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Short Note

Equations for three-dimensional dip moveout in depth-variable velocity media

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INTRODUCTION

Artley (1992) introduced an original method to perform $v(z)$ dip moveout in a two-dimensional earth model. The process uses ray tracing tables and solves a system of equations accounting for the location and the dip of the reflection point. This paper describes an extension of Artley's method and derives a new set of equations for the 3-D case.

Preamble: the ray parameter in three dimensions

In a two-dimensional earth model, the ray parameter is given at any point of the ray by the relationship

$$p = \frac{\sin\theta}{v}, \tag{1}$$

where θ is the inclination of the ray path with respect to the vertical axis and v is the local velocity. We can see that the ray parameter is simply the projection length of the ray path vector $\mathbf{r}$ ($\|\mathbf{r}\| = r = 1/v$) on the earth surface. In a 3-D v(z) model of the earth, the rays travel in a vertical plane. In this case, the ray parameter, being the projection of the ray path vector, is a two-dimensional vector that can be expressed in either cartesian or polar coordinates, as follows:

$$\mathbf{p} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} = p \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}, \tag{2}$$

where ϕ is the strike of the vertical plane containing the ray and $p = \sqrt{p_x^2 + p_y^2}$.

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DERIVING THE SYSTEM OF EQUATIONS

Vectorial relationships on the earth's surface

In the triangle (PSG) in Figure (1), the following expression relates the horizontal traveling of the source and geophone rays with the offset:

$$\overrightarrow{SG} = \overrightarrow{SP} + \overrightarrow{PG} \ . \tag{3}$$

The vectors $\overrightarrow{PS}$ and $\overrightarrow{PG}$ are related to the table that gives the lateral distance traveled by the ray as a function of the ray parameter and the travel time, $\xi(p,t)$[2]. The projection on the earth's surface of a ray path (of parameter p and traveltime t) is the 2-D vector $\xi(p,t)\frac{\mathbf{p}}{p}$. Thus, relation (3) yields

$$\mathbf{x}_g - \mathbf{x}_s = \xi(p_g,t_g)\frac{\mathbf{p}_g}{p_g} - \xi(p_s,t_s)\frac{\mathbf{p}_s}{p_s}, \tag{4}$$

where $\mathbf{x}_s$ and $\mathbf{x}_g$ are the 2-D vectors of the source and geophone coordinates.

A second relation expresses the coordinates of the point of emergence of the zero-offset ray, E. In the triangle (PEM), we have :

$$\overrightarrow{ME} = \overrightarrow{MP} + \overrightarrow{PE} \ . \tag{5}$$

Again, the vector $\overrightarrow{PE}$ is related to the table of lateral distances, $\xi(p,t)$, yielding

$$\mathbf{x}_0 = -\frac{1}{2}\left(\xi(p_s,t_s)\frac{\mathbf{p}_s}{p_s} + \xi(p_g,t_g)\frac{\mathbf{p}_g}{p_g}\right) + \xi(p_0,t_0)\frac{\mathbf{p}_0}{p_0}, \tag{6}$$

where $\mathbf{x}_0$ is the 2-D vector of the emergence point coordinates.

Time relationships in the earth's interior

The following obvious equation relates the traveltimes along the ray paths to source and geophone, t_s and t_g, to the given total traveltime t_{sg}:

$$t_s + t_g = t_{sg}. \tag{7}$$

We can also state two more equations to account for the fact that the three ray paths, namely the zero-offset, source, and geophone ray paths, end at the same point in depth (R):

$$\tau(p_0,t_0) = \tau(p_s,t_s) = \tau(p_g,t_g), \tag{8}$$

where $\tau(p,t)$ is the table of vertical traveltimes.

[2]The ray tracing tables are fully described in Artley (1992)

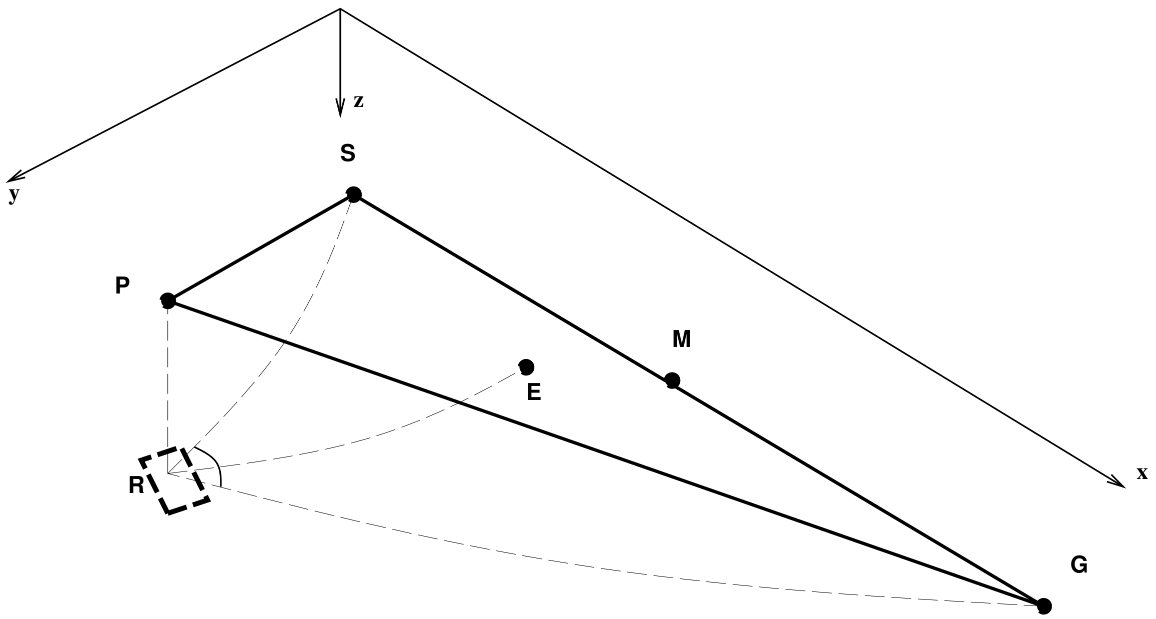

Figure 1: Three-dimensional view of the source, receiver, and reflection points for 3-D v(z) DMO. The dashed lines represent the ray paths in the earth, and the bold solid lines represent the distances on the surface of the earth. patrick3-Rayp3d [NR]

The relationship between the ray paths

At the reflector point R, the ray must obey Snell's law. In other words, the angle of incidence must equal the angle of reflection. In terms of ray paths, $\mathbf{r}_0$ fixes the position of the reflection plane, and $\mathbf{r}_s$ and $\mathbf{r}_g$ account for the angles of incidence and reflection. Since $\mathbf{r}_s$ and $\mathbf{r}_g$ have equal lengths $1/v$, Snell's law can be expressed in the following vectorial form:

$$\mathbf{r}_0 = \lambda(\mathbf{r}_s + \mathbf{r}_g), \tag{9}$$

where λ is a coefficient of proportionality. The first two coordinates are the ray parameter components, which have the following relation:

$$\mathbf{p}_0 = \lambda(\mathbf{p}_s + \mathbf{p}_g). \tag{10}$$

The third coordinate of the ray path vectors can be expressed as a function of the inclination angle and the vector length, $1/v$, as follows:

$$r_z = \frac{\cos\theta(p,t)}{v}, \tag{11}$$

where $\theta(p,t)$ is the table of angles along a ray of parameter p and time t. Then, after simplification of the $1/v$ factors (the velocity is the same for all rays at the reflector point), the third equation of relation (9) becomes

$$\cos\theta(p_0,t_0) = \lambda(\cos\theta(p_s,t_s) + \cos\theta(p_g,t_g)). \tag{12}$$

Substituting equation (12) into relation (10), we can eliminate the proportionality factor λ to obtain the following relation:

$$\mathbf{p}_0 \left(\cos\theta(p_s,t_s) + \cos\theta(p_g,t_g)\right) = (\mathbf{p}_s + \mathbf{p}_g)\cos\theta(p_0,t_0). \tag{13}$$

Sets of equations and unknowns

Our system of equations is constituted by collecting relations (4), (6), (7), (8), and (13):

$$\left\{\begin{array}{ccc} \xi(p_g,t_g)\frac{\mathbf{p}_g}{p_g} - \xi(p_s,t_s)\frac{\mathbf{p}_s}{p_s} & = & \mathbf{x}_g - \mathbf{x}_s \\ \xi(p_0,t_0)\frac{\mathbf{p}_0}{p_0} - \frac{1}{2}\left(\xi(p_s,t_s)\frac{\mathbf{p}_s}{p_s} + \xi(p_g,t_g)\frac{\mathbf{p}_g}{p_g}\right) & = & \mathbf{x}_0 \\ t_s + t_g & = & t_{sg} \\ \tau(p_0,t_0) & = & \tau(p_s,t_s) \\ \tau(p_0,t_0) & = & \tau(p_g,t_g) \\ \mathbf{p}_0\left(\cos\theta(p_s,t_s) + \cos\theta(p_g,t_g)\right) & = & (\mathbf{p}_s + \mathbf{p}_g)\cos\theta(p_0,t_0) \end{array}\right. . \tag{14}$$

Because the $\mathbf{x}$ and $\mathbf{p}$ vectors are two-dimensional, the first, second, and sixth relations of system (14) give 3×2 equations, yielding a set of nine equations.

The unknowns are $\mathbf{p}_s$, $\mathbf{p}_g$, t_s, t_g, t_0, and $\mathbf{x}_0$. As described in the preamble, the $\mathbf{p}$ vectors have only two unknown parameters, p_x and p_y (or p and ϕ). Similarly, $\mathbf{x}_0$ is a two-dimensional unknown vector. Therefore, system (14) relates nine unknowns with the known parameters $\mathbf{p}_0$, t_{sg}, $\mathbf{x}_s$, $\mathbf{x}_g$, and the ray tracing tables $\xi(p,t)$, $\tau(p,t)$, and $\theta(p,t)$.

Solving the system

The second relation of system (14) can be isolated because it simply adds two relations and two unknowns (the coordinates of $\mathbf{x}_0$). Similarly, the third equation can be substituted into the others by replacing t_s (or t_g) with $t_{sg} - t_g$ (or $t_{sg} - t_s$).

Thus, we obtain a reduced system of four relations (six equations):

$$\left\{\begin{array}{ccc} \xi(p_g,t_g)\frac{\mathbf{p}_g}{p_g} - \xi(p_s,t_{sg}-t_g)\frac{\mathbf{p}_s}{p_s} & = & \mathbf{x}_g - \mathbf{x}_s \\ \tau(p_0,t_0) & = & \tau(p_s,t_{sg}-t_g) \\ \tau(p_0,t_0) & = & \tau(p_g,t_g) \\ \mathbf{p}_0\left(\cos\theta(p_s,t_{sg}-t_g) + \cos\theta(p_g,t_g)\right) & = & (\mathbf{p}_s + \mathbf{p}_g)\cos\theta(p_0,t_0) \end{array}\right. . \tag{15}$$

and two additional relations to compute the remaining unknowns:

$$\mathbf{x}_0 = \xi(p_0,t_0)\frac{\mathbf{p}_0}{p_0} - \frac{1}{2}\left(\xi(p_s,t_s)\frac{\mathbf{p}_s}{p_s} + \xi(p_g,t_g)\frac{\mathbf{p}_g}{p_g}\right) \tag{16}$$

$$t_s = t_{sg} - t_g. \tag{17}$$

The six unknowns of system (15) are t_0, t_g (or t_s), $\mathbf{p}_s$ [including the two unknowns (p_{xs}, p_{ys}) or (p_s, ϕ_s)], and $\mathbf{p}_g$ [(p_{xg}, p_{yg}) or (p_g, ϕ_g)]. The parameters are t_{sg}, $\mathbf{p}_0$ [including the two parameters (p_{x0}, p_{y0}) or (p_0, ϕ_0)], $\mathbf{x}_s$, and $\mathbf{x}_g$. This system, like its 2-D counterpart, can be solved using the Newton-Raphson algorithm (Press et al., 1986).

AGREEMENT WITH THE EQUATIONS IN TWO DIMENSIONS

The 2-D system derived by Artley (1992) is a set of four equations with four unknowns. It can be retrieved by replacing the vectorial ray parameters with their scalar form (since the vectors are now in a common plane). The first relation of system (14) then becomes

$$\xi(p_g, t_g) - \xi(p_s, t_{sg} - t_g) = 2h, \tag{18}$$

where h is the half offset. The fourth relation becomes

$$\frac{\sin\theta(p_0, t_0)}{v}\left(\cos\theta(p_s, t_s) + \cos\theta(p_g, t_g)\right) = \left(\frac{\sin\theta(p_s, t_s)}{v} + \frac{\sin\theta(p_g, t_g)}{v}\right)\cos\theta(p_0, t_0) \tag{19}$$

and can be simplified to

$$\sin(\theta(p_0, t_0) - \theta(p_s, t_s)) = \sin(\theta(p_g, t_g) - \theta(p_0, t_0)). \tag{20}$$

Finally, we obtain the bisection condition as shown by Artley (1992), as follows:

$$\theta(p_0, t_0) = \frac{1}{2}\left[\theta(p_s, t_s) + \theta(p_g, t_g)\right]. \tag{21}$$

CONCLUSION

We derived a set of equations that constrains the travel path of a ray in a three-dimensional earth model with a depth-variable velocity. This system can be solved for the zero-offset traveltime and the emergence point of the ray, yielding an efficient method for dip moveout processing. A successful implementation of the equations have been realized by Artley at the Colorado School of Mines, showing the expected saddle-like impulse response.

ACKNOWLEDGMENTS

We began this work during Craig Artley's visit (February, 1993) by deriving a first system of equations. Then, after several discussions to improve the solving process, we came up with the final equations published in this paper. We are grateful to the sponsors of the Stanford Exploration Project and the Center for Wave Phenomena for making this collaboration possible.

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