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Interpretation of the acoustic properties of suspensions with the diffraction theory

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ABSTRACT

Though the viscoelastic theory suitably predicts the acoustic properties of suspensions, it does not help us understand the amplitude variability of the waves transmitted through media of different porosity. This paper attempts to explain two phenomena occurring in suspensions using the diffraction theory. First, lab experiments show that the amplitude of the transmitted waves decreases when the suspension becomes denser. The modeling of this experiment predicts the scattering effect on the amplitude of the waves in dense suspensions. The second phenomenon concerns the sigmoid increase of the measured P-velocities with the frequency of the measurement. This behavior can be qualitatively explained by the diffraction theory. Furthermore, a simple relationship exists between the critical frequency of the measurement and the size of the grains.

INTRODUCTION

The insights of diffraction theory are useful in the interpretation of P-velocity measurements of suspensions. After introducing the notion of suspension, this paper compares some experimental P-velocity measurements presented by Marion (1990) with an acoustic modeling of the same experiment on a computer. Finally, a diffraction-oriented study shows that in comparison to Biot's theory, the P-velocity measured at a low frequency is slower than the high-frequency measurement. Moreover, the critical frequency between the high-frequency and low-frequency domains is directly related to the radius of the grains.

BOUNDARY BETWEEN POROUS ROCK AND SUSPENSION

Though it is often difficult to make the distinction between a rock and a suspension when the sample is incoherent, the two terms have a clear definition. We can consider

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a sample to be a load-bearing rock when the constitutive grains have no freedom of movement, in other words, when the grains are in contact. Conversely, a suspension consists of grains separated from one another, moving freely in the medium.

Although suspensions usually have a higher porosity than load-bearing rocks, the critical porosity varies with the nature of the rock, its degree of compaction, the grain size, and so on. Figure (1) shows two types of granular arrangement of spherical isodiametric grains that have completely different porosities. The simple cubic arrangement is the loosest possible arrangement of spheres in a volume. In this case, a simple volume calculation yields the following porosity :

$$\rho_{\text{cubic}} = 1 - \frac{\pi}{6} \simeq 0.4764. \tag{1}$$

The tetrahedric arrangement is the densest known arrangement of spherical grains. A volumetric integration gives us the following porosity:

$$\rho_{\text{tetra}} = 1 - \sqrt{3}\frac{\pi}{6} \simeq 0.0931. \tag{2}$$

In both cases, the porosities do not depend on the radius of the grains composing the rock.

Assuming that a rock is composed of single-sized, monodisperse, spherical grains, a porosity higher than ρ_{cubic} implies a state of suspension and a porosity lower than ρ_{tetra} implies a load-bearing rock (all grains are in contact with their neighbors). Between these two values of porosity, the sample may be either a suspension or a load-bearing rock.

Obviously, knowing only the porosity is not enough to determine whether the sample behaves like a suspension or a rock. The boundary between these two states is uncertain. However, in practice, a sand starts to behave like a suspension beyond 40 or 45 percent porosity; suspensions are never encountered below this value.

AMPLITUDE OF THE WAVES TRANSMITTED IN SUSPENSIONS

Marion (1990) showed that suspensions behave in accord with Wood's velocity-porosity relation. Figure (2) displays in the top panel some experimental results obtained with glass spheres immersed in sodiumpolytungstate. Wood's relation fits the data perfectly down to a porosity of approximately 45 percent. However, the bottom panel shows that the first-peak amplitude of the arriving P-waves decreases drastically when the porosity goes down to 40 percent. This drop in amplitude is caused by the scattering effect of suspended spheres (Marion, 1990).

In order to verify this idea, I have attempted to model the propagation of P-waves in a rock sample during a lab experiment. The problem with modeling such data is that we are dealing with very high frequencies (1 MHz) and very short distances (centimeters for the model size and fractions of a millimeter for the grains). Since

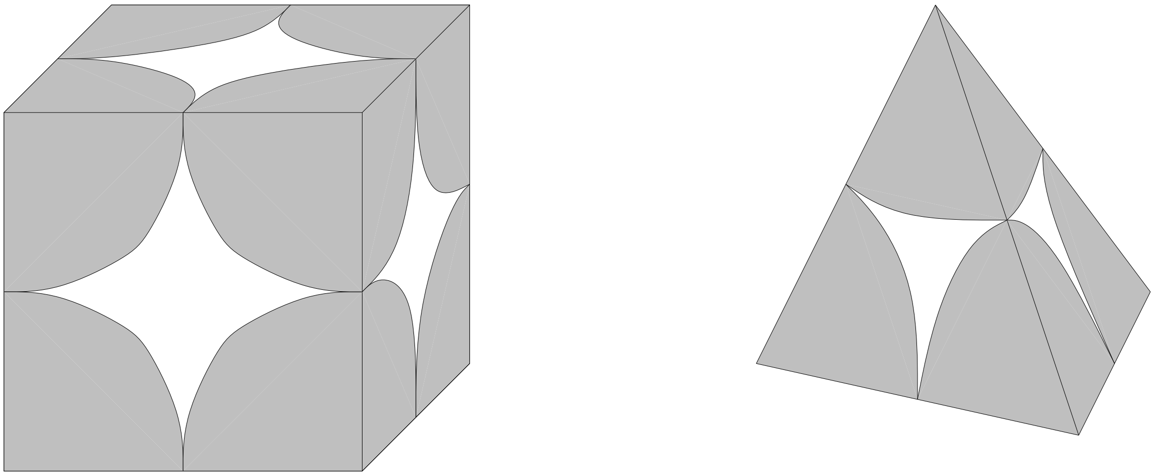


Figure 1: Two examples of granular arrangement. patrick2-Pores [NR]

seismic modeling programs are generally used at a completely different scale, care must be taken in the choice of parameters to keep the calculations stable. The 2-D acoustic modeling program used for the experiment, written by J. Etgen, is accurate to the second order in time and the sixteenth order in x and y. The model is 8 centimeters long and large enough to get rid of side reflections. The model itself is composed of two velocities: the velocity of the grains is 3000 ms^{-1}, and the pores are filled with water of velocity 1500 ms^{-1}. The grid spacing is 0.8 mm in both x and y. The velocity distribution in the model is generated randomly in order to achieve a given porosity (Popovici and Muir, 1989). The input wavelet has a maximum frequency of 600 kHz and a fundamental frequency of 500 kHz. These parameters yield a ratio between the wavelength and the grain size of

$$\frac{\lambda}{r} \sim 3, \tag{3}$$

which places the experiment in the range between the multiple scattering effect and the low-frequency domain. Several models with different porosities have been input to the program. Each velocity model yields a trace whose first arrival is more and more delayed when porosity increases (Figure 3).

Figure (4) displays the Reuss bound (equivalent to Wood's relation) for the velocities of the model. By mentally overlaying the curve to the synthetic seismogram in Figure (3), we can notice that the first peaks lie close to the curve. Unfortunately, they do not exactly fit the Reuss bound, mainly because the program only models P-velocities.

The interesting part of this modeling is the distribution of amplitudes. The first recorded peak has a large amplitude up to 40 percent porosity. Then the amplitude

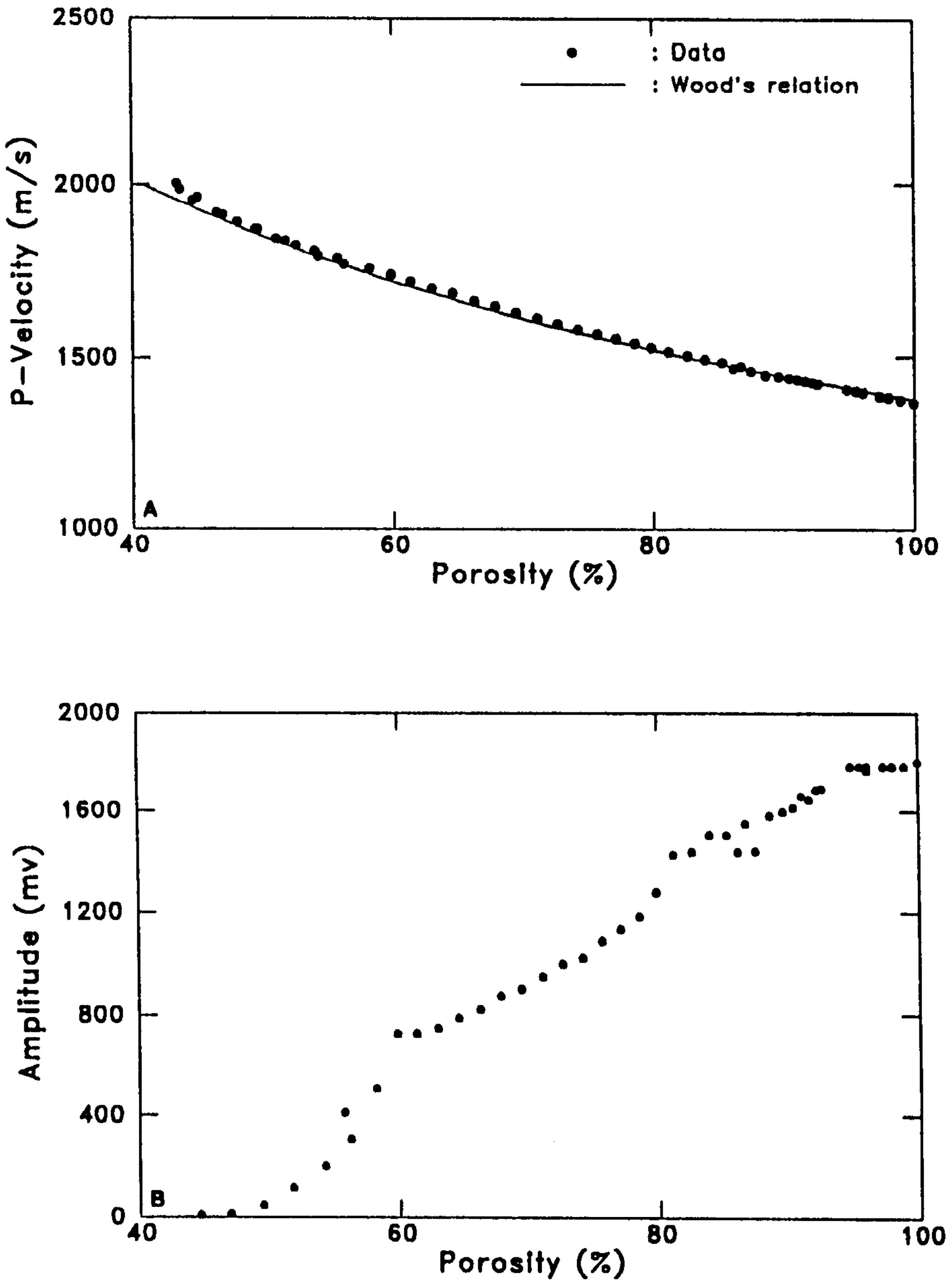


Figure 2: Compressional velocity versus porosity for a suspension of silver-coated glass spheres immersed in sodiumpolytungstate. Experimental data (•) are compared with the wood isostress relation (solid line) in the top panel. Measurements of first-peak amplitude (bottom panel) show that the quality and accuracy of the signal decreases with increasing volume fraction because of scattering effect of suspended spheres. Figure and caption taken from Marion (1990), Figure 2.3. patrick2-Fig2.3 [NR]

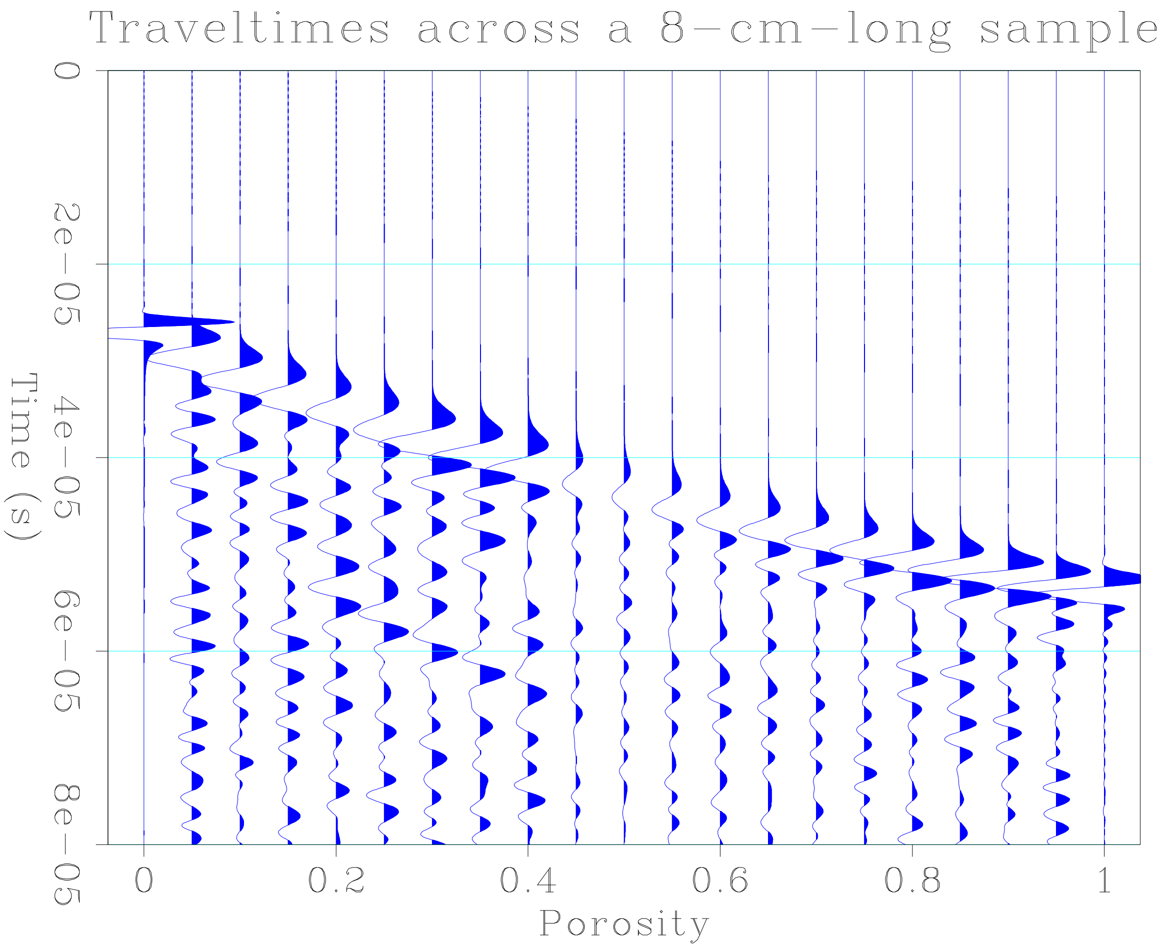


Figure 3: Synthetic seismograms using a high-frequency wavelet in random models of porosity (v_1 = 3000 ms^{-1}, v_2 = 1500 ms^{-1}). patrick2-Rec [CR]

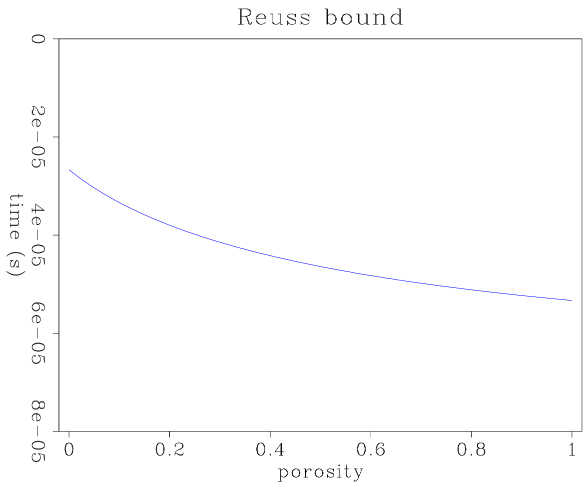


Figure 4: Wood's relation for v_{p1} = 3000 ms^{-1}, v_{p2} = 1500 ms^{-1}, $v_{s1} = v_{s2}$ = 1000 ms^{-1}, and $\rho_1 = \rho_2$ = 2 gcm^{-3}. Warning: the vertical axis is graduated in reverse values of traveltime across an 8-centimeter-long sample and not in velocity. patrick2-Reuss [ER]

suddenly drops and starts again to increase slowly in the suspension domain. Thus, its distribution agrees with that shown in the bottom panel of Figure (2).

Figure (5) shows how the velocity measurements disagree with Biot's theoretical curves in the region of 50 percent porosity. In this area, the medium is spatially highly varying: there is no large region with the same velocity. Therefore, the region around 50 percent porosity can be seen as the realm of diffraction. Because the diffracted waves travel in a spherical shape, they decay rapidly in amplitude. It is very hard to measure the velocity of such media because the amplitudes are very weak and we do not see the strong arrival caused by waves traveling through areas of constant velocity. Looking more closely at the synthetic seismogram, we can see that the strongest arrivals are not the first arrivals. Even when the porosity is close to unity, some small-amplitude precursors can be seen on the seismogram, attesting to the occurrence of some diffraction that allows shorter traveltimes but yields weaker amplitudes.

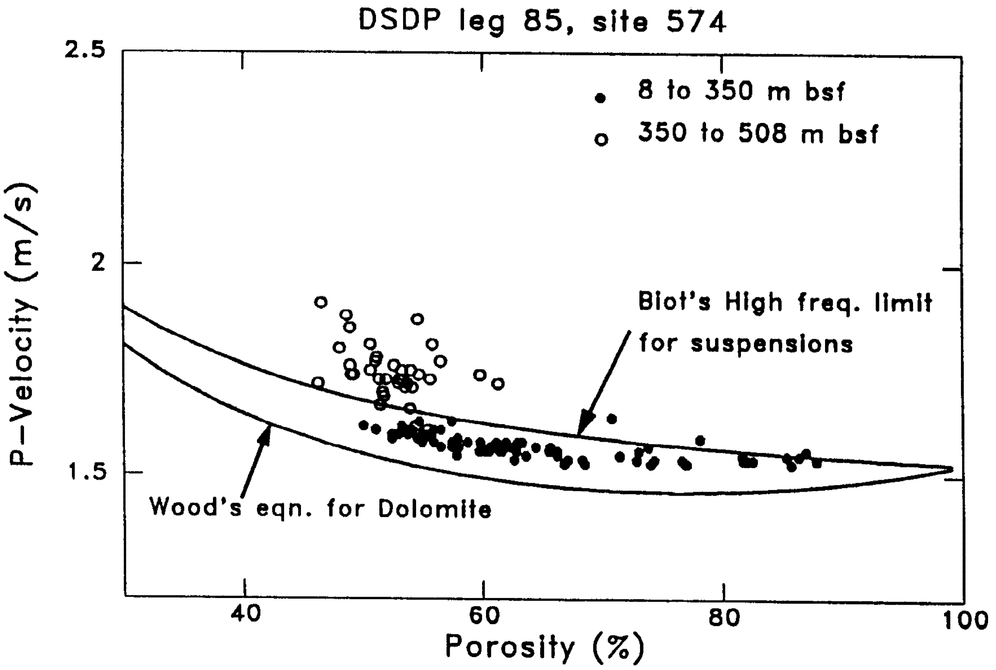


Figure 5: Velocity-porosity relationship in sediments at the rheological transition from suspension to load-bearing sediment. Data are from DSDP site 574 in carbonated sediments. Significant departure from Wood's equation is observed at a depth of 350 m below sea floor for porosities ranging from 60 to 45%. Velocity data at depths greater than 350 m lie above Biot's high-frequency limit for suspension, suggesting that the departure from Wood's relation is not caused by frequency but compaction. Figure and caption taken from Marion (1990), Figure 2.12. patrick2-Fig2.12 [NR]

We can conclude from these observations that the velocities measured in the realm of diffraction are closer to the average velocity across the medium than to the theoretical velocities predicted by Wood's and Biot's relations. But the important point is that the first peak is very hard to determine during the measurement and there is a high chance of being wrong.

P-VELOCITIES FOR DIFFERENT FREQUENCIES

There is a fundamental difference between lab measurements and in situ measurements (from sonic logs to shot-geophone surveys). The lab measurements are performed with devices emitting and recording frequencies on the order of 1 MHz. The dimensions of the rock samples hardly ever exceed 1 meter. Conversely, seismic surveys are operated at frequencies around 50 Hz and are designed to highlight layers more than 100 meters thick. Biot (1956) developed a theory for wave propagation that takes into account the frequencies involved in the measurement and described, using the theory of fluid mechanics, how velocities increase with frequency. The diffraction theory introduces an interesting interpretation of the sigmoid curves relating P-velocity to frequency.

The lens-grain analogy

The outstanding properties of lenses have been well known since Gallileo Gallilei. In the early nineteenth century, the birth of Fourier optics theory gave a better understanding of optical waves and their behavior with respect to lenses. One of the most important properties is the existence of a focal plane behind the lens where the energy of a normally incident plane wave focuses. A grain can be thought of as a spherical lens whose focus is the center of the grain. From this point, the diffracted wave radiates spherically as shown in Figure (6). The amplitude of this wave is weak and decays with distance as $1/r$, whereas the incident plane wave travels without amplitude decay.

The Fresnel zone

Several grains generate different diffracted waves that interfere and may raise a new wave. Huygen's principle asserts that a plane wave results from the superposition of a set of spherical diffractions. However, each spherical pattern contributes to the plane wave reconstruction only to a limited extent. Figure (7) represents the zone where the spherical waves add constructively, the Fresnel zone. The length of this zone depends on the frequency (related to the wavelength λ) of the spherical wave. Its width, $\mathbf{f}$, depends on both the wavelength and the radius $\mathbf{R}$ of the spherical wave.

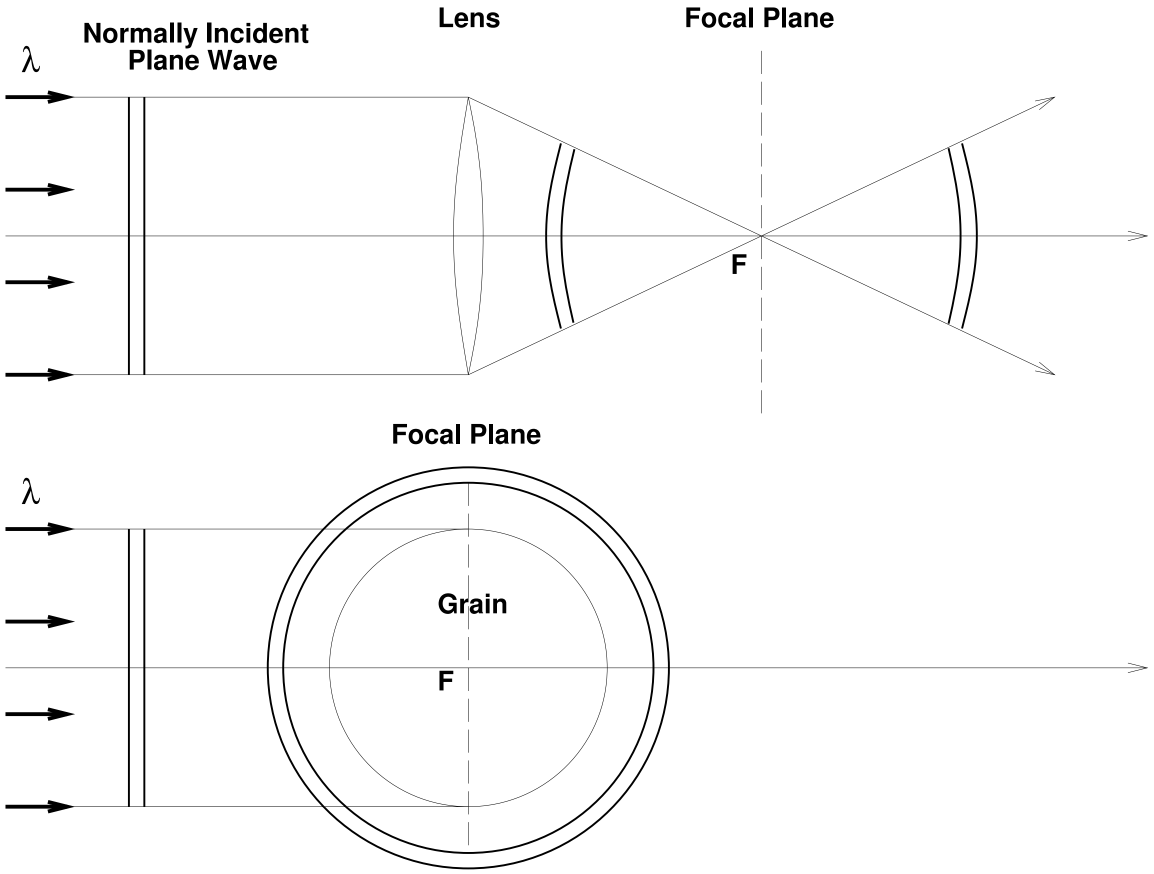
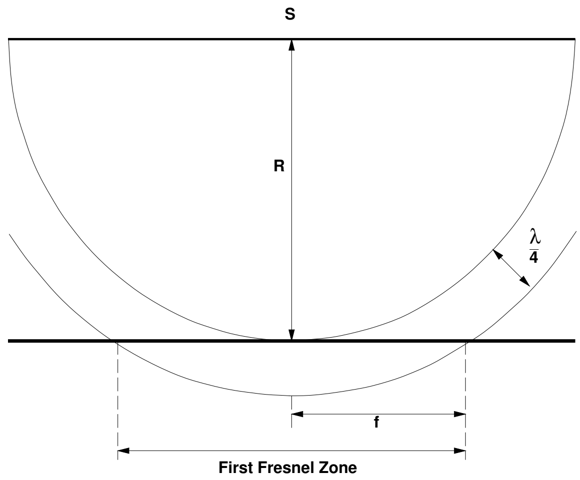


Figure 6: The lens-grain analogy. The focus of a thin lens comes closer to the lens when the radii of curvature decrease. At the limit, the focus of a spherical lens is the center of the lens. patrick2-Foptics [NR]

Figure 7: The Fresnel zone. Its width corresponds to the range within which another spherical wave will add constructively. patrick2-Fresnel [NR]



Diffracted waves

The idea of the Fresnel zone is useful in understanding when the the waves diffracted from scattering grains add constructively and what their relative importance is. Figure (8) shows two scattering responses of a spherical grain to different frequencies of the arriving wave at a given time t. The velocity of the grain, v_g, is assumed to be higher than the velocity of the host medium (water), v_w. The left drawing represents the high-frequency response and shows that the transmitted plane wave (slightly curved in the proximity of the grain because of the diffraction effect) is far from the purely diffracted spherical wave centered on the grain. At a lower frequency (right drawing), the planar and spherical waves add constructively in the region around point **A**. In the high-frequency case, the point of highest amplitude behind the grain on the axis of propagation is located on the diffracted spherical wave, although this is not a zone of constructive interference (point **a** on the drawing). The distance d_s from point **S** (the first point met by the planar wave incident to the grain) to point **a** is

$$d_s = 2r + \left(t - \frac{2r}{v_g}\right) v_w, \tag{4}$$

where r is the radius of the grain. In the low-frequency case, the point of highest amplitude behind the grain is not located exactly on the spherically diffracted wave but is closer to the center of the grain: the constructive interferences have moved the point of highest amplitude backward. As a consequence, the velocity appears higher in the high-frequency measurement than in the low-frequency measurement.

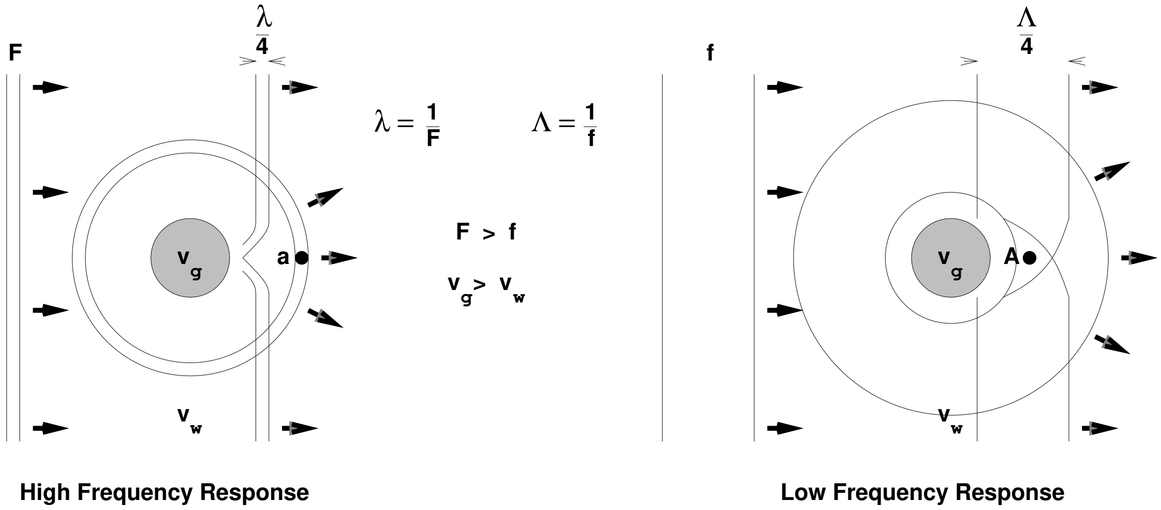


Figure 8: The scattering response of a spherical grain to different frequencies of the arriving wave. $\boxed{\text{patrick2-Freqvel}}$ [NR]

The low frequency-high frequency transition

What can we consider a low frequency and what a high frequency? The preceding little study of diffracted waves suggests that we have the low-frequency case as soon as the diffracted spherical wave and the transmitted plane wave interfere constructively. In terms of distances, this condition can be written as

$$|d_s - d_p| < \frac{\lambda}{4}, \tag{5}$$

where λ is the wavelength of the waves, and d_s and d_p are the distances traveled by the spherical and planar waves, respectively, during a time t. The expression of d_s is given by equation (4), and the distance d_p is simply

$$d_p = tv_w. \tag{6}$$

Then the condition (5) becomes

$$2r\left|1 - \frac{v_w}{v_g}\right| < \frac{\lambda}{4}. \tag{7}$$

Finally, substituting $\lambda f = v_w$ into the latter equation, we obtain the following low-frequency condition:

$$f < \frac{v_w}{8r}\left|1 - \frac{v_w}{v_g}\right|. \tag{8}$$

Biot's sigmoid velocity versus frequency curves

Figure (9) displays some theoretical velocity-frequency curves for different materials (sand, silt, and clay). The shape of these curves is sigmoid: the velocity is constant for low frequencies until it reaches a transitional region where it strongly increases to reach eventually another constant-velocity level corresponding to the high frequencies. As explained in the section on diffracted waves, using diffraction theory we can predict the low and high levels of velocity for low and high frequencies. Moreover, in the preceding section we arrived at a condition that locates the limit between low frequencies and high frequencies. If we apply this condition to the three curves by picking the frequency at which the sigmoid pattern occurs, we can estimate the radius of the grains. For the clay, the estimate of the radius is 0.1 μm. The silt and the sand give us approximately 10 μm and 2 mm, respectively. The silt and clay samples can be considered as suspensions because of their high porosity (60% and 80%). Therefore, the estimation of the grain size gives some realistic results. Unfortunately, since the sand with 40 percent porosity cannot be considered a suspension, condition (8) is no longer valid. However, the condition gives interesting predictions of the grain size.

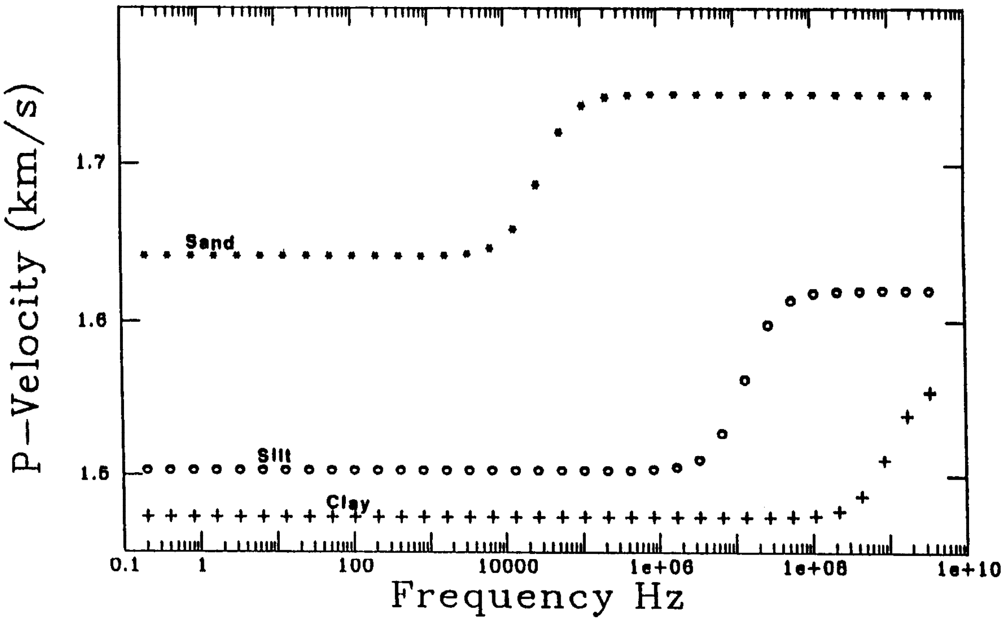


Figure 9: Calculated velocity versus frequency for sand, silt, and clay. For a sand of 40% porosity and 10 Darcy permeability, critical frequency is in the sonic-ultrasonic range (10-100 kHz). For a silt of 60% porosity and 10 mD permeability, critical frequency is in the high ultrasonic frequency range (10 MHz). In clay with 80% porosity and permeability of the order of 0.5 mD, critical frequency is in the GHz range. Figure and caption taken from Marion (1990), Figure 2.7. patrick2-Fig2.7 [NR]

CONCLUSION

Diffraction theory teaches us that there is not a single P-wave traveling in a porous medium. The arrival time picked in the lab measurements is generally the most "beautiful" amplitude: the highest and earliest. However, near the limit between suspension and load-bearing rock, picking the highest and earliest arrival is a very hard task, and, instead of picking a single peak, it is probably more interesting to pick a range of arrival times with equivalently high amplitudes that would yield a range of possible velocities.

I have derived a very simple expression relating the diameter of the grains to the frequency of the measurement. The validity of this expression is limited to very small grain sizes since we assume a perfect scattering behavior. However, the expression has not yet been experimentally tested.

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