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3-D wavefield depth extrapolation by rotated McClellan filters

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ABSTRACT

The application of McClellan transformations considerably reduces the computational cost of 3-D wavefield depth extrapolation by explicit convolutional methods. The accuracy of migration methods based on McClellan transformation depends on how well the transformation filter ($\cos \mid \vec{k} \mid$) is approximated; errors in this approximation cause anisotropy in the extrapolator operator. This anisotropy can be greatly reduced by rotating the approximate filter by 45 degrees, and averaging the rotated filter with the original filter. The application of the rotated filter yields a migration method that images correctly very steep dips, without additional computational cost.

INTRODUCTION

McClellan transformations offer a computationally efficient scheme for the depth extrapolation of 3-D seismic data in the frequency domain (McClellan and Chan, 1977; Hale, 1991; Palacharla, 1993). The computational efficiency is achieved by exploiting the circular symmetry of the wavefield extrapolator. Using McClellan transformations the convolution with the 2-D ($N \times N$) filter that is required for depth extrapolating the wavefield (Blacquiere et al., 1989) is reduced to N convolutions with a compact 2-D "transformation" filter. The accuracy of the McClellan transformation depends on the accuracy of the approximation used for the "transformation" filter; this transformation filter is defined in the wavenumber domain $\vec{k}$ as $\cos \mid \vec{k} \mid$. Hale (1991) gives two different approximations: the original McClellan filter, which has nine terms in the space domain, and a higher-order approximation with seventeen terms. However, the spectrum of even the higher-order McClellan filter deviates from the circular response at high wavenumbers. The error depends on the azimuthal direction and it is maximum along the 45° line to the orthogonal coordinate axes. This azimuthal dependence of the error causes the wavefield extrapolator to be anisotropic.

Blacquiere (1991) suggested a way for improving the accuracy of the transforma-

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tion by designing optimum McClellan filters as a function of the frequencies and dips to be imaged. The filters that are designed by this approach are optimum within the limitations of the stencil-pattern used for the optimization and they do not take advantage of the particular structure of the filter to be approximated. The most accurate ones are defined on a 5×5 stencils, thus they are more expensive to apply than the higher-order McClellan filter proposed by Hale.

In this paper we suggest a method for improving the accuracy of the transformation filter, without increasing its computational cost. The method takes advantage of the particular structure of the $\cos \mid \vec{k} \mid$ filter; namely, its being close to a separable filter. In order to improve the accuracy, we rotate the 9-point McClellan filter by 45 degrees, and then rescale the wavenumber axis so that the rotated filter matches the desired filter exactly along the orthogonal axes. The average of this rotated filter with the original McClellan one is a filter which is more isotropic, and closer to the ideal transformation filter.

Hedley (1992) has suggested an improvement in the circular response of the transformation filters by migrating the data on a hexagonal grid (Woodward and Muir, 1983). The hexagonal grid leads to a more accurate filter response for higher wavenumbers, because the hexagonal grid is more isotropic than the Cartesian grid. We think that our idea of rotating the McClellan filter for correcting the anisotropy could be extended to hexagonal grids, as well. We expect it to be more isotropic on the hexagonal grid compared to the corresponding McClellan filter.

DERIVATION OF ROTATED MCCLELLAN FILTERS

The McClellan filters proposed by Hale (1991) to approximate the $\cos \mid \vec{k} \mid$ filter have the important advantage of being compact, and consequently inexpensive to apply as convolutions in space domain. The lower-order filter is applied by convolving the data with a 9-point stencil, while the higher-order one by convolving the data with a 17-point stencil. However, both filters have an anisotropic impulse response; that is, their spectra vary with azimuth. They fit very well the desired circular spectrum of $\cos \mid \vec{k} \mid$ along the axes, but differ from it at high wavenumbers. The error is dependent on the azimuthal direction and is maximum for azimuths rotated 45 degrees with respect to the axes. In this section we derive a filter that is more isotropic than either of the McClellan filters and has a computational cost comparable to the 17-point one. The new filter is obtained by averaging the 9-point McClellan filter with a 45 degrees rotated version of itself.

The representation of the 9-point McClellan filter in the wavenumber domain is

$$\cos \sqrt{k_x^2 + k_y^2} \approx MC_0\left(k_x, k_y\right) = -1 + \frac{1}{2}\left(1 + \cos k_x\right)\left(1 + \cos k_y\right). \tag{1}$$

In the wavenumber domain, this filter can be easily rotated by an angle θ by applying the change of variables,

$$\begin{aligned} k_x^\theta &= k_x \cos\theta - k_y \sin\theta \\ k_y^\theta &= k_x \sin\theta + k_y \cos\theta. \end{aligned} \tag{2}$$

Applying this rotation to the McClellan filter we obtain the general expression for a filter aligned with an azimuth rotated by θ with respect to the axes,

$$\begin{aligned} & MC_\theta\left(k_x, k_y; \theta\right) = \\ & \quad = -1 + \tfrac{1}{2}\left[1 + \cos\left(k_x \cos\theta - k_y \sin\theta\right)\right]\left[1 + \cos\left(k_x \sin\theta + k_y \cos\theta\right)\right]. \end{aligned} \tag{3}$$

For an arbitrary θ this filter cannot be approximated in the space domain with a compact convolutional operator, but in the special case of θ being equal to 45 degrees, the general expression of equation (3) simplifies to

$$MC_{45}\left(k_x, k_y; \theta = 45^\circ\right) = -1 + \frac{1}{2}\left[\cos\left(\frac{k_x}{\sqrt{2}}\right) + \cos\left(\frac{k_y}{\sqrt{2}}\right)\right]^2. \tag{4}$$

The spectrum of the the rotated operator is shown in Figure **??**. As expected, it is the rotation of the spectrum of the 9-point McClellan filter (dotted line). The spectrum of the rotated filter matches the spectrum of the desired filter (solid line) along the diagonals, but not along the axes. Before averaging the two filters we scale the rotated filter such that it matches $\cos \mid \vec{k} \mid$ along the axes. This scaling is a non-linear stretch of the wavenumbers axes, that is defined by the relationships

$$\begin{aligned} \cos\overline{k_x} &= -1 + \tfrac{1}{2}\left[\cos\left(\tfrac{k_x}{\sqrt{2}}\right) + 1\right]^2 \\ \cos\overline{k_y} &= -1 + \tfrac{1}{2}\left[\cos\left(\tfrac{k_y}{\sqrt{2}}\right) + 1\right]^2, \end{aligned} \tag{5}$$

where the bars indicate that the wavenumber axes have been remapped. Solving for $\cos\left(k_x/\sqrt{2}\right)$ and $\cos\left(k_y/\sqrt{2}\right)$ in equation (5) leads to the relationships

$$\begin{aligned} \cos\left(\tfrac{k_x}{\sqrt{2}}\right) &= -1 + 2\cos\left(\tfrac{\overline{k_x}}{2}\right) \\ \cos\left(\tfrac{k_y}{\sqrt{2}}\right) &= -1 + 2\cos\left(\tfrac{\overline{k_y}}{2}\right), \end{aligned} \tag{6}$$

which upon substitution in equation (4) yield the desired filter

$$\overline{MC_{45}}\left(\overline{k_x}, \overline{k_y}\right) = -1 + 2\left[\cos\left(\frac{\overline{k_x}}{2}\right) + \cos\left(\frac{\overline{k_y}}{2}\right) - 1\right]^2. \tag{7}$$

The spectrum of the new operator is shown in Figure **??** with a dashed line. As expected, it matches the spectrum of $\cos \mid \vec{k} \mid$ along the axes, but not along the diagonals. However, the arithmetic and the geometric averages of the original 9-point McClellan filter $MC_0\left(k_x, k_y\right)$ with the newly derived rotated filter $\overline{MC_{45}}\left(k_x, k_y\right)$ are

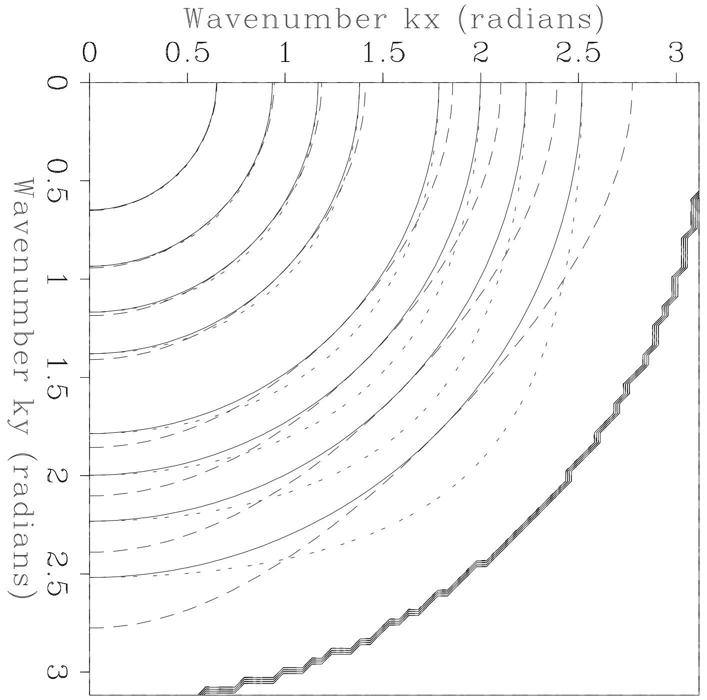

Figure 1: Contours of constant amplitude for the spectra of the 9-point McClellan transformation filter (double dots), the 45 degree rotate 9-point McClellan filter (dash) and the ideal circularly symmetric filter (solid). biondo1-rot [ER]

filters with almost isotropic spectrum. Figure **??** shows the spectrum of the averaged filters (dot dash for the arithmetic average and double dash for the geometric average) compared with the ideal filter (solid line) and the 17-point McClellan filter proposed by Hale (fine dash). The spectra of the averaged filters are similar and both match exactly the ideal spectrum along the axis and deviate from it considerably less than the spectrum of the 17-point McClellan filter across the whole range of azimuths.

The filter $\overline{MC_{45}}\,(k_x, k_y)$ can be efficiently applied to the data because the operator $[\cos{(k_x/2)} + \cos{(k_y/2)} - 1)]$ is separable; its space-domain representation is a cross-shaped convolutional operator. The arms of the cross are the space-domain representation of the $\cos{(k/2)}$ operator. This filter can be approximated in different ways; for computing the migration examples shown in the next section we windowed its ideal impulse response with a 7-point Kaiser window (Oppenheim and Shafer, 1975). Consequently, to apply $\overline{MC_{45}}\,(k_x, k_y)$ we convolved the data twice with a 13-point cross; longer filters can be used if more accuracy is required.

The application of the arithmetic averaged filter requires a convolution with the 9-point McClellan filter followed by two convolutions with a 13-point cross. The total cost of this operation is higher than convolving the data with the 17-point McClellan filter; though it yields more accurate results. However, the geometric average is as close to the ideal filter as the arithmetic average, and thus a very good approximation is also achieved by applying $MC_0\,(k_x, k_y)$ and $\overline{MC_{45}}\,(k_x, k_y)$ alternately between depth

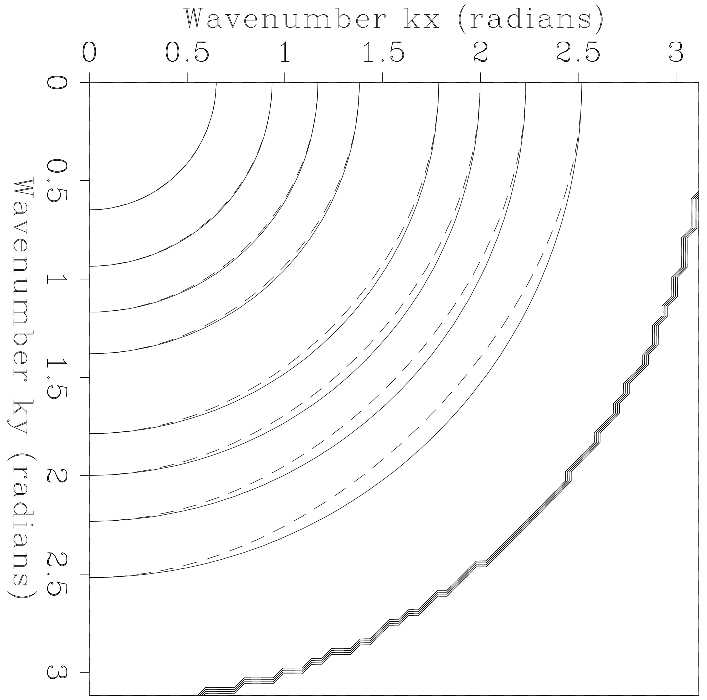

Figure 2: Contours of constant amplitude for the spectra of the scaled McClellan ($\overline{MC_{45}}$ transformation filter (dash) and the ideal circularly symmetric filter (solid). biondo1-rotscal [ER]

steps. The total computational cost of this procedure is about the same as applying the 17-point McClellan filter.

RESULTS OF 3-D POSTSTACK MIGRATION

We have tested the newly derived rotated McClellan filter for depth extrapolating a 3-D wavefield in a poststack migration program. We migrated a band-limited (10-100 Hz) impulse delayed .48 seconds assuming a constant propagation velocity of 2,250 meters per second. The spatial sampling was 5 meters in both in-line and cross-line directions and the depth step was constant for all frequencies and equal to 5 meters. The coefficients of the depth-extrapolation filters were derived by windowing the ideal impulse response of the operator with a Gaussian window (Nautiyal et al., 1993). To test the accuracy of the proposed method at high dips we used a fairly long extrapolator with half-length equal to 80 samples. As explained in the previous section, during the downward propagation we alternated between depth steps the application of $MC_0\,(k_x, k_y)$ and $\overline{MC_{45}}\,(k_x, k_y)$ as transformation filters. Further, we applied a spatial lowpass filter to the input data to attenuate waves with wavenumbers outside the circle defined by $\mid \vec{k} \mid = \pi$. These waves would be not correctly propagated in any case because the McClellan transforms cause the replication of the extrapolation operator along the radial directions in the wavenumber domain.

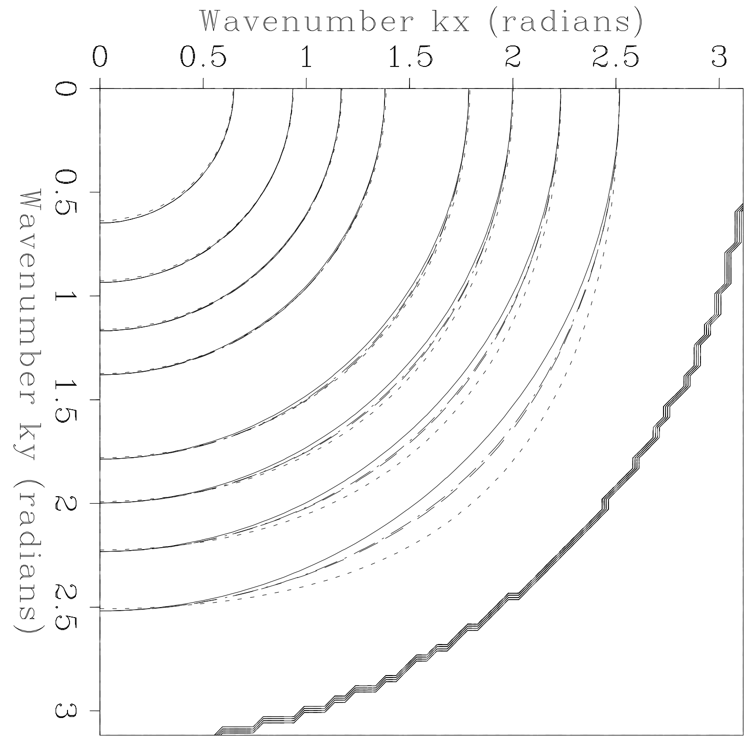

Figure 3: Contours of constant amplitude for spectra of the 17-point modified McClellan filter (double dots), the arithmetic averaged filter (dot dash), the geometric averaged filter (double dash) and the ideal circularly symmetric filter (solid). biondo1-avg [ER]

Figure ?? and Figure ?? show respectively the depth sections obtained by migrating the impulse using the 17-point McClellan filter and the proposed averaged filter. These sections were obtained by cutting the migrated cube along one of the vertical diagonals; that is, along the direction with largest error for both migration schemes. Both sections show the typical semi-circular migration impulse response; however the reflector computed using the 17-point filter is frequency dispersed above a depth of about 300 meters, equivalent to a dip of about 55 degrees. On the contrary, very little dispersion is present in the result computed using the proposed method. The reflector is correctly migrated up to the maximum dip propagated by the extrapolator operator used for the test; that is, a dip of about 78 degrees.

Figure ?? and Figure ?? show a depth slice of the migrated results, respectively computed using the 17-point McClellan filter and the proposed filter. The depth slices were cut at a depth of 200 meters, equivalent to a dip of about 68 degrees. The results of migration using the 17-point McClellan filter are highly dispersed. On the contrary, the reflector migrated using the proposed method appears to be perfectly circular, and very little dispersion of the wavefield is present.

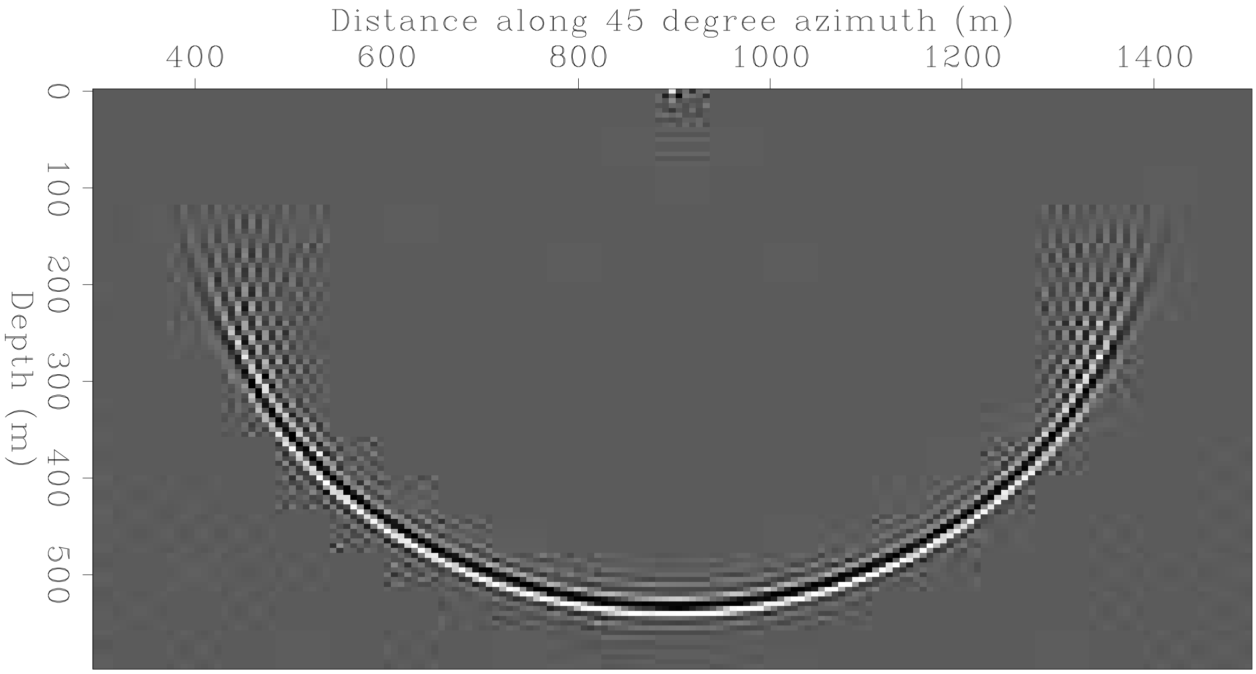

Figure 4: Depth section along the diagonal direction of the impulse response of 3-D migration using 17-point McClellan filter. biondo1-diagmcl17 [ER]

CONCLUSIONS

The accuracy of the McClellan transformation filter can be improved by rotating the 9-point McClellan filter by 45 degrees, and then averaging the rotated filter with the original filter. The spectrum of the averaged filter matches the spectrum of the

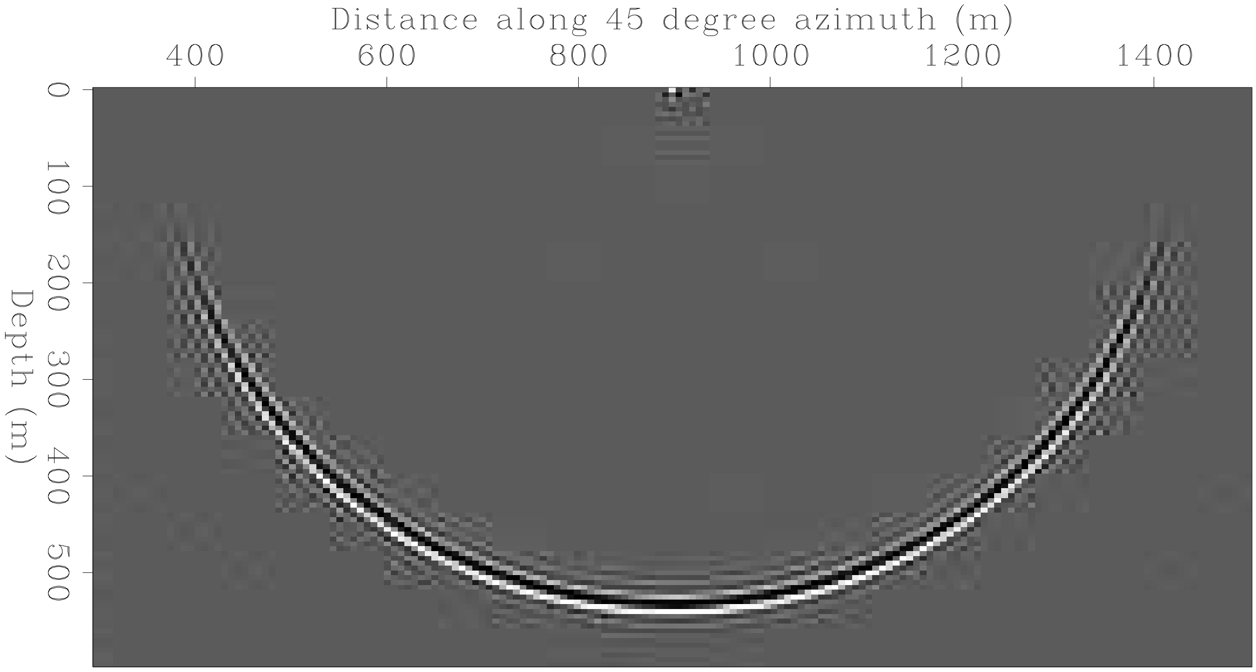

Figure 5: Depth section along the diagonal direction of the impulse response of 3-D migration using the proposed method. biondo1-diagnew [ER]

ideal transformation filter more accurately than the higher-order 17-point McClellan filter. The averaged filter can be applied by alternately convolving the data with the two filters; this procedure is about as computationally intensive as extrapolating the wavefield using the 17-point McClellan filter.

The application of the proposed transformation filter to 3-D poststack migration yields a migration algorithm that correctly images steep dips without dispersion of the wavefield.

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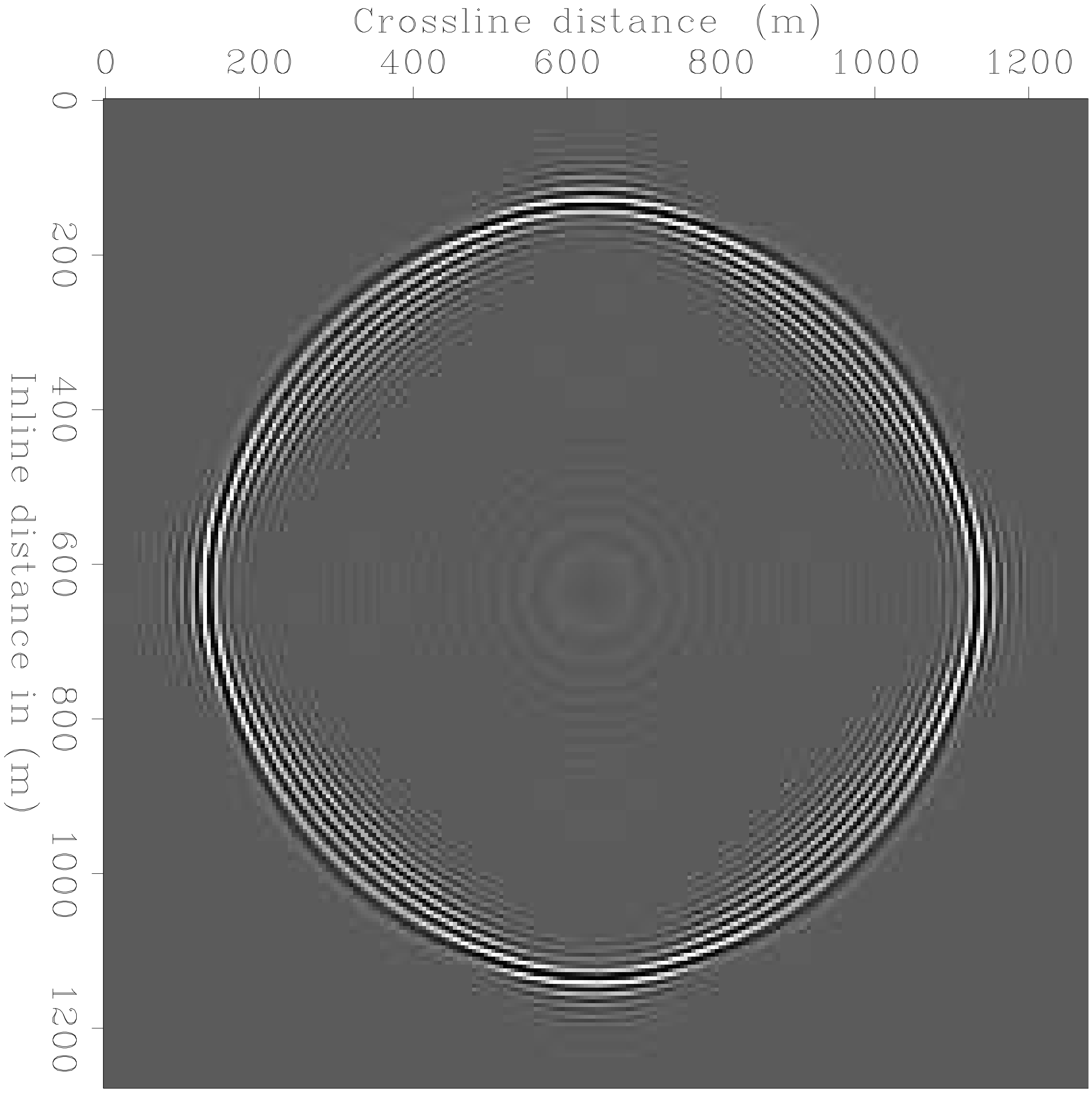

Figure 6: Depth slice of the impulse response of 3-D migration using 17-point McClellan filter. biondo1-sli17 [ER]

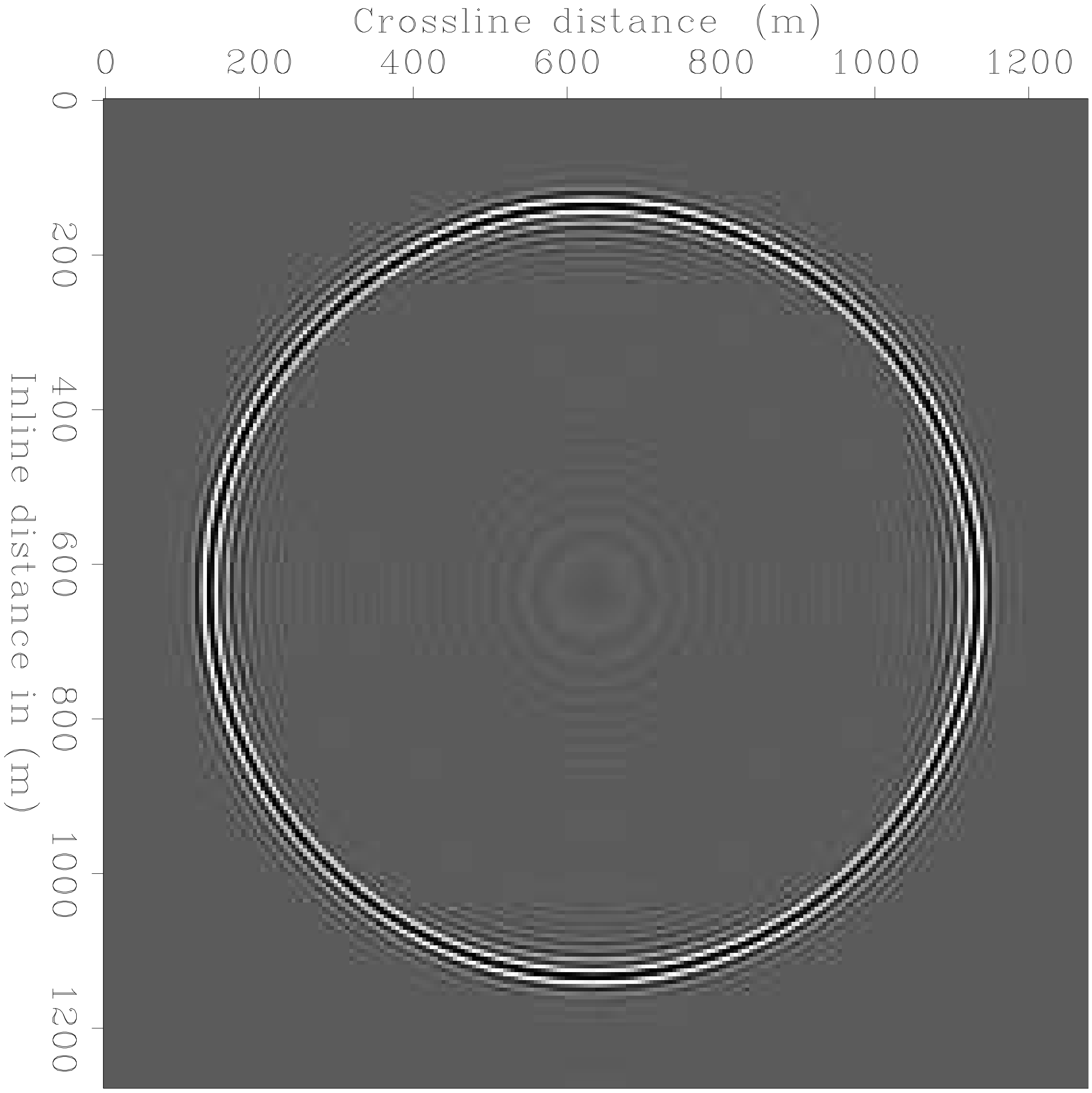

Figure 7: Depth slice of the impulse response of 3-D migration using the proposed method. biondo1-slinew [ER]

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