Chapter 1

Traveltime in homogeneous elliptically anisotropic media with a nonvertical axis of symmetry

The expression for the ray velocity in a medium with elliptical velocity dependency is given by the expression

$$\frac{1}{V^2(\alpha)} = \frac{\cos^2\alpha}{V_{||}^2} + \frac{\sin^2\alpha}{V_{\perp}^2}, \tag{1.1}$$

where α is the ray angle measured from the axis of symmetry (positive counterclockwise), and $V_{||}$ and $V_{\perp}$ are the velocities in the directions parallel and perpendicular to the axis of symmetry (Figure ??a). When the axis of symmetry is vertical, the angle that measures the direction of propagation of the ray with respect to the vertical is the same as the group velocity angle.

When the axis of symmetry is rotated an angle γ (Figure ??b), the expression for the ray velocity (for the same ray direction) becomes

$$\frac{1}{V^2(\alpha)} = \frac{\cos^2(\alpha-\gamma)}{V_{||}^2} + \frac{\sin^2(\alpha-\gamma)}{V_{\perp}^2}, \tag{1.2}$$

where $\alpha - \gamma$ is the angle from the axis of symmetry to the ray (the group velocity angle).

If the ray travels a distance d between two points (Figure ??),

$$d = \sqrt{\Delta x^2 + \Delta z^2}, \tag{1.3}$$

Figure 1.1: Ray velocity as a function of direction in an elliptically anisotropic medium: (a) Vertical axis of symmetry (ray angle = group velocity angle). (b) Tilted axis of symmetry (ray angle = α; group velocity angle = ϕ).

the corresponding traveltime t is

$$\begin{aligned} t^2 &= \frac{(d\cos(\alpha-\gamma))^2}{V_{||}^2} + \frac{(d\sin(\alpha-\gamma))^2}{V_{\perp}^2} \qquad (1.4) \\ &= \frac{(d\cos\alpha\cos\gamma + d\sin\alpha\sin\gamma)^2}{V_{||}^2} + \frac{(d\sin\alpha\cos\gamma - d\cos\alpha\sin\gamma)^2}{V_{\perp}^2}. \end{aligned}$$

To further simplify this equation we need to know the values of $d\cos\alpha$ and $d\sin\alpha$. In order to do this, we need to be careful about the sign of α (clockwise or counter-clockwise) for the given ray direction. We also need to be careful about the signs of Δx and Δz. It turns out that regardless of how the signs of these quantities are defined (as long as they are consistent) the final expression for t^2 is always, as expected, the same. The result is

$$t^2 = \frac{(-\Delta z\cos\gamma + \Delta x\sin\gamma)^2}{V_{||}^2} + \frac{(\Delta x\cos\gamma + \Delta z\sin\gamma)^2}{V_{\perp}^2}. \qquad (1.5)$$

This is the expression for the traveltime of a ray that travels between two points separated by a distance d in a homogeneous elliptically anisotropic medium with the axis of symmetry forming an angle γ with the vertical. This equation is the heart of the inversion procedure that I propose in chapter **??**.