Chapter 1

Introduction

The effect of velocity anisotropy on wave propagation in homogeneous and heterogeneous media has been the subject of numerous publications. Careful forward modeling has helped interpreters understand how velocity anisotropy manifests itself in field data. Fewer attempts have been made, however, to solve the inverse problem, namely, the estimation of the parameters that describe the complexity of velocity anisotropy. These parameters are the elastic constants. The estimation of elastic constants is important because it can aid lithologic discrimination and fracture orientation, reveal anisotropic properties of the medium not obvious in the data, and provide further imaging or full waveform inversion algorithms with background models that can be refined iteratively. This dissertation will focus on one solution to the inverse problem, namely, the estimation of elastic constants from seismic measurements.

1.1 Anisotropy or heterogeneity?

Rocks can be anisotropic for a variety of reasons. Some rocks, minerals for example, can be inherently anisotropic depending on how their molecules arrange themselves in an orderly way to form a crystalline structure. Other rocks can become anisotropic after a sufficiently large stress has altered the crystalline structure of their constituents. Sufficiently large stresses can also cause preferentially oriented cracks in isotropic rocks or in rocks with otherwise randomly oriented cracks to make them look anisotropic at seismic frequencies. Other types of rocks can be formed by arrangements of elongated grains or fine isotropic layers that make the rocks look anisotropic at seismic frequencies. A more detailed explanation of possible causes of rock anisotropy can be found in Crampin et al. (1984).

Backus (1962) showed that a region composed of thin isotropic layers is equivalent in the long wavelength limit to a homogeneous transversely isotropic medium. More recently, Schoenberg and Muir (1989) extended Backus's conclusion to arbitrary, anisotropic, thin layers. These results have two important implications. The first is that, for scales much smaller than the seismic wavelength, there is no way to distinguish intrinsically anisotropic materials from materials with preferentially oriented

heterogeneities (e.g., fine layering), which means that, from seismic measurements alone, it is not possible to identify the causes of anisotropy. The second implication is that, for scales smaller than a fraction of the minimum wavelength in the data, there is no way to distinguish whether the medium is heterogeneous or homogeneous anisotropic. This fundamental equivalence between anisotropy and heterogeneity in the wave propagation problem has its counterpart in the inverse problem.

Both heterogeneity and anisotropy affect wave propagation. The goal of the inverse problem in heterogeneous anisotropic media is to transform this coupled effect in the data into a model that is simultaneously heterogeneous and anisotropic. At first glance, this problem may seem insoluble because of the fundamental equivalence between anisotropy and heterogeneity that I mentioned before. We should remember, however, that the equivalence in the forward problem occurs at scales that are small compared to the wavelength. At larger scales, the effects of heterogeneity and anisotropy are distinguishable, and therefore, as long as the scale of interest in the inverse problem is not too small, it should be possible to set up an inverse problem that separates such effects.

The problem of the equivalence between anisotropy and heterogeneity worsens when only a fraction of the information contained in the wavefield (e.g., first arrival traveltimes) is used for the inversion. In this case, anisotropy and heterogeneity may be equivalent every time they are described with the same number of parameters, or, even worse, with different combinations of parameters in anisotropy and heterogeneity that fit the data equally well. Figure 1.1 (taken from Babuska and Cara, 1991) illustrates this difficulty. In this example, two different models are used to fit the same pair of traveltimes: model (a) is heterogeneous isotropic and model (b) is homogeneous elliptically anisotropic. Since both sets of parameters can be used to explain the observations, there is a complete equivalence between anisotropy and heterogeneity in this case. Increasing the number (finite) of measurements and the angles of observation is one way to resolve the ambiguity by selecting the model that fits the particular data better. However, if the model is subdivided into smaller cells, it is always possible to find a scale where the equivalence occurs because only two rays cross the small-size cells. If the ambiguity remains after increasing the number of measurements, additional information is necessary to resolve it.

The additional information necessary to resolve the ambiguity comes from the prior information about the medium, which tells us what model to use. That information can be used in the example of Figure 1.1 to choose between models (a) or (b) and estimate only the corresponding pair of parameters. However, if the prior information is not correct, anisotropy in the real medium is transformed into heterogeneities that do not exist, or, conversely, heterogeneities in the real medium are explained by unreal anisotropic solutions. Therefore, the selection of the proper model is crucial.

Figure 1.1: Simple velocity models that show the heterogeneity/anisotropy equivalence that needs to be considered when solving inverse problems in heterogeneous anisotropic media (after Babuska and Cara, 1991). (a) Heterogeneous isotropic model. (b) Homogeneous anisotropic model. These two models fit the data equally well. hete-vs-ani [NR]

1.2 Estimation of elastic constants in heterogeneous media: previous work

Elastic constants have been estimated primarily in the lab where the small rock samples can be assumed to be homogeneous. When the assumption of homogeneity is not valid, the problem of estimation is more difficult because the effects of anisotropy and heterogeneity are coupled in the data. This coupling problem has been addressed by making prior assumptions about both effects. For example, by assuming that the rock samples are homogeneous, Arts et al. (1991) estimate, from lab measurements, the 21 elastic constants that characterize a general triclinic system. By assuming isotropic velocities, McMechan (1983) estimates tomographically arbitrary spatial variations using 2-D and 3-D models. These two papers are examples of different trade-offs between the complexity of the models in regard to velocity and heterogeneity. On the one hand, the first paper uses the simplest model for heterogeneities (homogeneous) without making any assumption about the type of velocity anisotropy. On the other hand, McMechan's paper uses the simplest model for the velocity (isotropic) without making any assumption about the heterogeneity. It is interesting to notice that even though the only model that does not assume anything about the medium is 3-D triclinic, the models used in the previous papers are usually presented as examples of models that make no assumptions about the medium.

Previous studies that have estimated variations of anisotropy with position have used "intermediate" models that simplify anisotropy and heterogeneity. The selection of the model is based on two factors: the prior information available about the medium and the geometry used to record the data. When selecting the model for anisotropy, these two factors make transverse isotropy (TI) a good candidate because, on the one hand, TI is a common form of anisotropy in the subsurface, and, on the other hand, 3-D multicomponent information that is necessary to study more complex symmetries is not usually recorded. For analogous reasons, layered models (1-D) have routinely been used to describe the heterogeneities. Therefore, not surprisingly, several authors have chosen the combination of 1-D and TI to describe their models. White et al. (1983) estimate the five TI elastic constants of a homogeneous formation using a VSP geometry. Hake et al. (1984) approximate the traveltimes curves of layered models with a three-term Taylor series expansion in which the coefficients are a function of the elastic constants. Winterstein and Paulsson (1990) estimate the elastic constants from VSP and cross-well measurements in a medium with velocity increasing linearly with depth. Byun and Corrigan (1990) propose an iterative, model-based, optimization scheme to invert traveltimes for the five elastic constants of a layered TI medium. More recently, Sena (1991) has proposed a variant of this method in which all the calculations are done analytically, without having to go through the semblance analysis needed with Byun and Corrigan's method.

When using ray theoretic tomographic methods, the 1-D assumption about the heterogeneity can easily be relaxed. Moreover, the formulation of the problem is almost identical for any type of heterogeneity, regardless of its complexity. The simplicity of the formulation of the problem for general heterogeneity may explain

why most papers on ray theoretic traveltime tomography describe the heterogeneities of the medium using the most complex and general model: a fine 2-D grid (Dines and Lytle, 1979; McMechan, 1983; White, 1989). The basis function required to describe a 2-D grid is orthogonal. Although other basis functions that don't have the property of orthogonality have been recently proposed (Van Trier, 1988; Harlan, 1989; Michelena and Harris, 1991), those that have such a property are still the most widely used to represent the heterogeneities in tomographic traveltime inversions. At the same time, however, the model assumed for velocities has been, until recently, the most simple: isotropic.

Recent papers have addressed the problem of estimating velocity anisotropy from cross-well measurements using ray theoretic traveltime tomography. These papers have focused on the problem of eliminating the artifacts obtained when isotropic tomography is used to invert data recorded in anisotropic environments (Carrion et al., 1992). McCann et al. (1989) show how an isotropic inversion improved after assuming a fixed amount of anisotropy. Stewart (1988) and Williamson et al. (1993) describe the heterogeneities with nonoverlapping square cells and the velocity anisotropy using Thomsen's (1986) expression for P-wave phase velocity in weakly anisotropic media. Saito (1991) and Lines (1992) propose to separate the effects of anisotropy and heterogeneity by first "removing" the anisotropy effects from the data so that conventional isotropic tomography in heterogeneous media can later be applied. The elimination of the anisotropy effect is partial and depends on the model of heterogeneities that is assumed. Qin et al. (1992) describe the model as a superposition of two parts, one isotropic and the other anisotropic. The heterogeneities in the isotropic part are described by small, square cells and the heterogeneities in the anisotropic part are described by large cells. By using ray paths computed in isotropic models for the anisotropic inversion, Qin et al. implicitly assume also that the medium is weakly anisotropic. Chapman and Pratt (1992) and Pratt and Chapman (1992) also assume weak anisotropy but make no assumption about the type of anisotropy or heterogeneity. Pratt and Chapman's procedure is more complex for shear waves than for compressional waves, and it requires, as do all the previous methods, wide-aperture data.

The preceding papers (except Saito's) assume weakly anisotropic media in order to justify ray tracing in isotropic models. The assumption of weak anisotropy also justifies the use of Thomsen's equation for P-wave phase velocity to approximate the corresponding group velocity (in the papers by Stewart, Lines, and Williamson et al.). No assumptions are made about the heterogeneities (except in Saito's paper) which, as I show in appendix **??**, makes more difficult an accurate estimation of spatial variations in velocity anisotropy. Although all these papers show how to alleviate artifacts in the tomograms by estimating parameters that describe variations of velocity with direction, none of them (except Chapman and Pratt's) show how to transform those parameters into the five elastic constants that describe a general TI medium. Chapman and Pratt, however, don't show examples that illustrate the use of their transformation.

1.3 The goal of this thesis

This thesis focuses on the estimation of the elastic constants that describe heterogeneous TI media when the measurements, first arrival traveltimes, have narrow aperture around one axis of symmetry. I start by solving the problem for homogeneous media. Then I show that the generalization to heterogeneous media consists of fitting the data with heterogeneous elliptically anisotropic models that can be transformed into models for elastic constants. No assumptions are made about the weakness of the anisotropy or the heterogeneity and rays are traced in heterogeneous anisotropic models. Synthetic and field data examples show that the technique works well for simple structures, but, since only limited-aperture data are used, the technique produces less accurate results when the medium contains arbitrary 2-D structures. A detailed summary of each chapter follows.

1.3.1 Estimation of elastic constants in homogeneous TI media

In chapter 2, I explain how to estimate the elastic constants that describe a homogeneous TI medium from measurements near one or both axes of symmetry. The procedure consists of fitting *P*- and *SV*-wave traveltimes with elliptical velocity functions. The result of this fitting is four elliptical velocities (direct and normal moveout) that can be transformed into four elastic constants by solving analytically a system of four equations and four unknowns. No assumptions are made about the weakness of the anisotropy. When the traveltimes correspond to *SH*-waves, the transformation between elliptical velocities and elastic constants is immediate because *SH*-wave slowness surfaces are already elliptical.

The equations developed in this chapter are valid near the axes of symmetry, regardless of their inclination. In particular, for vertical axis of symmetry, the procedure is more accurate for cross-well than for VSP geometries.

The procedure proposed in this chapter for homogeneous media is generalized to heterogeneous media in the rest of the thesis. The generalization consists of describing the heterogeneous medium as a superposition of homogeneous nonoverlapping regions. When the model is described in this way, two new methods are needed in order to calculate accurately the elliptical velocities at each region: ray tracing in anisotropic models and anisotropic traveltime tomography. The next two chapters describe these new procedures.

1.3.2 Kinematic ray tracing in anisotropic layered media

In chapter 3, I describe the details of the implementation of a kinematic ray tracing algorithm in transversely isotropic media with variable inclination of the axes of symmetry. The heart of the algorithm is the numerical solution of Snell's law at each

interface.

Snell's law tells us how a plane wave changes direction when it hits an interface between two different media. When the media are isotropic, the wave (phase) velocity is the same as the ray (group) velocity, and, therefore Snell's law also tells us how to bend a ray at a given interface. When the media are anisotropic, the procedure is more complicated because rays and waves travel with different velocities in different directions, and it is thus necessary to change from one to the other either to propagate a ray through a region or to cross a boundary between two regions.

The traveltimes computed by this ray tracing agree, as expected, with the ones calculated by finite-difference elastic modeling in TI media.

The ray tracing algorithm developed in this chapter is used in chapter 4 to do anisotropic traveltime tomography for elliptically anisotropic models.

1.3.3 Tomographic estimation of elliptical velocities

Chapter 4 explains how to fit traveltimes with heterogeneous elliptically anisotropic models. The procedure is identical for any wave type, and it makes no assumptions about the weakness of anisotropy or heterogeneity because the rays are traced in heterogeneous anisotropic models.

To show the behavior of the algorithm when using limited view measurements, I present two simple synthetic examples where both anisotropy and heterogeneities are weak. These examples show that when we use iterative techniques such as conjugate gradients to solve the linearized problem, different components of the velocity converge at different speeds with different types of artifacts, which can alter the amount of anisotropy in the solution.

Different models for describing the heterogeneities are presented in this chapter. I conclude that for cross-well geometries, the estimation of elliptical velocities in 1-D models is accurate and stable, whereas in general 2-D models it is less accurate and unstable, because of the small singular values introduced in the problem by high-frequency variations in the vertical component of the slowness. As a result, the resolution of the different components of the slowness is also different, and, therefore variations in velocity anisotropy cannot be estimated at the same resolution as variations in the velocity. In appendix **??**, I explain in more detail the issue of resolution of the different components of slowness by examining the results of the singular value decomposition of the related matrices.

The P-wave field data examples presented in chapter 4 show that, even when only one wave type is available, useful results can be obtained by fitting the data with elliptically anisotropic models. If more wave types are available, all we need to do is apply the same algorithm separately to each of them and, finally, transform the results at each region into elastic constants, as described in chapter 2. The next chapter explains how to do that.

1.3.4 Estimation of elastic constants in heterogeneous TI media

In chapter 5, I use all the tools developed in the previous chapters in order to estimate spatial variations in the elastic constants that describe a TI medium. The procedure consists of two steps. First, compressional and shear-wave traveltimes are fitted using heterogeneous elliptically anisotropic models, as described in chapter 4. Second, the elliptical parameters are transformed locally into elastic constants, as described in chapter 2. The technique is illustrated with synthetic and field data examples for cross-well geometries.