The integral formulation of residual depth migration

In this appendix, I derive an integral formula for residual depth migration using a kinematic approximation. Residual depth migration is a linear transformation from an image migrated with one velocity model to an image migrated with another velocity model. Suppose I migrate surface data $P(t, x_s, x_g)$ with velocity $v(x, z)$. Equation (**??**) of Chapter **??** gives the integral formula as follows:

$$Q(x, z; \eta) = \int_\xi \int_t R_{s,g} A_s A_g \delta(t - \tau_s - \tau_g) w(t) \odot D_{\frac{1}{2}}(t) \odot P(t, x_s, x_g) dt d\xi, \qquad (1)$$

where $\odot$ stands for the cross-correlation in time, $D_{\frac{1}{2}}(t)$ is the impulse response of the half-derivative operator, $w(t)$ is the source wavelet, $R_{s,g}$ is the directivity function defined in equation (**??**), and $A_c = A(x, z; x_c)$ and $\tau_c = \tau(x, z; x_c)$ for $c = s$ or g are the geometrical amplitude and traveltime fields in the velocity field, $v(x, z)$, respectively, with the source at the surface position x_c. Two variables ξ and η are determined by the data geometry. For common-shot migration, $\xi = x_g$ and $\eta = x_s$; for constant-offset migration, $\xi = 0.5(x_g + x_s)$ and $\eta = 0.5(x_g - x_s)$. Similarly, if the surface data are migrated with a different velocity model $\hat{v}(\hat{x}, \hat{z})$, the image is then

$$\hat{Q}(\hat{x}, \hat{z}; \eta) = \int_\xi \int_t \hat{R}_{s,g} \hat{A}_s \hat{A}_g \delta(t - \hat{\tau}_s - \hat{\tau}_g) w(t) \odot D_{\frac{1}{2}}(t) \odot P(t, x_s, x_g) dt d\xi, \qquad (2)$$

where the geometrical amplitude and traveltime fields are calculated from the corresponding velocity model. Residual depth migration is a mapping from image $\hat{Q}(\hat{x}, \hat{z})$ to image $Q(x, z)$.

If the migration operator is assumed to be close to a unitary operator (Claerbout, 1992), then rearranging equation (2) yields

$$P(t, x_s, x_g) = \int_{\hat{x}} \int_{\hat{z}} \hat{R}_{s,g} \hat{A}_s \hat{A}_g D_{\frac{1}{2}}(t) * w(t) * \delta(t - \hat{\tau}_s - \hat{\tau}_g) \hat{Q}(\hat{x}, \hat{z}; \eta) d\hat{z} d\hat{x} \qquad (3)$$

Substituing the expression on the left of equation (3) into equation (1) gives

$$\begin{aligned} Q(x,z;\eta) &= \int_\xi \int_t R_{s,g} A_s A_g \delta(t-\tau_s-\tau_g) w(t) \odot D_{\frac{1}{2}}(t) \odot \\ &\quad \int_{\hat{x}} \int_{\hat{z}} \hat{R}_{s,g} \hat{A}_s \hat{A}_g D_{\frac{1}{2}}(t) * w(t) * \delta(t-\hat{\tau}_s-\hat{\tau}_g) \hat{Q}(\hat{x},\hat{z};\eta) d\hat{z} d\hat{x} dt d\xi \\ &= \int_\xi d\xi R_{s,g} A_s A_g \int_{\hat{x}} \int_{\hat{z}} \hat{R}_{s,g} \hat{A}_s \hat{A}_g B(\tau_s+\tau_g-\hat{\tau}_s-\hat{\tau}_g) \hat{Q}(\hat{x},\hat{z};\eta) d\hat{z} d\hat{x} d\xi, \end{aligned} \tag{4}$$

where $B(t) = w(t) \odot D_{\frac{1}{2}}(t) \odot D_{\frac{1}{2}}(t) * w(t)$ is an autocorrelation function that is compacted around $t = 0$.

Define

$$I(x,z;\eta,\xi) = \int_{\hat{x}} \int_{\hat{z}} \hat{R}_{s,g} \hat{A}_s \hat{A}_g B(\tau_s+\tau_g-\hat{\tau}_s-\hat{\tau}_g) \hat{Q}(\hat{x},\hat{z};\eta) d\hat{z} d\hat{x}. \tag{5}$$

For each fixed (x,z) and ξ, $B(\tau_s+\tau_g-\hat{\tau}_s-\hat{\tau}_g)$ is zero except along a curve defined in $(\hat{x},\hat{z})$ by the following nonlinear equation:

$$\tau_s+\tau_g-\hat{\tau}_s-\hat{\tau}_g = 0 \tag{6}$$

Therefore, the area-integral in equation (5) becomes actually a line-integral along the curve defined in equation (6). For each ξ, this line-integral sums data samples along the corresponding curve, weighted by amplitude factor $\hat{R}_{s,g}\hat{A}_s\hat{A}_g$. The integration of $I(x,z;\eta,\xi)$ over ξ, weighted by amplitude factor $R_{s,g}A_sA_g$, accumulates the sums from all curves. Figure 1 shows a set of curves. If one integrate data samples along all these curves, data samples interfere and cancel with one another everywhere except along the envelope formed by the set of curves. Therefore, the effective operation is to integrate data samples along the envelope.

The envelope of the curves defined in equation (6) is implicitly defined in the system of nonlinear equations, as follow:

$$\left\{ \begin{array}{l} \tau(x,z;x_s) + \tau(x,z;x_g) = \hat{\tau}(\hat{x},\hat{z};x_s) + \hat{\tau}(\hat{x},\hat{z};x_g) \\ \\ p(x,z;x_s)\dfrac{\partial x_s}{\partial \xi} + p(x,z;x_g)\dfrac{\partial x_g}{\partial \xi} = \hat{p}(\hat{x},\hat{z};x_s)\dfrac{\partial x_s}{\partial \xi} + \hat{p}(\hat{x},\hat{z};x_g)\dfrac{\partial x_g}{\partial \xi}, \end{array} \right. \tag{7}$$

where p and $\hat{p}$ are the surface horizontal slownesses defined in equation (**??**) of Chapter **??**.

Suppose I have solved this nonlinear system and obtained the parametric functions of the envelope, as follow

$$\left\{ \begin{array}{lcl} \hat{x} &=& \hat{x}(x,z;\xi) \\ \\ \hat{z} &=& \hat{z}(x,z;\xi) \end{array} \right. \tag{8}$$

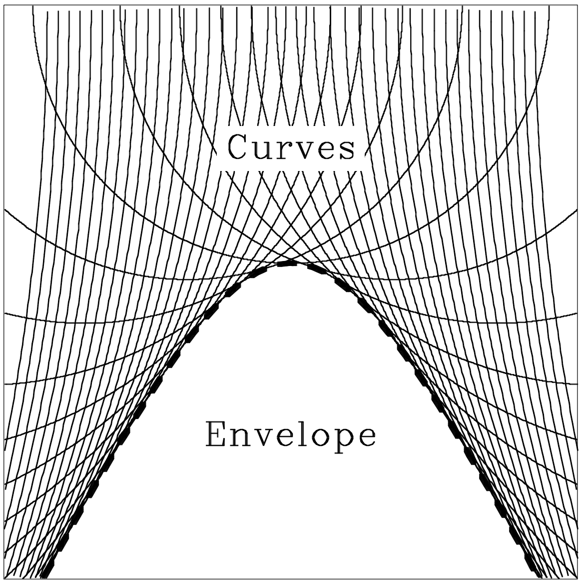

Figure 1: A set of curves and their envelope. The envelope is indicated by the fat dash line. rmgapdb-vprmg [ER]

With the assumption that the amplitude factors vary smoothly in space, I can approximate equation (4), as follows:

$$Q(x,z;\eta) \approx \int_{\xi} \left[R_{s,g} A_s A_g \hat{R}_{s,g} \hat{A}_s \hat{A}_g B(0) \hat{Q}(\hat{x},\hat{z};\eta) \right] \Big|_{\begin{array}{ccc} \hat{x} & = & \hat{x}(x,z;\xi) \\ \hat{z} & = & \hat{z}(x,z;\xi) \end{array}} d\xi. \tag{9}$$

which is equation (**??**) of Chapter **??**. Using the stationary phase approximation, Etgen (1990) derived a similar formula in the frequency domain.