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Short Note

Transmissivity simulation in fine layered media

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INTRODUCTION

In a fine layered media of cyclic sedimentation, the first arrival is attenuated very quickly due to the transmission loss. As a result, the seismic information is carried by the short path multiples(O'Doherty and Anstey, 1971). In this note, I studied the average effect of many thin isotropic layers by simulating the impulse response in one dimensional random sequences of two states.

MODELING LAYERS

The sedimentation process has a tendency towards randomness as well as blockiness. A Markov chain can be a good stochastic model to simulate these features. The merit of the model is its simplicity, for the layers of M states, it can be described only by M parameters(Godfrey et al., 1980).

Markov Chain

To construct a Markov chain, all we need to know are the probability for each state ($M-1$ parameters) and the blockiness of the sequence(the single remaining parameter). In case of $M=2$, telegraph matrix can be made from those parameters as follows.

$$P_T = \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + (1-\lambda) \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_1 & \alpha_2 \end{pmatrix} \tag{1}$$

where λ =blockiness, α_j =the probability of state j. The blockiness λ is obtained from the exponential coefficient of well logs' correlation data. The left term of the right hand side describes the tendency to remain in the same state as before, and the right one is for the tendency to change into another state. Markov chains depend

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only on the last state. Once the current state is determined, it forgets all the previous states and choose the next state only from the current information. If the first state is state 1, the probability state vector $\pi = (1, 0)$. The probability for the continuous state α becomes

$$\begin{pmatrix} \alpha_1 & \alpha_2 \end{pmatrix} = P_T \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} \tag{2}$$

After obtaining α, a random number (from 0 to 1) is used to choose new state and we get the new π for the next lottery.

TRANSMISSIVITY SIMULATION

To model a one dimensional medium, additive quantities such as density and compliance are assigned to the random sequences made in the previous section. In this case, only two states are considered. The equations needed for simulation are taken from the paper by Muir in this report. The equations for crossing the boundary of the two different states are,

$$\begin{pmatrix} v \\ \sigma \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{\rho/s}} & -\frac{1}{\sqrt{\rho/s}} \end{pmatrix} \begin{pmatrix} v^+ \\ v^- \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} v^+ \\ v^- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{s/\rho} \\ 1 & -\sqrt{s/\rho} \end{pmatrix} \begin{pmatrix} v \\ \sigma \end{pmatrix} \tag{4}$$

and the translation equation within a layer of thickness z, density ρ and compliance s is,

$$\begin{pmatrix} v^+(z) \\ v^-(z) \end{pmatrix} = \begin{pmatrix} e^{-i\omega z\sqrt{\rho s}} & 0 \\ 0 & e^{+i\omega z\sqrt{\rho s}} \end{pmatrix} \begin{pmatrix} v^+(0) \\ v^-(0) \end{pmatrix} \tag{5}$$

These three equations can be combined to form a basic building block

$$\begin{pmatrix} v(z) \\ \sigma(z) \end{pmatrix} = \begin{pmatrix} \cos(\omega z\sqrt{\rho s}) & -i\sqrt{s/\rho}\sin(\omega z\sqrt{\rho s}) \\ -i\sqrt{\rho/s}\sin(\omega z\sqrt{\rho s}) & \cos(\omega z\sqrt{\rho s}) \end{pmatrix} \begin{pmatrix} v(0) \\ \sigma(0) \end{pmatrix} \tag{6}$$

where

v : velocity field

σ : traction field

v^+ : upgoing wave amplitude

v^- : downgoing wave amplitude

z : layer thickness

ρ : density of each layer

s : compliance of the layer

Let $\mathbf{A}_i$ a building block matrix in i-th layer. Then,

$$\mathbf{A}_i = \begin{pmatrix} \cos(\omega z_i\sqrt{\rho_i s_i}) & -i\sqrt{s_i/\rho_i}\sin(\omega z_i\sqrt{\rho_i s_i}) \\ -i\sqrt{\rho_i/s_i}\sin(\omega z_i\sqrt{\rho_i s_i}) & \cos(\omega z_i\sqrt{\rho_i s_i}) \end{pmatrix} \tag{7}$$

By multiplying the matrices $\mathbf{A}_i$ for all layers, we get the following solution of the transmissivity T in the frequency domain.

$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{s/\rho} \\ 1 & -\sqrt{s/\rho} \end{pmatrix} \mathbf{A}_N\mathbf{A}_{N-1}\ldots\mathbf{A}_2\mathbf{A}_1 \begin{pmatrix} 1 & 1 \\ \sqrt{\rho/s} & -\sqrt{\rho/s} \end{pmatrix} \begin{pmatrix} I \\ R \end{pmatrix} \tag{8}$$

An impulse of amplitude 1 is used as an input. The output is averaged over 32 different random sequences with the same statistical properties. The experiment is done for various numbers of layers to see how the number of layers affects the output. The numbers used here are powers of 2, say 32,64,128. To make problem simpler, the velocities of the two states are chosen to be the same. The time sampling interval is equal to the difference between the arrival time of the primary and that of the shortest path multiple. The frame is retarded by the amount of the first arrival time. Figure 1-4 show the impulse responses for the different four numbers of layers.

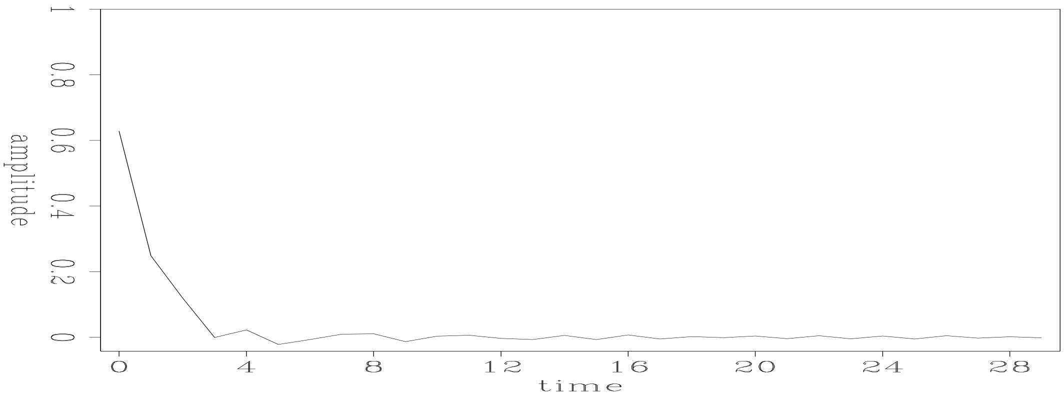

Figure 1: Impulse response for 16 layers hyangim-avrg16 [NR]

As expected, the impulse broadens as the number of layers increases. The peak arrival time is retarded accordingly. The peak amplitude decays approximately in proportion to the square root of the number of layers. Figure 5 shows this relationship. The retarded time of the peak amplitude from the primary is proportional to the number of the layers.

Further Study

One of the problems of this experiment is that the output in frequency domain diverges to very large values when there are more than 250 layers. It's not simply caused by round-off errors or bad choice of units. It's because of the term $\sqrt{\rho/s}$

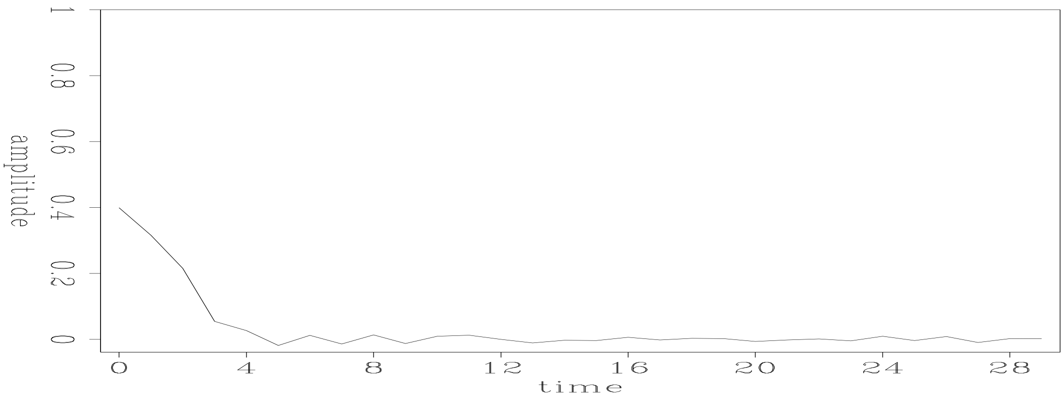

Figure 2: Impulse response for 32 layers $\boxed{\text{hyangim-avrg32}}$ [NR]

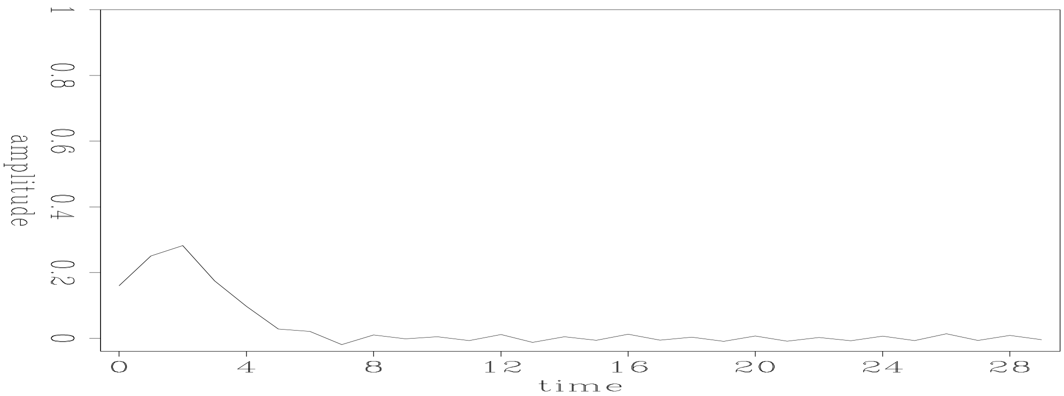

Figure 3: Impulse response for 64 layers $\boxed{\text{hyangim-avrg64}}$ [NR]

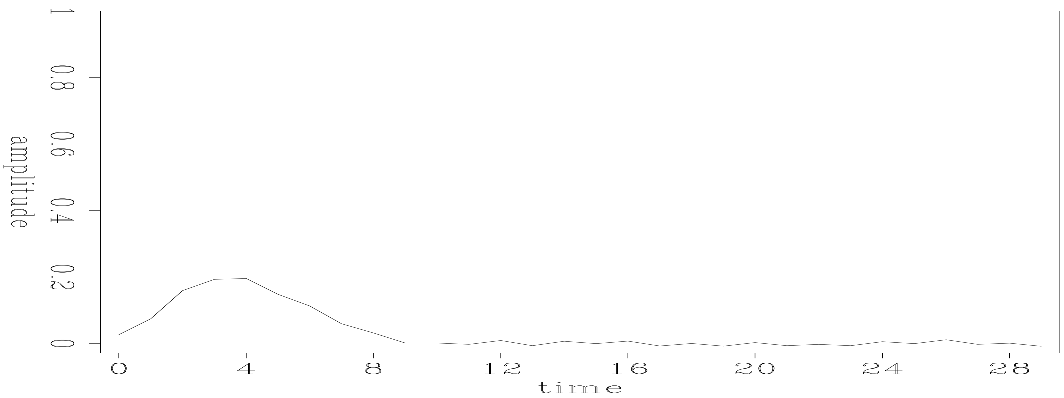

Figure 4: Impulse response for 128 layers $\boxed{\text{hyangim-avrg128}}$ [NR]

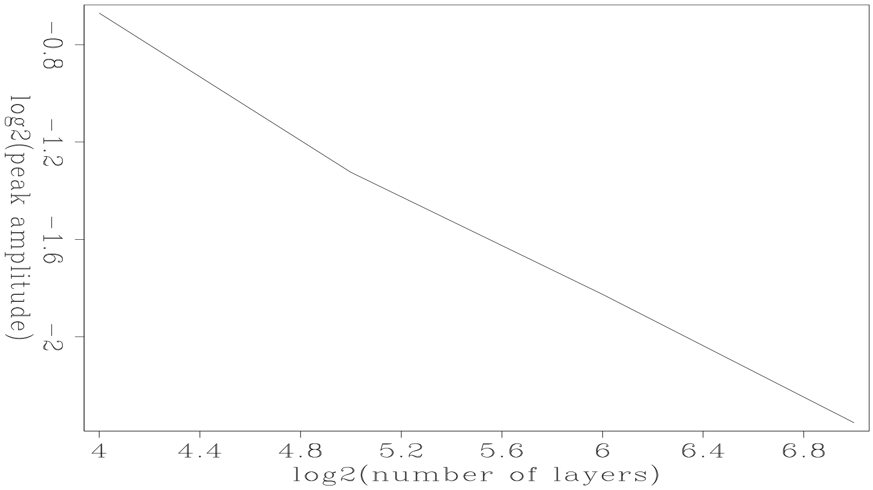

Figure 5: The peak amplitude decays in proportion to approximately square root of number of layers. hyangim-amp-num [NR]

in equation (6). As long as it is not equal to 1, the matrix **A** has a large element resulting from the successive multiplication of $\sqrt{\rho/s}$. Does it mean that it's wrong to use the equation (6) for this experiment? I don't know yet. After figuring out this problem, more examples should be studied.

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