Stanford Exploration Project, Report 77, November 17, 1997, pages 127–141

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Shot gather continuation

Matthias Schwab[1]

keywords: *continuation, velocity, amplitudes, modeling*

ABSTRACT

Standard offset continuation transforms a constant offset gather to a gather at a different offset. The new gather is designed to yield the same migrated subsurface image as the original one. Similarly, the shot continuation operator introduced in this article extrapolates shot gathers to new shot locations, while preserving the associated migrated subsurface image. The derivation of the shot continuation method is analogous to the derivation of the offset continuation process. Unfortunately, the shot continuation operator is space and time variant and therefore more expensive than the space invariant offset continuation operator. Synthetic examples demonstrate that the shot continuation method yields shot gathers whose kinematics are correctly predicted. A novel approach for $v(x,t)$ velocity analysis motivates the development of the shot continuation operator.

INTRODUCTION

This paper presents a constant velocity shot continuation algorithm. The new operator extrapolates a shot gather along its shot profile to a different shot location. I develop this operator to facilitate a new velocity $v(x,t)$ analysis scheme.

Velocity analysis

A diffractor in the subsurface scatters the energy of an incident wave. For a constant velocity subsurface, such a diffractor causes a perfect hyperbola in the recorded time section. The hyperbola's apex is at the lateral position of the diffractor independent of the the actual shot location. The hyperbolic shape of the event is caused by the differences in traveltime of rays from the diffractor to individual geophones. Varying the shot location at the surface changes the traveltime of the ray from the shot to the diffractor, shifting the hyperbola vertically without affecting its shape or lateral position.

[1]**email:** not available

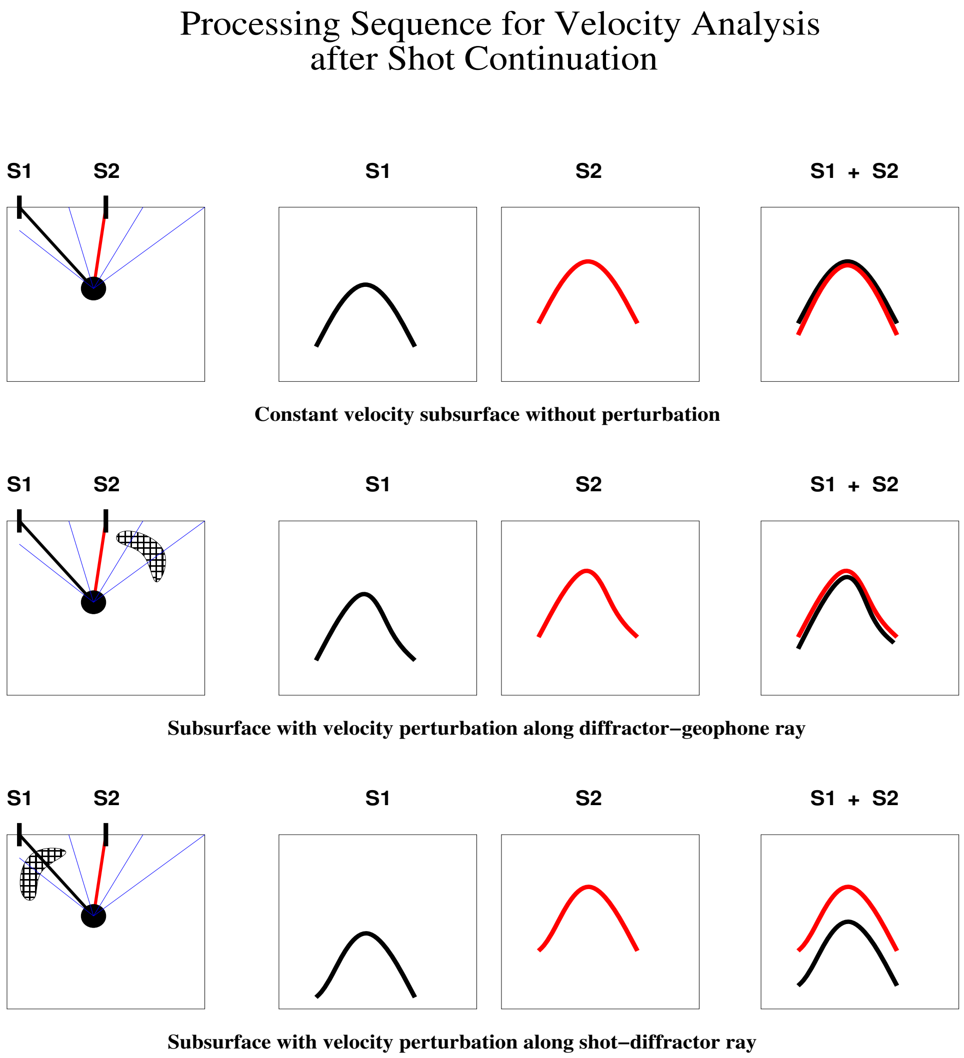

Figure 1: Velocity analysis. The different rows of frames describe the same processing sequence for different velocity cases. In all rows, the left frame depicts a subsurface section, two surface shot positions, and several raypaths. The center frames show the diffractor response in each shot gather. The right frame overlays the shot gather s_1 and s_2 after extrapolation. matt1-cv1 [NR]

The left uppermost frame of Figure **??** depicts schematically the raypaths connected with two shots, s_1, s_2, in a constant velocity medium with a single diffractor. The two center frames of the top row show the two recorded shot gathers. After extrapolation of shot gather s_1 to shot gather s_2 using shot continuation, the hyperbolic events coincide, as it is indicated by overlaying the two gathers in the fourth frame.

In the center row of Figure **??**, a velocity perturbation along geophone rays distorts the hyperbolic moveout in both shot gathers. Since the events in both shot gathers are distorted identically, the shot gathers nevertheless coincide after shot continuation has been applied. In the bottom row of Figure **??**, however, a velocity perturbation along a shot ray causes an additional relative time shift of the two hyperbolic events. After continuation of shot s_1 to shot location s_2, the two events are parallel but shifted in time.

In the case of a more complex subsurface, these shifts are detected by choosing a vertex position, extracting a local hyperbolic window in both equivalent gathers and correlating them by shifting along the time axis. The time shift τ which yields the maximum correlation is stored at the vertex position.

Since only velocity perturbations along the shot-diffractor ray segment cause these τ shifts, a simple first step to velocity inversion is back projecting these τ values along the line between vertex and shot position. Superposing the resulting time sections for different shots yields a map displaying the difference between the actual subsurface velocity distribution in (x, t) and the constant velocity model.

In a velocity variable medium, the outlined velocity analysis scheme only approximates the true velocity. Tests on complex models and real data will have to prove its usefulness.

Dip moveout

The presented derivation of the shot continuation operator resembles Forel's (1988) formulation of his dip moveout (DMO) operator. This close link promises a potentially swift improvement of the shot continuation method applying the experience gained in research concerning offset continuation and dip moveout. Especially, investigations into amplitude properties of the offset continuation (Deregowski and Rocca, 1981) and dip moveout operator (Black et al., 1992; Blondel, 1993) should help to refine the amplitude behavior of the shot continuation operator.

Agenda

In the following section, I compare some extrapolated shot gathers with shot gathers actually recorded at these positions. Then I outline the computational algorithm of the shot continuation program. Finally, I derive the operator from basic traveltime considerations. The Mathematica script in the appendix 'automatically' derives the operator symbolically. Thereby, it documents reproducibly the mathematical derivation of the continuation operator, which otherwise will only be briefly sketched.

EXAMPLES

Having recorded a shot gather at shot location s_1, a geophysicist could ask himself what he would have recorded laying out a slightly different shot gather with shot location s_2. The process introduced in this paper provides an answer by shifting a given shot gather s_1 along the profile line assuming a constant velocity subsurface.

Figure **??** depicts a simple subsurface model comprising two horizontal layers, one dipping layer, and two diffractors.

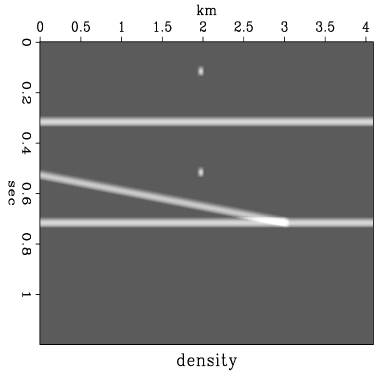

Figure 2: Input subsurface model. The model is used for modeling shot gathers. The geophones are at fixed surface locations, but the shot positions vary. matt1-refl [ER]

The left and middle frame of Figure **??** show the result of a simple Kirchhoff modeling corresponding to shots at surface positions $s_1 = 2.048km$ and $s_2 = 2.096km$. The geophone locations are identical in both shots. The third frame shows the output of the shot continuation process after shifting shot s_1 to the location of s_2. If you press the pushbutton in the electronic document accompanying this report, a movie illustrates the success of the shift: while the screen switches rapidly switching between the two original shot gather frames, the reflections from the horizontal layers move sideways and the hyperbolae corresponding to the diffractor move vertically. In comparison, the events do not move when the movie alternates between the continued shot gather and the shot gather s_2 recorded at that location.

Figure **??** is based on the same subsurface model as the previous shot gather set (Figure **??**). A more sophisticated finite difference code modeled the shots using a Ricker source wavelet and including not only reflections but also a direct wave and multiples. Even having primarily been designed to handle reflection hyperbolae correctly, the shot continuation algorithm succeeds in shifting more complex gathers.

The nearly vertical lines in the shifted shot are artifacts of the operator. As I discuss in the paragraph 'Operator', I believe them to be connected with the special case of reciprocal raypath. Furthermore, the subroutine computing the amplitude along my operator requires an aperture cut off, which is manifested by the disappearance of the flanks of the earliest hyperbola.

The shifted shot profiles show a wedge of zero data at the right bottom corner. This wedge contrasts with the strong arrival of the two deeper layers in shot s_2 at this position. The missing event is readily explained by observing that the original shot gather s_1 does not include reflections from the reflector points at the far right of the model. Since the shot location s_1 lies to the left of shot location s_2, reflections from the far right of the model just arrive within s_2's recording window, but are not recorded in the shot gather s_1. In general, the continued shot only includes reflections from the subsurface illuminated by both shots.

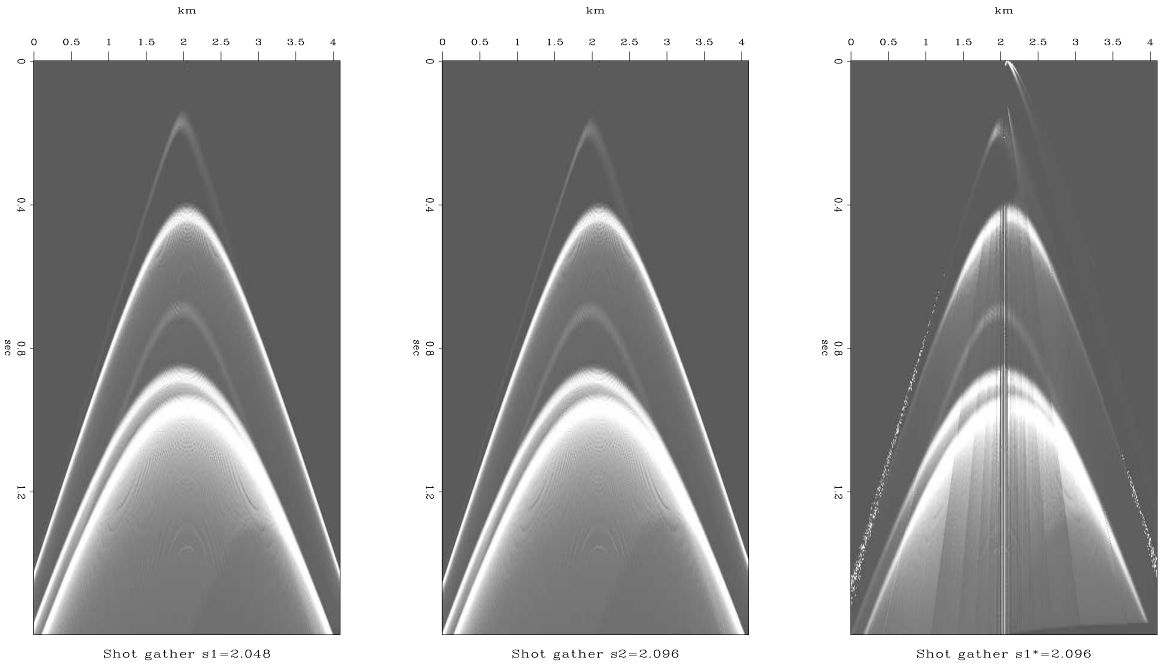

Figure 3: Shot Gathers. A shot at $2.048km$ generates the left gather, a shot at $2.096km$ the middle gather. Shot continuation extrapolated the shot at $2.048km$ to the shot location at $2.096km$. The hyperbolic events in the extrapolated gather at the right correspond to the middle frame's events recorded at this location. matt1-shfkr [CR]

THE OPERATOR

The shot continuation operator constructs a shot gather s_1 from a neighboring shot gather s_2. Both gathers are to be equivalent in the sense that both data sets result in the same subsurface picture after migration.

Traveltimes

For a given impulse at (h_1, t_1) the length of the raypath that connects the shot and receiver with any possible reflector point is constant, in a constant velocity medium.

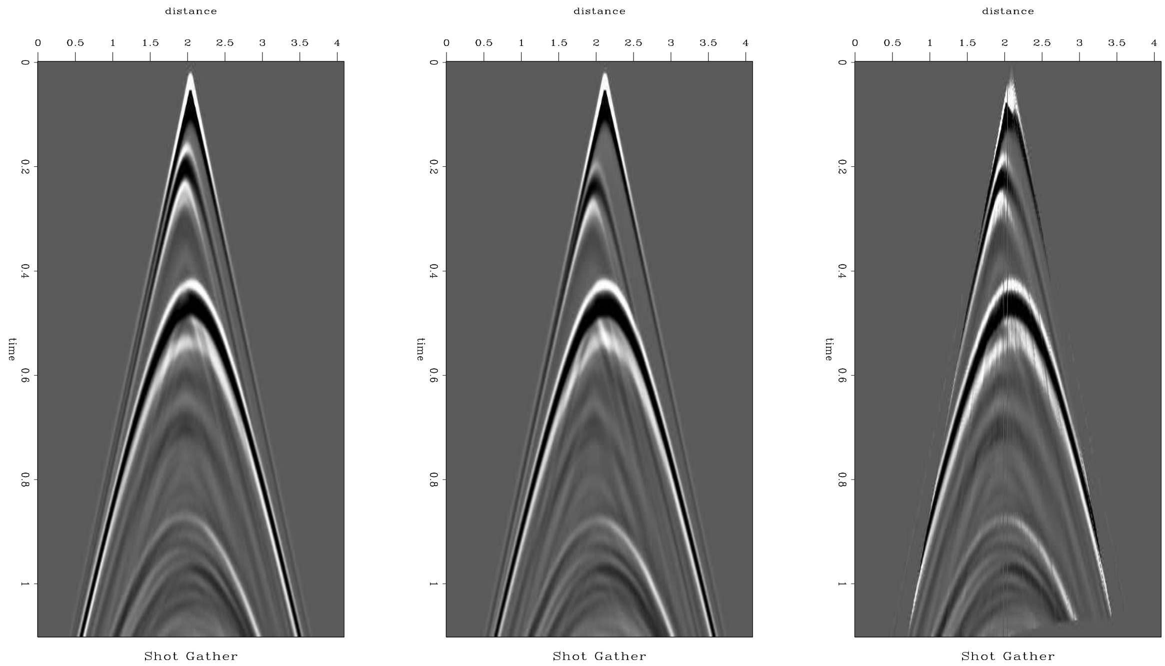

Figure 4: Shot Gathers. These shots were modeled by a more sophisticated program. Despite the direct wave and the multiples, the subsequent shot continuation extrapolates the left shot correctly. matt1-shffd [CR]

The locus of reflector points constitute an ellipse, the foci of which coincide with the source and receiver location:

$$0 = \frac{x^2}{a_1^2} + \frac{z^2}{b_1^2} - 1. \tag{1}$$

The lengths a_1, b_1 of the major and minor axis are defined as

$$a_1^2 = \frac{V^2\, t_1^2}{4} \tag{2}$$

$$b_1^2 = a_1^2 - h_1^2\ , \tag{3}$$

where V refers to the constant subsurface velocity.

Similarly, the migration of an impulse at $(t2, h2)$ in shot gather s_2 yields an ellipse of the form

$$0 = \frac{(x+d)^2}{a_2^2} + \frac{z^2}{b_2^2} - 1, \tag{4}$$

where

$$a_2^2 = \frac{V^2\, t_2^2}{4}, \tag{5}$$

$$b_2^2 = a_2^2 - h_2^2. \tag{6}$$

The variable d denotes the difference between the midpoints of the two ellipses:

$$d = (s_2 + h_2) - (s_1 + h_1). \tag{7}$$

Extrapolation of shot gather s_1 requires the construction of a shot gather s_2 which when migrated yields the same subsurface image as the migration of the s_1 impulse: the ellipse of (1). Such a shot gather s_2 consists of a family of events, migrated impulse responses of which are tangent to the s_1 ellipse (1).

When shot gather s_2 is migrated, this tangency condition provides constructive superposition along the original isochron of s_1 (1). Anywhere else the events will cancel each other.

The tangency of the ellipses, (1) and (4), requires that the ellipses coincide and have equal slope at some point (x, z). Differentiation of equations (1) and (4) with respect to x and elimination of dx/dz yields an expression for x:

$$d = (1 - \frac{a_2^2\, b_1^2}{a_1^2\, b_2^2})\, x. \tag{8}$$

After multiplication of equations (1) by b_1^2, and (4) by b_2^2, z is eliminated by subtracting the two equations. Substitution of x from equation (8) yields

$$d^2 = \frac{b_1^2 - b_2^2}{b_1^2\, b_2^2}\, (a_1^2\, b_2^2 - a_2^2\, b_1^2). \tag{9}$$

Finally, replacement of a_2, b_2 by expressions (5) and (6) provides the desired quadratic relation for t_2^2:

$$0 = a\, t_2^4 + b\, t_2^2 + c, \tag{10}$$

where

$$a = a_1^2 - b_1^2 \tag{11}$$

$$b = -a_1^2\, b_1^2 + b_1^4 + b_1^2\, d^2 - 2\, a_1^2\, h_2^2 + b_1^2\, h_2^2 \tag{12}$$

$$c = a_1^2\, b_1^2\, h_2^2 - b_1^2\, d^2\, h_2^2 + a_1^2\, h_2^4. \tag{13}$$

In the derivation above, transforming equations (8) to (9) requires a division by $a_1^2\, b_2^2 - a_2^2\, b_1^2$. If

$$0 = a_1^2\, b_2^2 - a_2^2\, b_1^2 \tag{14}$$

then it follows from equation (8) that the midpoints of the ellipses (1) and (4) coincide ($d = 0$). Because of this symmetry, the point of tangency lies at the hyperbolae's minor vertex. Since the lengths b_1, b_2 of the ellipses' minor axes are equal, condition (14) requires that also the major axes of the two ellipses agree: the two ellipses have to be identical. Hence in the case of identical or reciprocal positioning of geophone and source, the original impulse at (t_1, h_1) does not generate a set of events in shot gather s_2, but results in an identical impulse at (t_2, h_2).

Note, that in the case $b_2 = a_2 = R$ the second ellipse (4) collapses to a circle. The relationships described in equation (8) and (9) then describe the partial prestack migration to zero offset as described by Forel [2] (1988).

The Appendix discusses a Mathematica script which derives this result 'automatically'. This Mathematica script starts from the basic ellipse equations and performs the transformations as they are described above.

Amplitudes

The previous paragraph derived the kinematic behavior of the shot continuation operator, but not its amplitude properties.

As a first amplitude approximation I chose an operator amplitude proportional to the local operator curvature. It should be possible to improve the amplitude accuracy by applying approaches similar to recent investigations into amplitude properties of the offset continuation and dip moveout operator (Black et al., 1992).

Impulse response

The form of the shot continuation operator varies dramatically depending on the distance the shot is shifted. For illustration, Figure **??** depicts a shot gather s_1, which is shifted various distances. The original shot gather contains a single event at $(t_1 = 1.0, h_1 = 0.25)$ and it is shifted to s_2 at the center of the geophone spread. The distance shifted varies from frame to frame as the shot location of s_1 advances from the right to the left of the geophone spread.

Each thin line in Figure **??** represents the kinematic solution as it is derived and rendered by Mathematica. The background raster plot is computed by the Ratfor routine `shct.rst` incorporating amplitude variations along the operator.

The operator is a symmetric DMO-like smile when s_2 coincides with the midpoint associated with the input impulse at (t_1, h_1). However, the operator skews sideways

[2]The presented derivation is essentially a generalization of Forel's DMO derivation.

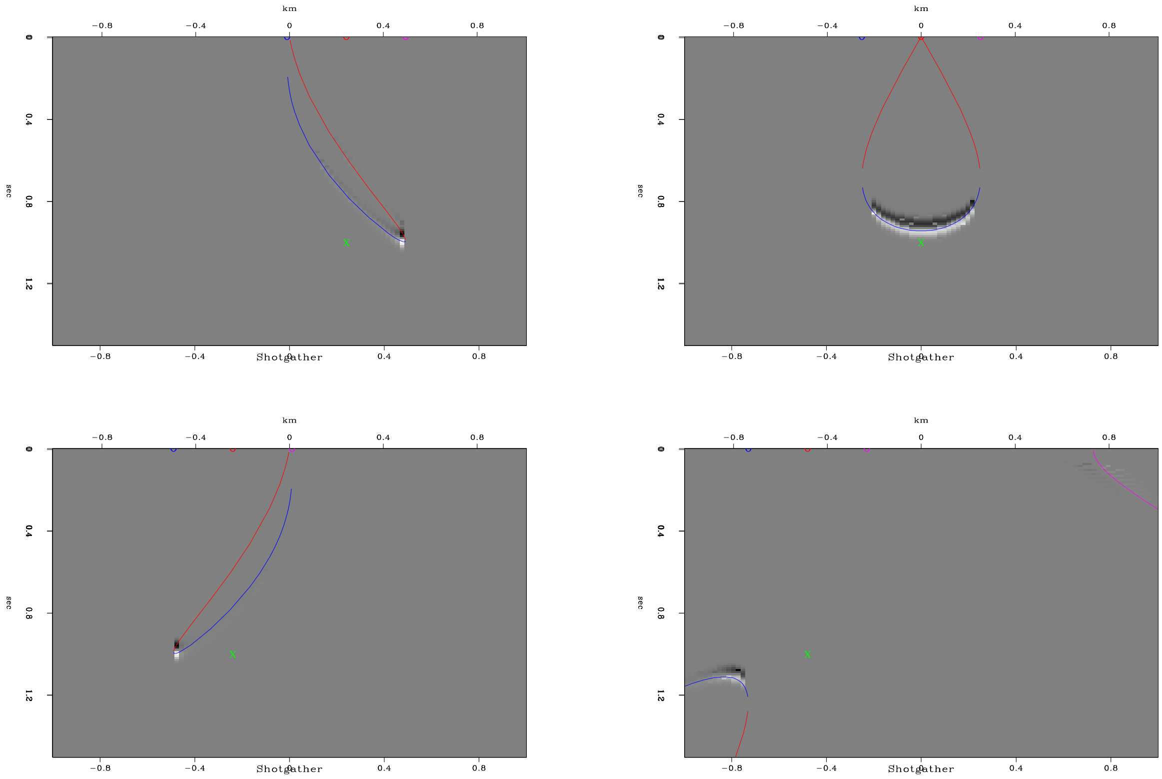

Figure 5: Impulse Response. The kinematic operator derived by the Mathematica script **smile.ma** agrees with the raster-plot output of the corresponding Ratfor program **shct.rst**. The operator varies strongly with the relative position of the original and the extrapolated shot. The impulse response collapses to a point when the original and extrapolated shot position is identical or reciprocal. matt1-shct [CR]

and finally collapses to a point as the shot location s_1 or the geophone g_1 approaches s_2. Beyond the identical and reciprocal shot position, the operator turns into a frown. In addition to the variations induced by varying shift distances, the operator also varies with different impulse event times t_1.

Algorithm

The algorithm which shifted the shot gathers in the previous paragraph computes the roots of a simple quadratic expression while looping over the samples of the input shot gather and the geophone locations of the output shot gather. For each sample at time t_1 and offset[3] h_1 in the original gather s_1, a_1, b_1 are computed according to equations (2) and (3). To construct the shot gather s_2 the algorithm loops over the offset h_2 computing t_2 by solving the quadratic polynomial (10). Since our implementation `shct.rst` of this algorithm allows different layout parameter for the input and output shot gather, the shot continuation qualifies as a binning operator.

DISCUSSION

The examples shown in this article demonstrate that the shot continuation scheme shifts shots successfully to different locations along the shot profile. Reflection events in an extrapolated shot gather agree kinematically with events recorded at the location.

The comparison of two shots after extrapolating them to the same location, offers new approaches to exploit seismic information. For example, discrepancies in the source wavelets of the two shots could be analyzed and resolved by a two-dimensional deconvolution. Kinematic differences in the two equivalent shot gathers are probably related to perturbations from the assumed constant velocity model. In the introduction, I explained how the correlation of extrapolated shot gathers can be used for velocity analysis. Since the geometry of the shifted shot gather can be freely chosen by the processor, the operation might additionally provide a tool for regularizing or binning seismic field data.

But before many of these applications can be successfully implemented, the amplitude of the shot continuation impulse response has to be improved, following the approaches utilized in the derivation of a sophisticated offset continuation operator.

Generalizing the shot continuation operator to variable velocity media will probably prove to be as difficult a task as generalizing the dip moveout operator. But as in DMO, a constant velocity approximation should be acceptable for a wide range of applications.

[3] The offset is directional along the shot profile. Negative offsets refer to one side of the shot,positive offsets to the other side.

ACKNOWLEDGMENTS

Fabio Rocca deserves to be an author of this paper. He originally derived a shot continuation operator for small dips and suggested it as a tool for velocity analysis. I extended the operator to all dips and implemented the ideas in Mathematica and Fortran. Additionally, I thank Martin Karrenbach and Steve Cole for their constructive discussions. Converting Mathematica's Graphics primitives to vplot would have not been possible without Martin Karrenbach's superb shell programming skills.

REFERENCES

Abma, R., 1992, Seismic data processing with Mathematica: SEP–**73**, 73–96.

Black, J. L., Schleicher, K. L., and Zhang, L. ., 1992, True-amplitude imaging and dip moveout: Geophysics, **58**, 47–66.

Blondel, P., 1993, Constant-velocity anti-aliasing three-dimensional integral dip moveout: SEP–**77**, 49–58.

Cole, S., and Nichols, D., 1992, Xtpanel: An interactive panel builder: SEP–**75**, 497–520.

Cook, D. M., Dubisch, R. ., Sowell, G. ., Tam, P., and Donnelly, D., 1992, A comparison of several symbol-manipulating programs: Computer in Physics, **6**, no. 5, 411–419.

Deregowski, S. M., and Rocca, F., 1981, Geometrical optics and wave theory of constant offset sections in layered media: Geophys. Prosp., **29**, no. 3, 374–406.

Forel, D., and Gardner, G. H. F., 1988, A three-dimensional perspective on two-dimensional dip moveout: Geophysics, **53**, no. 5, 604–610.

Karrenbach, M., 1993, How to organize an interactive document: SEP–**77**, 417–426.

APPENDIX A

SYMBOL-MANIPULATING COMPUTER LANGUAGES

Introduction

Besides establishing a theorem's correctness, mathematical communication serves to illustrate and motivate its application. Often, the reader of a mathematical proof may believe its soundness but studies the chain of arguments to clarify its mechanism and its implications. However in applied sciences, like geophysics, theorems are usually further supplemented by relevant applications of the derived algorithm. In complex applications, adequate examples are often considered more convincing than dogmatic proof or theory.

In his publication's Claerbout (Karrenbach, 1993) automates the reproduction of discrete seismic processing results using a vast array of computer and program tools. This reproducibility safeguards that no step in the data processing goes undocumented. Additionally, Claerbout's true multimedia environment allows users to change the input parameter or the code to study the algorithm's characteristics. Especially since the introduction of Xtpanel (Cole and Nichols, 1992), the interactivity enables the audience to learn about an algorithm heuristically by easily supplying their own test cases.

Contrary to the production of illustrating data examples, Claerbout publishes only outlines of his theorem's mathematical derivations. Most of of these theorems are easily understood, but potentially the verification of the published argument chains can be time consuming and difficult.

Previous work (Abma, 1992; Cook et al., 1992) suggested symbol-manipulating computer programs, like MACSYMA, Maple, Mathcad, Mathematica, or Theorist, as convenient research environment for small scientific problems.

In this appendix, I demonstrate how symbolic mathematical computer languages (SMCL) can document and reproduce mathematical discussions. Integrating mathematical derivations, they offer tools to extend Claerbout's concept of reproducibility upstream from computer algorithms.

Reproducible mathematics

Symbolic mathematical computer languages, like MACSYMA, Maple, Mathcad, Mathematica, or Theorist, usually comprise a symbol-manipulating language, numerical routines, a general purpose programming language, graphics tools, and a user interface which incorporates text. In the following paragraph, I annotate the Mathematica script `smile.ma`, which derives the mathematical results outlined in the paper above.

The script starts by defining the ellipses (1) and (4) in `eq1` and `eq2`:

```
eq1 = x^2 / a1^2 + z^2 / b1^2 == 1 ;
eq2 = (x-ds)^2 / a2^2 + z^2 / b2^2 == 1 ;
```

Next, I apply the tangency conditions. First, I eliminate z from `eq1` and `eq2` and yield `eq3`:

```
eq3 = Eliminate[{eq1,eq2},z][[3]] ;
```

I then define the parameter of `eq1` and `eq2` as independent of x, take the derivative of both equations and finally eliminate the slope `p`. The resulting equation `eq4` corresponds to equation (8) in the article:

```
a1/: Dt[a1,x] = 0 ;
a2/: Dt[a2,x] = 0 ;
b1/: Dt[b1,x] = 0 ;
b2/: Dt[b2,x] = 0 ;
ds/: Dt[ds,x] = 0 ;
d1x = Dt[eq1,x] /. Dt[z,x] -> p ;
d2x = Dt[eq2,x] /. Dt[z,x] -> p ;
eq4 = Eliminate[{d1x ,d2x},{p,z}][[1,3]] ;
```

Combination of the two tangential conditions, `eq3, eq4`, eliminates x from the equation system, and yields equation `eq5`, which is represented by equation (9) in the article:

```
eq5 = eq3 /. Solve[eq4, x][[1]] ;
```

After introducing the definition (6) of b_2 in `eq9`, the program rearranges the terms by operating on both sides of any equation simultaneously. Finally, equation `eq10` represents the desired quadratic polynomial (10). Mathematica then isolates the coefficients of the polynomial, which correspond to expression (11), (12), (13) in the paper above. They are the same coefficients implemented in the Ratfor program `shct.rst`:

```
den = (a2^2 b1^2 - a1^2 b2^2) ;
eq6 = Thread[Simplify[Plus[-(ds^2 + 2 a1^2 b2^2 ds^2 / den),eq5]], Equal];
eq7 = Thread[Simplify[Times[- den b2^2 / a2^2 ,eq6]],Equal] ;
eq8 = Thread[Simplify[eq7],Equal] ;
eq9 = Thread[ReplaceAll[eq8,b2 -> Sqrt[a2^2 - h2^2]],Equal] ;
eq10= Thread[Collect[ExpandAll[eq9],a2],Equal] ;
(* The polynomial *)
ex1 = Simplify[Coefficient[eq10[[2]],a2,0]] ;
ex2 = Simplify[Coefficient[eq10[[2]],a2,2]] ;
ex3 = Simplify[Coefficient[eq10[[2]],a2,4]] ;
eq11= 0 ==  ex3 as^2 + ex2 as + ex1 ;
```

The rest of the mathematica script `smile.ma`, which I will not discuss here, illustrates the shot continuation operator by computing its impulse responses for a series of cases (see Figure **??**).

The script is executed by piping it into the mathematica start command 'math', or by calling it from within Mathematica. The semicolon behind an expressions prevents Mathematica from printing the value of this expression on the screen. Without the semicolon mathematica renders this line's intermediate result during execution.

Discussion

Potentially, such an homogeneous research medium is indeed very powerful: It offers a homogeneous, integrated software environment for scientists. Reproducibility and interactivity are usually easily achieved and very transparent. Integrating mathematical derivations, they offer tools to extend Claerbout's concept of reproducibility upstream from computer algorithms.

Naturally, today's symbol-manipulating languages exhibit shortcomings in particular areas when compared to more dedicated systems. Especially, the shear size of a typical geophysical problem exhausts the capability of these programs. It is the strength of Claerbout's multimedia approach to flexibly incorporate symbol-manipulating programs, as I have in this case of shot continuation.

Since the packages are commercial products the reproducibility and interactivity is restricted to owners of a software license. Yet, some of these languages are accompanied by a freely distributed 'reader' program, which permits the reproduction of a result, but disables interactivity.