Chapter 1

Partial derivatives of the traveltime with respect to the model parameters

In this appendix, I show the expressions for the partial derivatives of the traveltime t_i [equation (**??**)] with respect to the model parameters m_k, where m_k is a component of the vector $\boldsymbol{m}$ as follows:

$$\begin{aligned} \boldsymbol{m} &= (m_1, ..., m_N, m_{N+1}, ..., m_{2N}, m_{2N+1}, ..., m_{3N}, m_{3N+1}, ..., m_{4N}, m_{4N+1}, ..., m_{5N}) \\ &= (S_{\perp_1}, ..., S_{\perp_N}, S_{\|_1}, ..., S_{\|_N}, \gamma_1, ..., \gamma_N, b_1, ..., b_N, a_1, ..., a_N). \end{aligned}$$

First, the derivatives with respect to the interval parameters $S_{\perp_j}$, $S_{\|_j}$, and γ_j $(1 \leq j \leq N)$ are

$$\frac{\partial t_i}{\partial m_k} = \begin{cases} \frac{\Delta X_{ik}^2 S_k}{t_{ik}} & \text{if } 1 \leq k \leq N \\ \frac{\Delta Z_{ik}^2 S_k}{t_{ik}} & \text{if } N+1 \leq k \leq 2N \\ \frac{\Delta X_{i,k} S_k^2 (-\Delta x_{i,k} \sin\gamma_k + \Delta z_{i,k} \cos\gamma_k)}{t_{i,k}} + & \\ \frac{\Delta Z_{i,k} S_{k+N}^2 (\Delta z_{i,k} \sin\gamma_k + \Delta x_{i,k} \cos\gamma_k)}{t_{i,k}} & \text{if } 2N+1 \leq k \leq 3N. \end{cases}$$

Note that, for the interval parameters, the derivatives with respect to the j^{th} variable depend only upon the properties of the j^{th} layer.

The derivatives of the traveltime with respect to the boundary parameters (a_j and b_j) depend on the properties of the medium above and below that boundary, as follows:

$$\frac{\partial t_i}{\partial m_k} = \begin{cases} \frac{-\sin\gamma_k \Delta X_{i,k} S_k^2 + \cos\gamma_k \Delta Z_{i,k} S_{k+N}^2}{t_{i,k}} + & \\ \frac{\sin\gamma_{k-1} \Delta X_{i,k-1} S_{k-1}^2 - \cos\gamma_{k-1} \Delta Z_{i,k-1} S_{k-1+N}^2}{t_{i,k-1}} & \text{if } 3N+2 \leq k \leq 4N \\ \frac{\partial t_i}{\partial m_{k-N}} x_{i,k} & \text{if } 4N+2 \leq k \leq 5N. \end{cases}$$

Since the traveltime t_i is not affected by the position of the top boundary, $\frac{\partial t_i}{\partial a_1} = \frac{\partial t_i}{\partial b_1} = 0$.

When a ray travels horizontally, only the first N components of the vector of partial derivatives are nonzero. For the forward modeling this is not problem; it simply means that the horizontal component of the velocity is the only parameter that affects the traveltime of a horizontally traveling ray. Problems arise, however, when we try to invert these traveltimes, because there are infinite combinations of the other parameters (null space) that satisfy the data equally well. This problem casues instability in the inversion procedure. For this reason, the inversion procedure described in chapter **??** does not use rays that travel exactly along the horizontal.

Making the appropriate simplifications in the above equations for the case of isotropic media results in a set of equations similar to the ones obtained by Lee (1990), except for two misprints in Lee's equations.