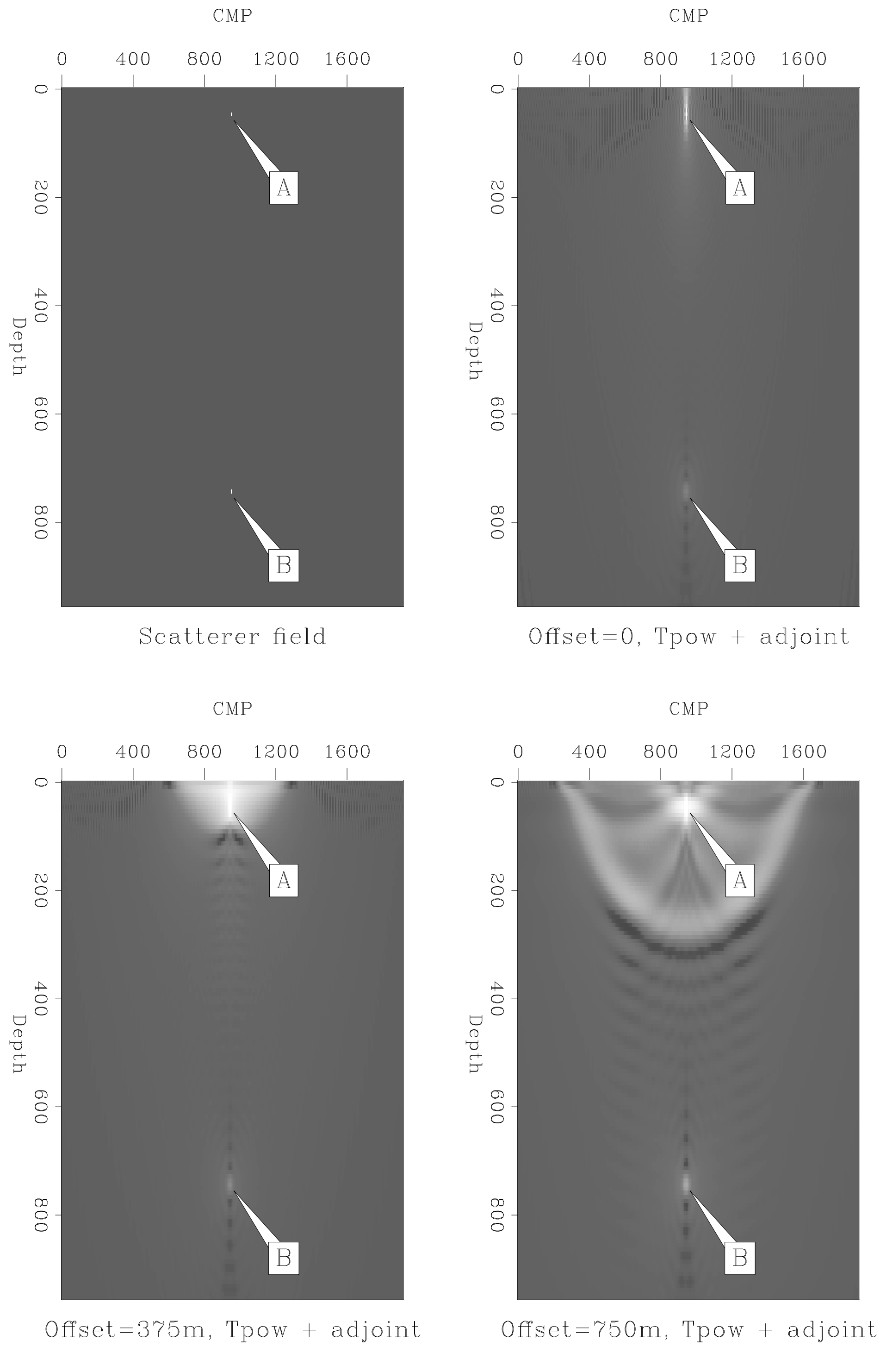

Figure 1.5: Adjoint modeling applied to the constant offset panels after spherical spreading correction. The deeper events have still not been imaged at the correct amplitude. A square-root gain function has been applied to enhance weak events. VZmodmig-tadj-mig [CR]

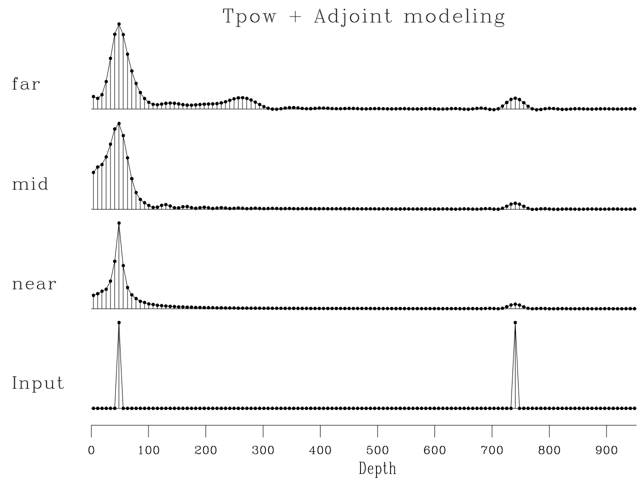

Figure 1.6: The central trace extracted from each panel of figure 1.5. The bottom trace is the true scattering function that was the input to the modeling. VZmodmig-tadj-mig2 [CR]

In many applications it is relatively simple to calculate (Diag($A'A$)). For the modeling algorithm used here it is,

$$\mathrm{Diag}(A'A)(x,z) = \int\int G^{\dagger}(x-y,z,\omega;h)G(y,z,\omega;h)\ dy\ d\omega.$$

This is simple and cheap to calculate in a migration scheme. Instead of just accumulating the product of the adjoint Green's function and the data, I also accumulate the energy of the Green's function. The cost is less than twice that of the simple adjoint modeling. If the modeling is not limited in spatial aperture then the weight is just a function of depth and offset panel. It is a depth and offset-dependent scaling applied after adjoint modeling rather than the time-dependent scaling often applied in seismic processing before migration.

Figure **??** shows the constant offset migration/inversions calculated using this weight function. Figure **??** shows the central traces from each frame. The peak amplitudes of the two scatterers, A and B, have been correctly recovered but the shape of the impulses has not. The diagonal weighting only corrects the relative amplitudes of the result, it does not correct the spectrum of the result; the full inverse operator $(A'A)^{-1}A'$ would correct the spectrum as well. In many cases a true amplitude reflectivity estimate is not the desired result. All that is needed is a structural image. In this situation the adjoint operator, or some scaled version of it, applied after spherical spreading correction, can be the best choice. An inversion operator will amplify noise present in the input data. The adjoint operator is a "matched filter" that will optimize the signal to noise ratio in the output section.

1.3 Summary

I have shown that a single set of Green's functions calculated in a polar coordinate frame can be used to model and migrate overturned waves in a V(z) medium. If

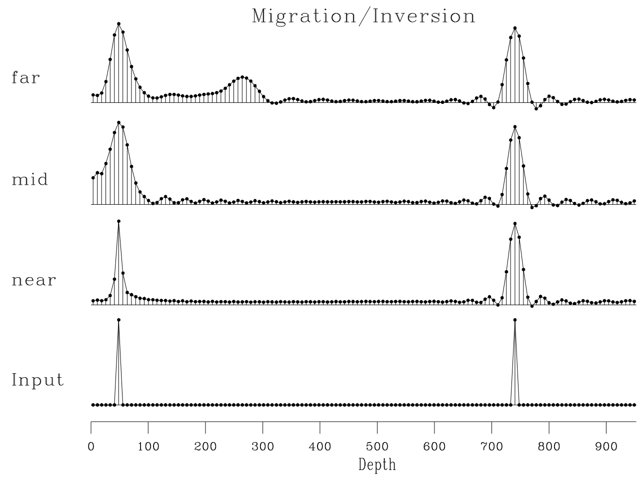

Figure 1.7: The central trace extracted from each panel of figure **??**. The bottom trace is the true scattering function that was the input to the modeling.The peak amplitudes have been correctly recovered but the shape has not. VZmodmig-ata-mig2 [CR]

the Green's function contains the true amplitude information, then the migration or migration/inversion algorithm must take account of that amplitude information if a true amplitude image is desired.

If the medium has a velocity that is a function of horizontal coordinate as well as depth then the Green's function must be calculated for each surface location and the convolution in space becomes a banded-multiplication. This procedure is too expensive to implement on today's computers so an alternative scheme must be used. In the following chapters I describe a method for calculating a parametric representation of the Green's function, and its use in Kirchhoff imaging.

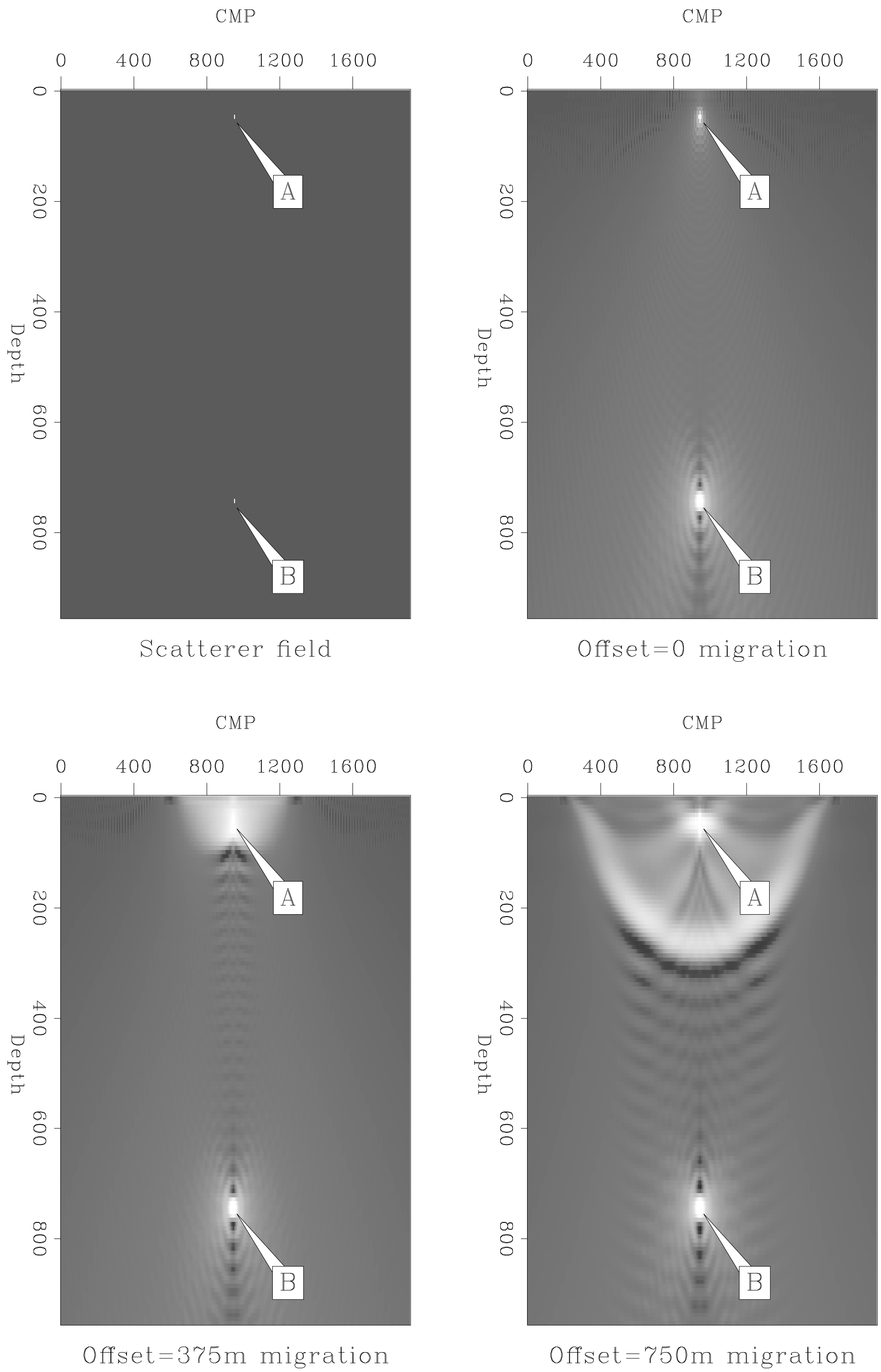

Figure 1.8: Constant offset migration/inversion in a V(z) medium. VZmodmig-ata-mig [CR]