Stanford Exploration Project, Report 77, November 17, 1997, pages 325–343

Relating seismic measurements on different scales

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keywords: *velocity, diffraction, NMO, elastic*

ABSTRACT

Measurements carried out at different scales may not agree in their observations. An equivalent medium approach can link measurements carried out at different scales. I apply high frequency (Dix) and low frequency (Schoenberg&Muir) averages to a well log and compare the results to a conventional velocity analysis of surface seismic data.

INTRODUCTION

Each physical experiment of finite size and duration carried out in nature implies a certain scale over which the measurement takes place. The actual value of the scale is determined by the discrete sampling of a natural phenomenon. For surface seismic measurements, the time scale is determined by the length of recording time. The frequency scale is determined by the temporal sampling interval. Both scales are related and dual to each other. They serve as an upper limit of observable scales. What scale is actually observed depends on the experiment itself; for example one factor determining the experiment is the frequency content of a seismic source radiating energy into the subsurface. These scale dimensions can vary to a limited degree, depending on the type of seismic experiment. Given that the subsurface structure can be heterogeneous on any scale, the spatial correlation of material properties is directly related to the scale of heterogeneities. When elastic waves propagate in the subsurface, the subsurface velocity and the given temporal frequency determine the spatial wave number locally at any given point in space. The wave equation describing the propagation, couples spatial and temporal resolution.

Imagine a high frequency seismic surface source which radiates energy into the subsurface. Let frequency and bandwidth go to infinity and let the spatial sampling interval shrink infinitely small, then reflected energy could be recorded and resolved from infinitely small scale obstacles. Lowering the frequency content of the source and increasing spatial sampling intervals only enables us to record wave propagation effects (reflection, diffraction) at a larger scale. Thus we can only hope to resolve ob-

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stacles (layers, diffractors) properly, that are of the order of that minimum/dominant wave length. However, if there are material discontinuities, which are much smaller in scale than our minimum/dominant wavelength, then a wave, while propagating through a medium, will "feel" some average effect. Folstad et al. (1992) showed synthetic comparisons of fine and large scale wave propagation effects, using a Backus averaging of elastic constants. Thus, for a given finite resolution, we can only measure the properties of the "equivalent medium" at that scale. Small scale variations still influence effects at a larger scale, but in some averaged sense. I introduce one application of equivalent medium theory in form of homogeneous overburden replacement. Using such a group-theoretical approach, I find NMO (or focusing) velocities by calculating an anisotropic elastic replacement medium and carrying out small offset approximations along the vertical and horizontal axes. I compare this approach (low frequency) to a Dix rms velocity calculation (high frequency).

THE EFFECTIVE MEDIUM

Dix defines an average medium in the infinite frequency/bandwidth limit, thus ending up with a paraxial approximation, the rms velocity. His exposition is based on scalar-wave traveltimes and is purely kinematic. Let this be the "ray average", because rays see well defined interfaces in the medium. Schoenberg&Muir (Schoenberg and Muir, 1989) on the other hand are operating at the very low frequency (long wavelength) spectrum of observations. Within this limit, their approach lends itself to anisotropic media and even dipping interfaces. Their theory can be cast in terms of an equivalent medium scheme that preserves well defined properties. It extends beyond kinematics. Let this be the "wave average", because it is more closely related to basic elastic quantities, namely stress and strain. In reality, observations lie in a frequency regime that might be closer to one or the other frequency limit. Here I compare both schemes.

TYPICAL SEISMIC EXPERIMENT SCALES

Seismic measurements at different scales have different objectives and may result different observations. However, the measured property still pertains to a unique medium. Thus in theory it should be possible to relate differently scaled measurements to one another. In the past, there seems to have been a lack of success in this regard. Typically, NMO velocities from surface seismics do not tie well with sonic logs. This has been a long recognized problem. For example, check shots are used to stretch the seismic log to a surface seismic scale. I assume that velocity varies not to drastically over the given frequency interval, such that velocity dispersion will not be accounted for. However, equivalent medium schemes suggest ways how to relate measurements carried out at different scales. Additionally one knows exactly which quantities are preserved in the rescaling process.

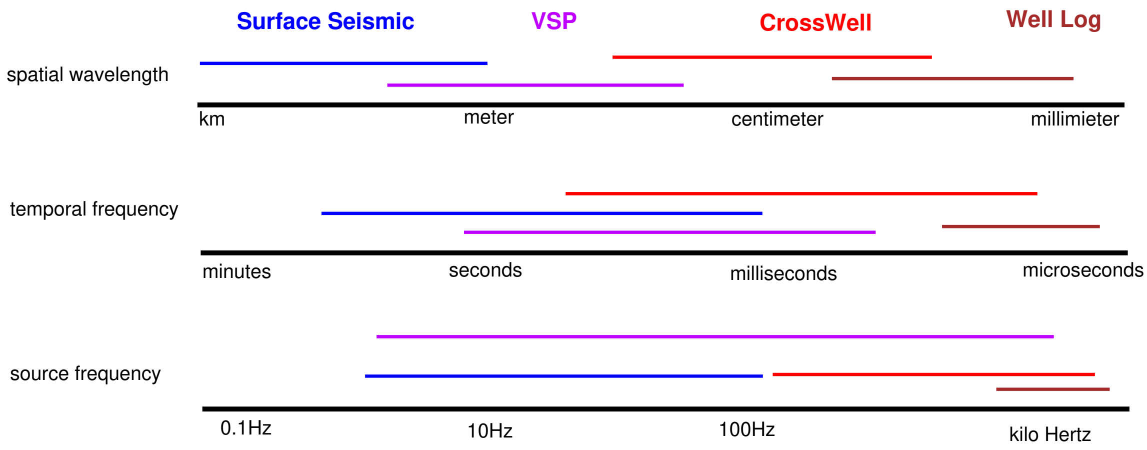


Figure 1: Schematic display of different types of measurements with spatial and temporal frequency. There can be a fair amount of overlap. martin1-cartoon [NR]

Surface seismic

The scale length in surface seismic measurements is generally in the range of kilometers or tens of meters. For exploration applications a typical scale length of tens/hundreds of meters is reasonable. The temporal scale is of the order of a few seconds, while the source frequency is usually between 1-100 Hz.

VSP

VSP experiments can show a large overlap with surface seismics; on the other hand the spatial scale typically ranges from meters down to decimeters. This includes Reversed VSPs. Depending on the type of source, surface source or bore hole source, the temporal frequency scale ranges from surface seismic values down to cross well values.

Cross Well

In Cross Well measurements scale overlaps are likely with VSP but not with surface seismics. On the other hand overlap with well log values is possible. This holds for temporal and spatial frequencies as well and stems from distances being in general smaller. The usable source frequency can be varied extensively.

Well Log

Here we consider well logs as being at the lowest spectrum of scales. In general there can be scale overlap with Cross Well measurements, but it is very unlikely that there is overlap with any other of the listed experiments. Obviously the most challenging

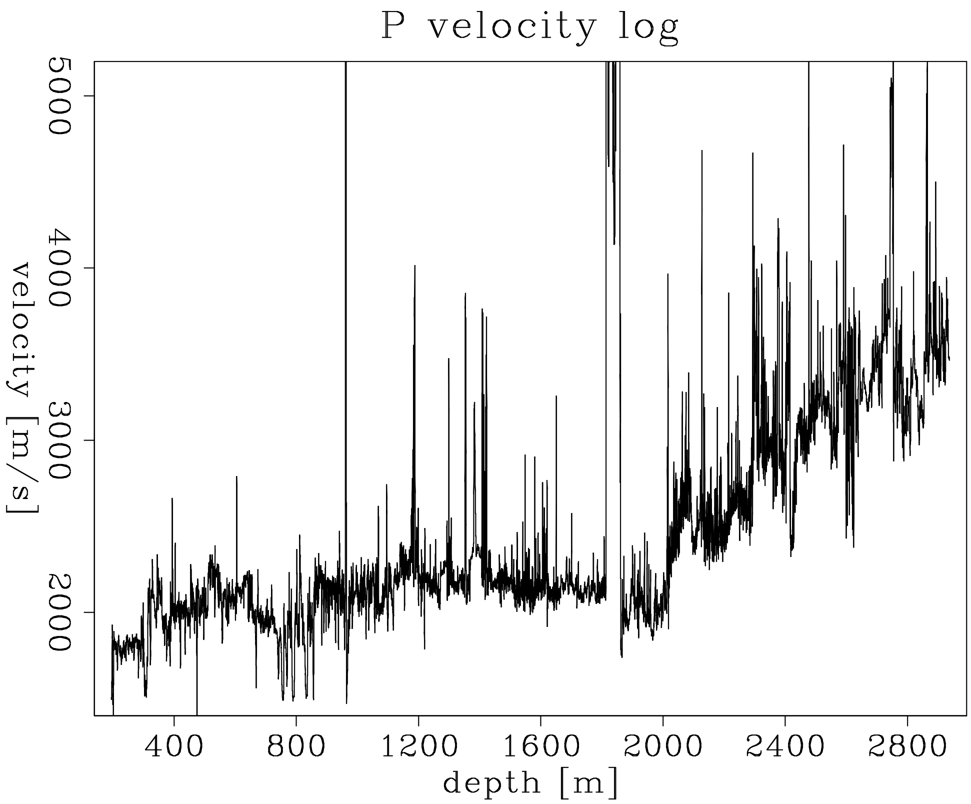


Figure 2: A well log, showing the P wave sonic velocity with depth. martin1-velp [CR]

task is then to relate well log with surface seismic measurements. Fig 2 and 3 show stacking velocity and sonic log obtained at the same surface location. We can clearly see how experimental scale differences can lead to quite different results in resolution and values, although they both measure the same medium. This well-known problem exists when comparing borehole sonic logs and surface seismic velocity estimations.

ASSUMPTIONS

All averaging schemes discussed here are based on a layered earth model. The S&M approach is valid in the long wavelength limit, where the layer thickness is much smaller than a wavelength. This approach averages horizontal dipping layers and comes up with an homogeneous equivalent medium. It is essentially a low frequency wave averaging scheme. Dix on the other hand applies to infinite frequency and bandwidth. It is essentially a ray averaging scheme. Obviously surface seismic data lie in between those two extremes, potentially with some bias towards the low frequency end. I compare both schemes and will outline its possibilities and pitfalls.

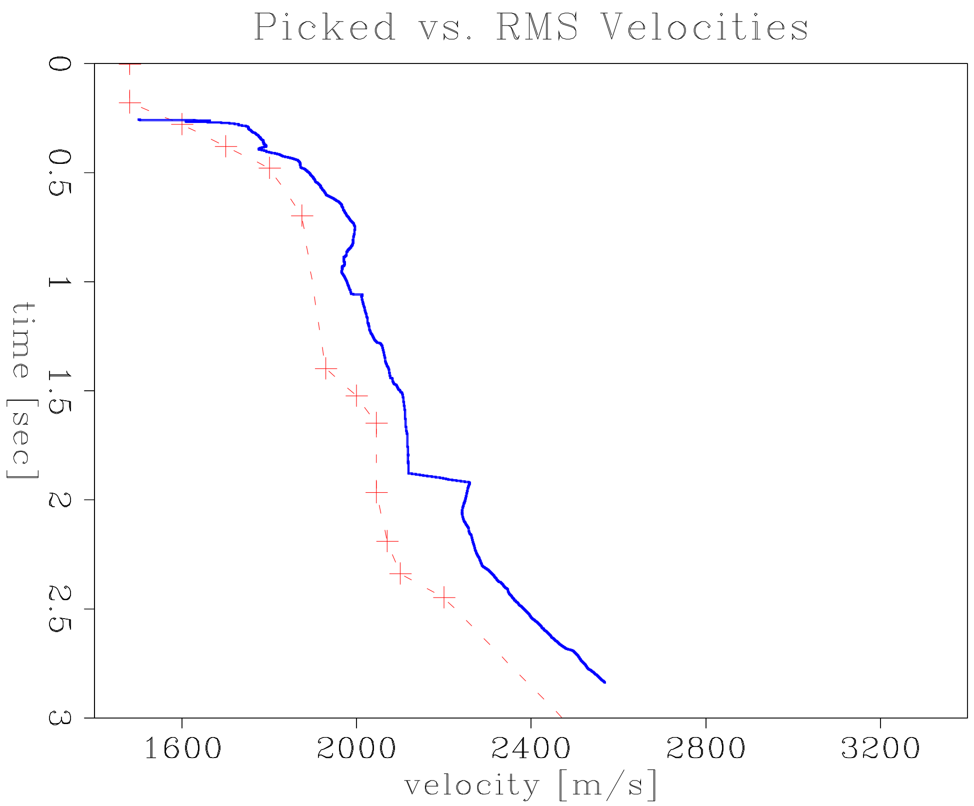


Figure 3: The stacking velocity (dashed line) picked from a conventional velocity analysis is compared to the rms velocity (solid line) calculated from the original P wave sonic in Fig 2. martin1-pickrms [CR]

METHODOLOGY

Given the premise that conventional sonic logs measure vertical travel time in the formation, we make the assumption that the elastic properties of the layer are isotropic. This may or may not be the case, but could only be determined with more additional measurements. An rms velocity calculation following Dix, uses that vertical velocity to come up with an NMO velocity. This velocity is isotropic (unless extended to anisotropy as shown by (Dellinger and Muir, 1993)). The S&M averaging works with hybrid matrices of stiffness and compliance. Thus it requires P wave and S wave velocities and density in a layer to be known. If we again make the assumption of an isotropic layer, these three parameters determine the elastic properties uniquely. However, using the S&M average, the resulting average layer, may or may not be isotropic. This is in contrast to the conventional Dix average and will manifest itself in differences between vertical velocity (depth mapping) and Move-out velocity (NMO velocity).

Nowadays, P and S wave sonics, and density measurements are more commonly recorded than in the past, where only P wave sonics and density were available. If

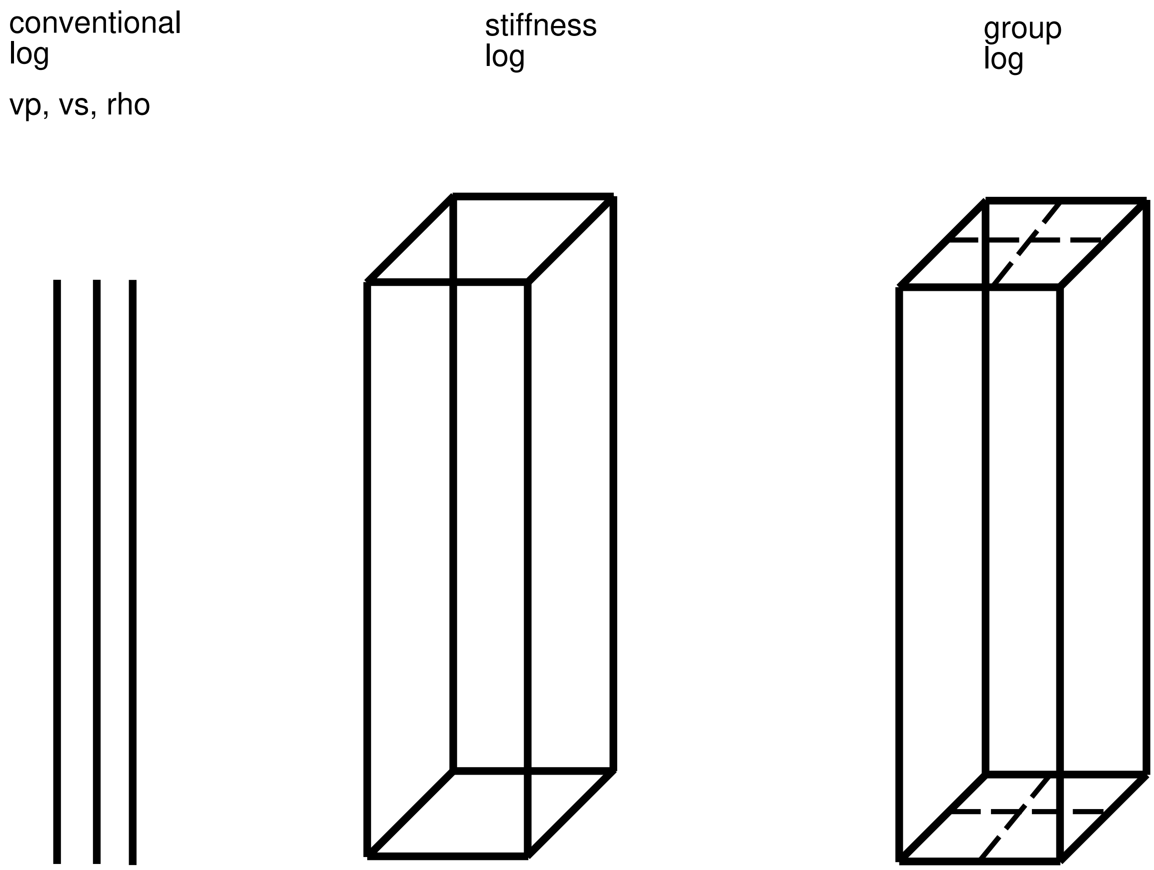


Figure 4: This schematic drawing illustrates the steps in generating the "elastic log". Assuming isotropy, stiffnesses can be calculated from v_p,v_s and ρ. The S&M group transformation rearranges the stiffness log in a hybrid matrix of stiffness and compliance. The result is "group log". martin1-groupcartoon [NR]

we do not have a shear wave log available, we would need to estimate one from other auxiliary information. In the examples in this paper I assumed a constant pressure/shear wave velocity ratio. Which is clearly in error in the target zone since the reservoir will have a different Poisson ratio than the embedding medium.

Some well logs include a dipmeter measurement, giving an estimate of the spatial orientation of a layer. The dip of a plane layer can be one more possible parameter in the S&M average, It has the effect that the coordinate system in which the average is carried out is rotated appropriately. Since the S&M average is based on the elastic properties, it can readily incorporate any information that might be possibly measured in the future, like velocities at different propagation angles and orientations. Fig. 4 illustrates the "elastic log" used during averaging. Having a reasonable method of scaling well log information up to surface seismic measurements may help in constraining stacking velocities during velocity analysis at points close to a seismic line. it may also help in estimating velocities for depth mapping (in general focusing and depth mapping velocities are not identical). In this paper I base my analysis on stacking velocities, but the application to focusing and depth mapping is straight forward extension.

AVERAGING THE LOG

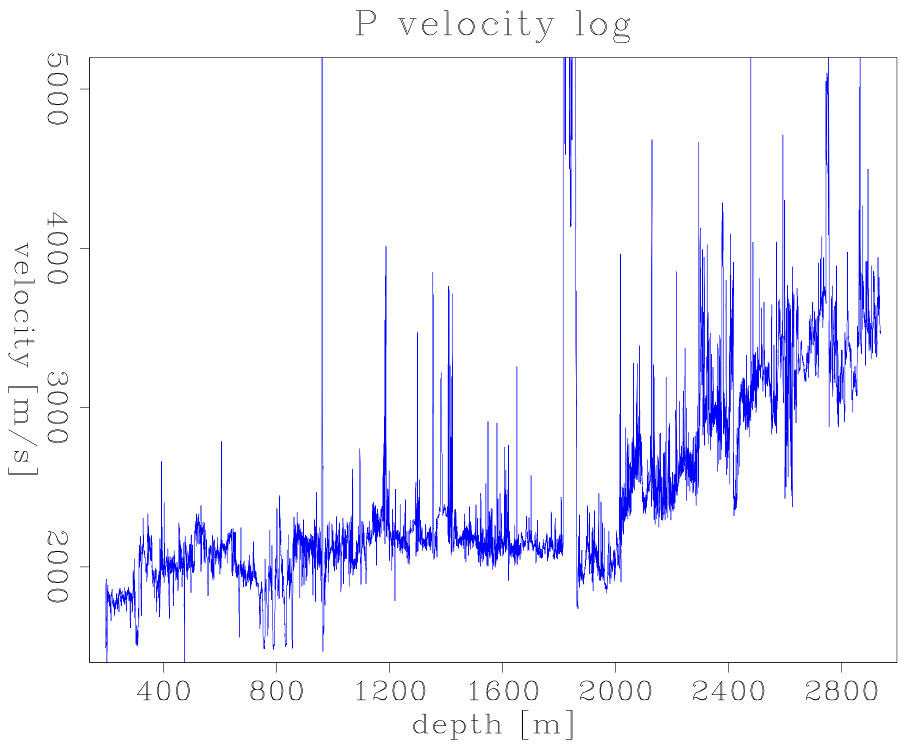
The justification for smoothing well logs using the S&M calculus is that we want to relate to surface seismic measurements by replacing the overburden with an equivalent homogeneous elastic medium. When using a smoothing method for "relating" well logs to surface seismic measurements, the length of the smoothing filter is the crucial parameter. One choice is to smooth over the smallest possible wave number determined by the surface method. This is the shortest wavelength we will see locally when a wave is propagating through a medium. We can average over that wave length with the S&M layer addition scheme, where average stress and strain components are calculated, which characterize the effective medium on a larger scale. It is important to notice that we are not smoothing traveltimes directly (like Dix more or less does), but rather assume that the measured traveltimes represent the vertical traveltimes only.

Assuming further isotropic layers (this is not necessary, if we have more information) the P and S wave sonic log together with the density log specify an isotropic stiffness matrix. For each well log point in depth, these measurements are transformed into the group domain (S&M, 1990). The important point is that all the smoothing is carried out in the group domain.

$$v_p, v_s, \rho \leftrightarrow c_{ijkl} \leftrightarrow g \leftrightarrow \text{Operator} \leftrightarrow g^* \leftrightarrow v_x, v_z, v_{xnmo}, v_{znmo}, ... \tag{1}$$

Equation (1) schematizes the steps from a conventional log to an "in some sense" equivalent log. The original log is converted to elastic stiffnesses c_{ijkl}, which in turn can be transformed in the S&M group domain, whose elements are g. An arbitrary operator can be applied in the group domain and the resulting group elements g^* can be converted into quantities relating to a particular wave type and propagation direction. Here I just list a few: $v_x, v_z, v_{xnmo}, v_{znmo}$. Since the S&M group transformation is a linear operation, we can apply any smoothing filter or other transform (Fourier transform, etc.) in this domain. All steps in equation (1) can be reversed, as long as the operator itself is reversible.

It is not readily clear what the best smoothing operator is. I chose to use a boxcar smoothing operator. After smoothing the data in the group domain, the resulting effective medium will in general not be isotropic, but instead transversely isotropic. A useful elliptic approximation, as derived by (Muir, 1991), introduces in this way an effective NMO velocity. Figure 5 shows the ingredients for generating the isotropic stiffness log. From this stiffness log the S&M group transformation is applied resulting in a group log. Using this group log an equivalent medium can be calculated by partially integrating all the layers up to a certain depth point in the log, given by



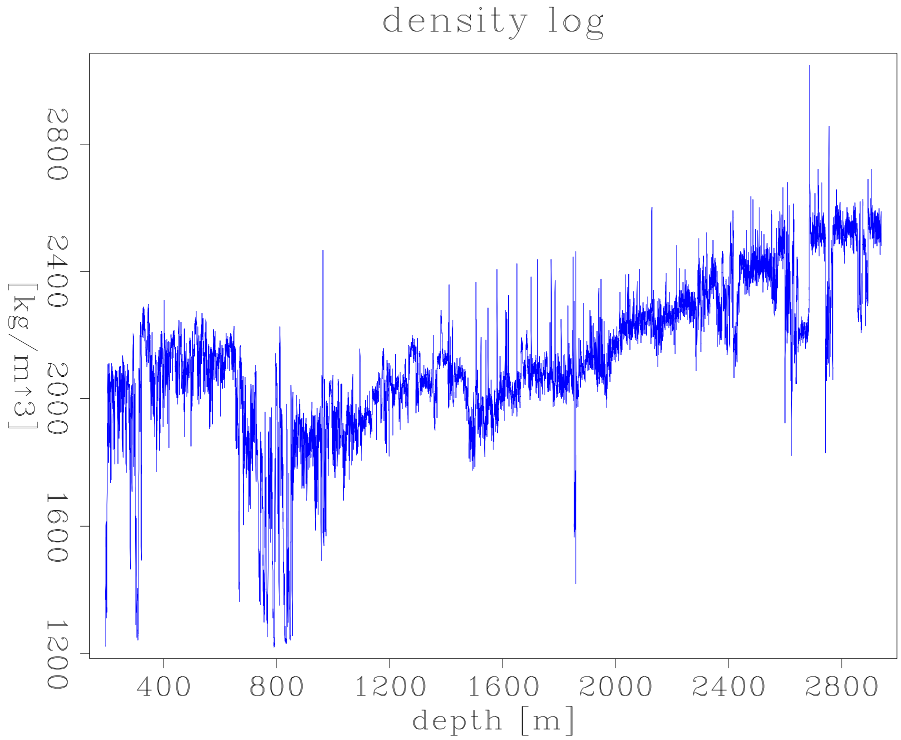


Figure 5: vp and rho log martin1-velprho [CR]

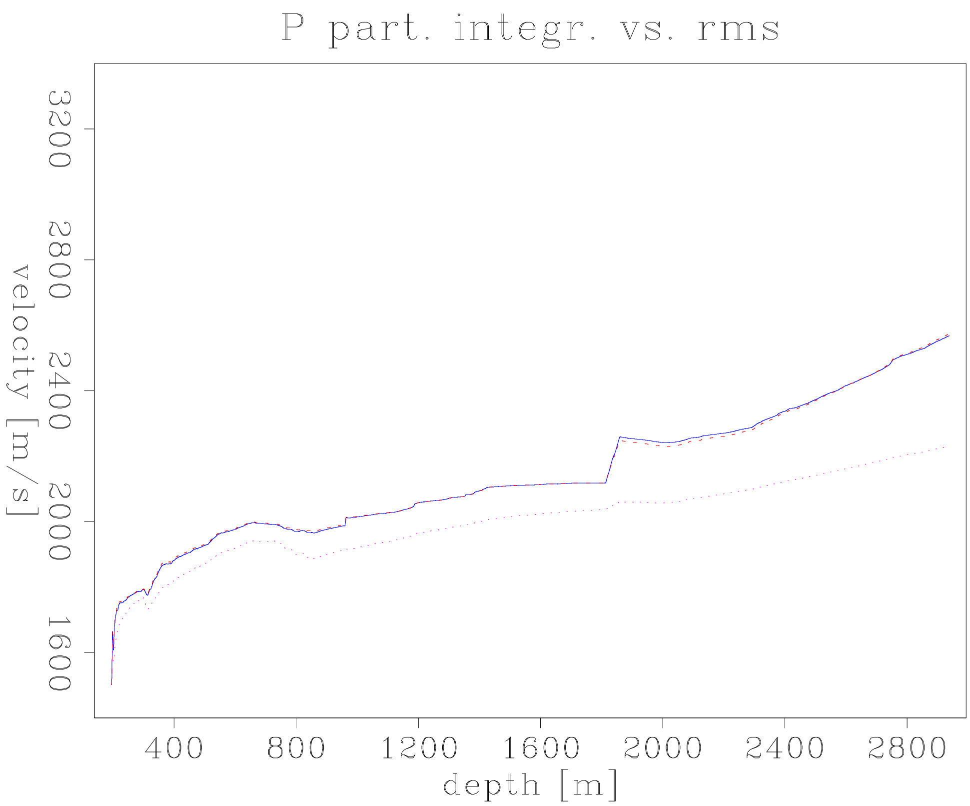


Figure 6: Dix rms velocity calculated from the well log(solid line). The two other curves are the S&M partially integrated horizontal and vertical propagation velocities. (v_z = dotted, v_x = dashed). The horizontal velocity is nearly identical with the rms velocity. martin1-velrmsp [CR]

$$S^{equiv}(z) = \int_{\zeta=0}^{\zeta=z} S(\zeta) d\zeta \tag{2}$$

This integration replaces the overburden at a specific depth point by a homogeneous equivalent medium. At the same time it preserves all stress and stain components across layer boundaries, honoring solid-solid boundary conditions exactly. For liquid layers different group elements have to be chosen, but the usage is identical. The elastic energy is conserved in this averaging process. See Nichols and Karrenbach (1990) for more details.

Figure 6 shows the well log derived Dix rms velocity and the horizontal and vertical velocities calculated using the S&M partial integration. The Dix rms velocity is the solid curve, which is nearly identical with the horizontal S&M velocity v_x. The vertical propagation velocity v_z is clearly distinct from these other two curves. It is not surprising that this partially integrated medium is no longer isotropic. It is a well known fact (Backus, 1962) that a sequence of thin layers produces anisotropic propagation effects for wavelengths much longer than the layering scale. Based on the assumption that the original material was thinly layered isotropic, the partially integrated medium is now transverse isotropic. I am only comparing P velocities since, I started out with only a P sonic log and I had to assume a shear wave velocity. In Figure 6 I extracted only direct wave velocities, but using a paraxial approximation ((Dellinger et al., 1993)) it is possible to extract "NMO" velocities. For relating elastic properties to moveout velocities see the paraxial approximation around the vertical axis for TI media in the Appendix.

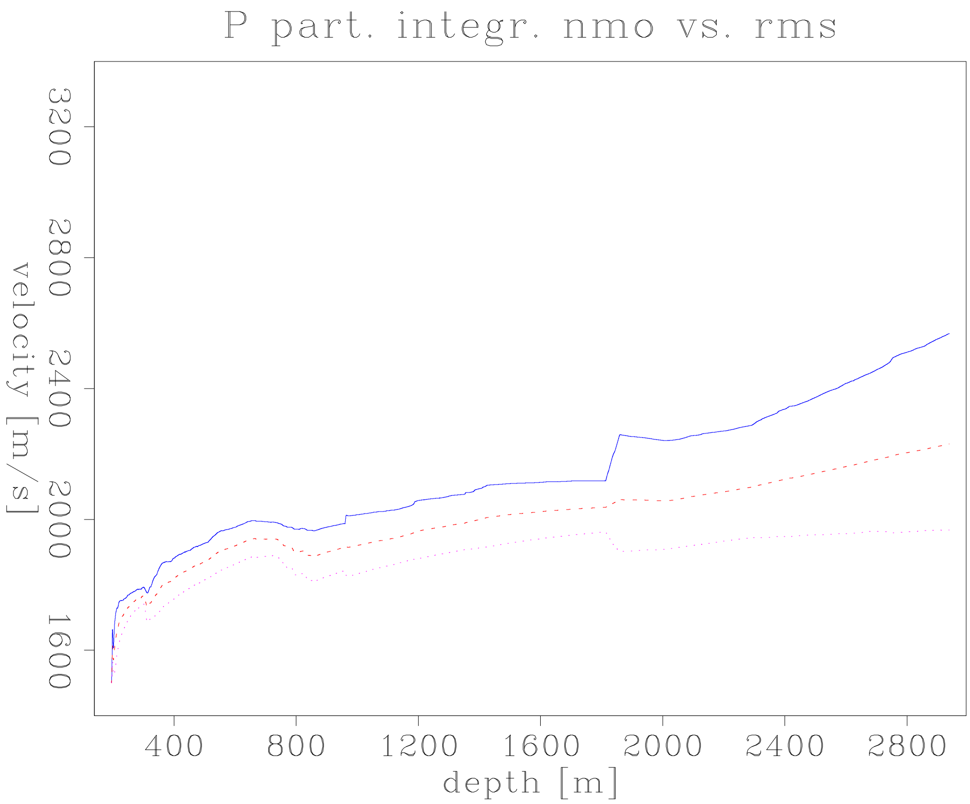


Figure 7: Dix rms velocity calculated from the well log (solid line). The two other curves are the S&M partially integrated horizontal and vertical NMO velocities. (v_{zpnmo} = dotted, v_{xpnmo} = dashed). Both NMO velocities clearly differ from the rms velocity. martin1-velrmspnmo [CR]

Figure 7 shows the Dix rms velocity and the "wave averaged" NMO velocity. The solid curve is the Dix rms velocity. The dashed curve is the horizontal NMO velocity v_{xpnmo} calculated using S&M partial integration. The dotted curve is the corresponding vertical NMO velocity v_{zpnmo}. A marine surface seismic observation measures vertical velocity at zero offset and normal moveout velocity v_{xpnmo} from a finite offset range. As we can see in Figure 7, there is only a small difference at small depth, but growing disagreement with depth. The S&M derived curves are always lower than the Dix rms velocity, as expected. The Dix rms velocity is a high frequency approximation satisfying Fermat's principle and thus provides us with the minimum travel time.

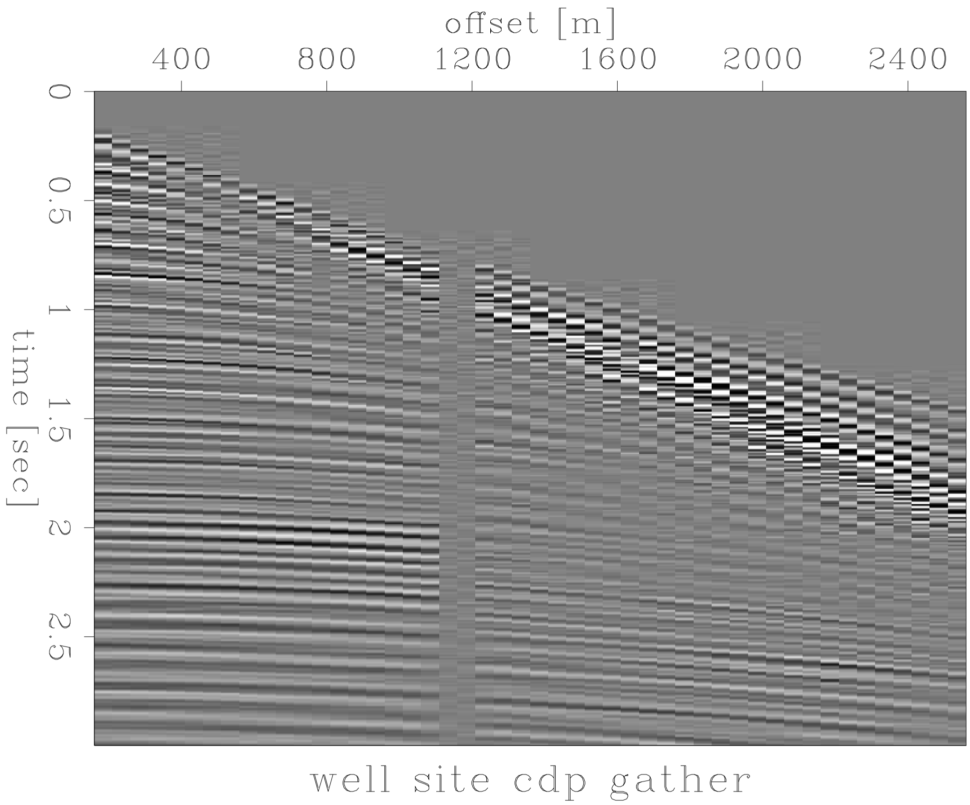


Figure 8: Marine cdp gather at the well site. martin1-cdp641 [CR]

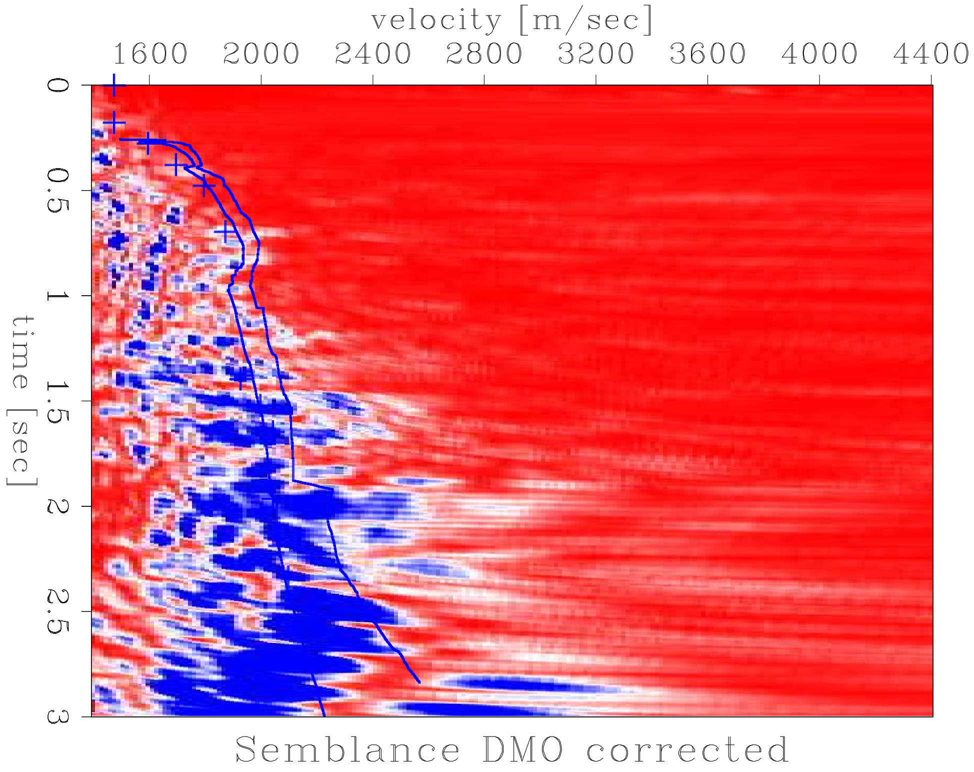


Figure 9: Semblance of a 2D DMO corrected gather with picks, partially integrated curve (left most) and rms curve (right most) overlain. martin1-sembdmo [CR]

The crucial test however is to compare those curves to actual surface seismic data. Figure 8 shows a CMP gather right at the well site. Figure 9 shows a semblance velocity analysis of that same gather. Overlain are the two estimated velocity curves. The left most is the NMO velocity derived from the S&M partially integrated stiffness log, while the right most curve is the Dix rms velocity. The velocity analysis is using only the raw data with a divergence correction, no other processing is applied. The multiples show up very strong in the semblance panel. Some primaries are visible and S&M derived NMO velocity curves follow pretty close some primary semblance peaks at times less than 2.0 seconds. The Dix rms velocity seems off the semblance peaks to higher values, except for the water bottom reflection and close arrivals. Both curves miss a primary peak at around 1.5 seconds. This is a reflection from a dipping interface. Thus the velocity is shifted towards higher values than predicted using a horizontal layer assumption. Below the target around 2.0 seconds the peaks are shifted again towards higher values. Layers at that depth exhibit progressively steeper dips. The high contrast at the target zone gives rise to strong interbed multiples.

I summarize the velocity relationships for matching surface seismic velocity analysis with well log observations. Relationships between direct propagation velocity are:

$$v_{rmsp} = v_{xp} > v_{zp} \tag{3}$$

The direct horizontal propagation velocity v_{xp} is essentially the same velocity component that is sampled by the P wave rms velocity. Thus their values are nearly identical. The horizontal direct propagation velocity v_{xp} is parallel to the layer interfaces, while the vertical direct propagation velocity v_{zp} perpendicular to the layering. The layering tends to lower the vertical with respect to the horizontal direct propagation velocity.

The rms velocity is derived from Fermat's principle (high frequency approximation) and thus has to fulfill the least travel time condition. The rms velocity is therefore higher than the vertical and horizontal NMO velocities. v_{xpnmo} and v_{xznmo} are not necessarily least traveltime, they are derived from low frequency elastic constants and relate to the wave front curvature at small offsets, but additionally those velocities include effects caused by the micro structure of the medium, like multiple bounces and they are not least travel time measurements. Relationships between NMO velocities are:

$$v_{rmsp} > v_{xpnmo} > v_{zpnmo} \tag{4}$$

Relationships between direct propagation and NMO velocities are:

$$v_{xp} > v_{xpnmo} \tag{5}$$
$$v_{zp} > v_{zpnmo} \tag{6}$$
$$v_{xp} > v_{zpnmo} \tag{7}$$
$$v_{zp} > v_{xpnmo} \tag{8}$$

Where the last relationship relates the two most interesting quantities for surface seismic, the direct vertical propagation velocity v_{zp} and the normal moveout velocity

v_{xpnmo}. In some special cases those two quantities can be made equal, but in general they are not (see the Appendix for such conditions). Figure 10 shows the results we

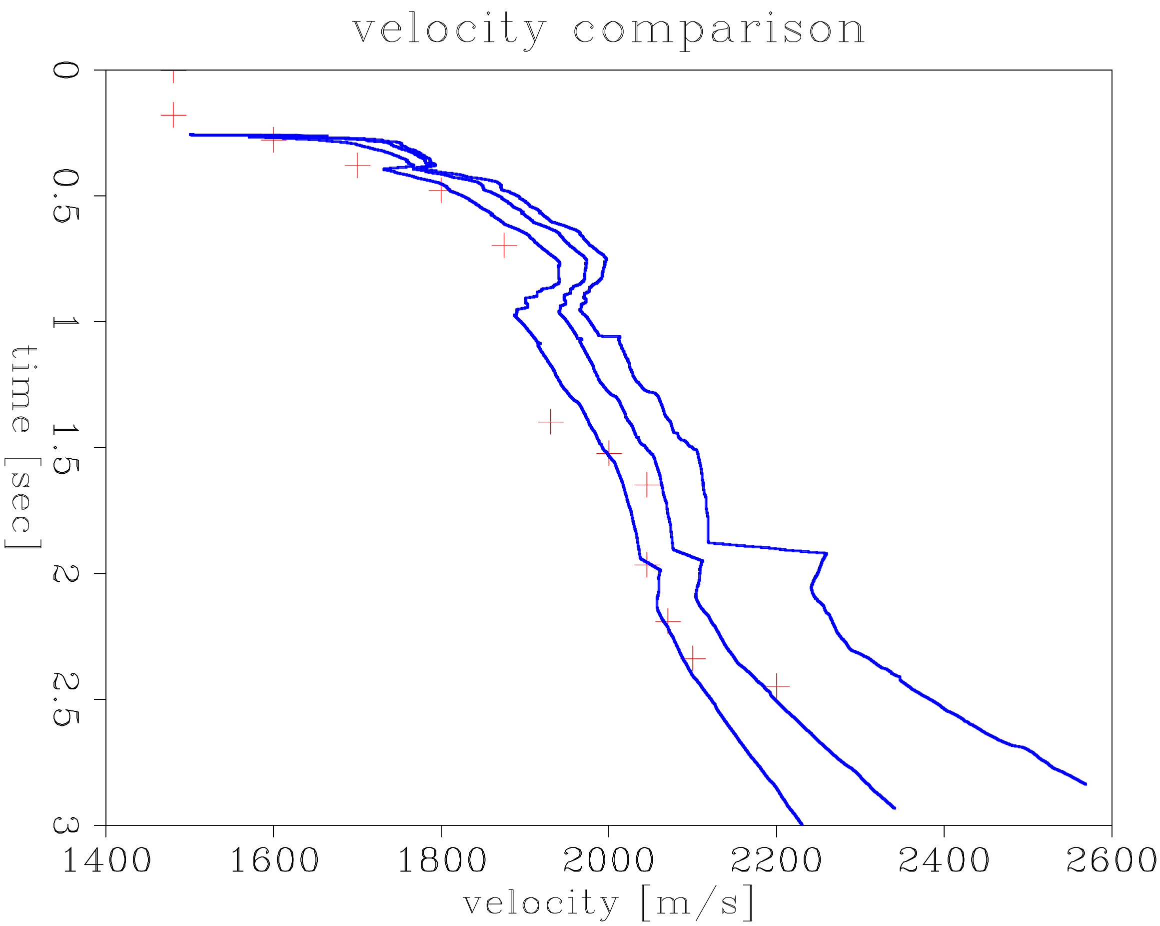


Figure 10: Picks compared with Dix rms and S&M predicted NMO velocities. In general the S&M predicted velocity is closer to the velocity picks. The largest difference in prediction by using rms velocities is 15 percent, while S&M predictions are in the order of a few percent. martin1-summarize [CR]

obtained using Dix rms and the S&M partial integration to relate well log measurements to surface seismic velocity analyses. Generally the S&M predicted velocity is closer to the velocity picks. The largest difference in prediction by using rms velocities is 15 percent, while S&M predictions are in the order of a few percent.

The previous examples all compare NMO and stacking velocities implying that the wave averages for that quantity with very large wave length. It appears from the results that for velocity analysis the wave propagation effects tend to be better described using a low frequency averaging approach than a high frequency averaging approach.

CONCLUSIONS

I chose here a particular example of relating velocity measurements at different scales, namely a well log and a surface seismic data set. I have applied Dix and Schoenberg&Muir averaging to the well log and compared it with the surface seismic velocity analysis. Both approaches replace the overburden with an homogeneous equivalent. However the sense of averaging is different. Overlaying velocity analysis, Dix rms velocity and S&M derived velocities show that Dix rms velocity seems consistently higher (up to 15%). The S&M average gives velocities that seem closer to the analysis picks (up to 5% deviation); At large depths however it is noticeably off the velocity picks. The S&M average is consistently lower than the Dix rms average, as we would expect since the high frequency average is based on the Fermat's principle and is thus least travel time.

ACKNOWLEDGMENTS

Thanks to Jim Berryman for careful review of the presented ideas. Thanks to SEP sponsors and in particular Norsk Hydro for giving us access to the prestack 3D seismic data set.

REFERENCES

Backus, G. E., 1962, Long-wave anisotropy produced by horizontal layering: J. Geophys. Res., **66**, 4427–4440.

Dellinger, J., and Muir, F., 1993, Dix revisited : A formalism for rays in layered media: Fifth international workshop on seismic anisotropy, Canadian Soc. Expl. Geophys., Proceedings of the Fifth International Workshop on Seismic Anisotropy.

Dellinger, J., Muir, F., and Karrenbach, M., 1993, Anelliptic approximations for TI media: Journal of Seismic Exploration, **2**, 23–40.

Folstad, P. G., and Schoenberg, M., 1992, Use of equivalent anisotropic medium theory to calculate synthetic seismograms: SEG Summer Research Workshop on AVO, Soc. Expl. Geophys., Expanded Abstracts.

Kennett, B. L. N., 1974, Reflections, rays and reverberations: Bull. Seis. Soc. Am., **64**, 1685–1696.

Muir, F., 1991, Various equations for transverse isotropic media: SEP–**70**, 367–372.

Nichols, D., and Karrenbach, M., 1990, What is equivalent in an equivalent medium ?: SEP–**67**, 1–4.

Schoenberg, M., and Muir, F., 1989, A calculus for finely layered anisotropic media: Geophysics, **54**, no. 5, 581–589.

APPENDIX A

Following Francis Muir's notation, I use the following symbols: W_{ij} is $(1/\rho)C_{ij}$, an elastic modulus divided by density; C is $\cos\theta$ and S is $\sin\theta$. More detailed and complete definitions for all wave types can be found in (Muir, 1991).

APPROXIMATING TI MEDIA WITH ELLIPTIC COEFFICIENTS

P waves only

$$W_{P,x} = W_{11} \tag{A-1}$$

$$W_{P,xnmo} = W_{44} + \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}} \tag{A-2}$$

$$W_{P,z} = W_{33} \tag{A-3}$$

$$W_{P,znmo} = W_{44} + \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}} \tag{A-4}$$

PSEUDO-ISOTROPIC CONDITIONS

If the paraxial move-out velocity equals the axial velocity, then the medium will be isotropic in kinematic approximation for the wavetype concerned.

Pseudo-isotropy about the Z axis

The condition is that

$$W_z = W_{xnmo} \tag{A-5}$$

For P waves this means

$$W_{33} = W_{44} + \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}} \tag{A-6}$$

which is

$$(W_{13} + W_{44})^2 = (W_{33} - W_{44})^2 \tag{A-7}$$