Introduction

Efficient and accurate imaging

In complex geologic environments, imaging of seismic reflection data requires depth migration driven by an interval velocity model. If the true velocity model of the earth is known, depth migration can back-propagate the seismic events recorded at the earth's surface to the subsurface positions where they were reflected. In recent years, industrial practices in oil exploration have proved that depth migration is a powerful tool for imaging complex subsurface structures in areas of strong lateral velocity variations (Hatton et al., 1981; Claerbout, 1985; Deregowski, 1985). However, studies have also shown that the result of such an imaging method is sensitive to errors in the velocity model (Parkes and Hatton, 1987). If the velocity model used in the migration process does not correctly model the propagation of reflection events, depth migration produces an erroneous image of the subsurface. Therefore, many efforts have been made to develop methods for estimating interval velocity models. Among most of the methods, iterative prestack depth-migration velocity analysis appears as a common thread (Al-Yahya, 1989; Deregowski, 1990; Van Trier, 1990; Etgen, 1990). In this method, seismic reflection data are first migrated with an initial guess of the true velocity model. Then velocity perturbations are estimated by optimizing the offset coherence of the initially migrated images. New images are subsequently obtained by migrating data with the updated velocity model. This process is repeated until the perturbations of interval velocities no longer improve the offset coherence of the images. Because depth migration has to be done as many times as the number of iterations in the optimization process, the cost of depth migration in each iteration is a concern. The kinematic accuracy of the depth migration is also important because it is related to the correct estimation of velocity perturbations. Thus, the development of accurate and efficient depth migration algorithms is essential to the practical application of such a velocity estimation method.

The goal of the methods presented in this thesis is to improve the efficiency and accuracy of Kirchhoff depth migration. I use two approaches to accomplish this goal. In the first approach, I develop two wavefront-propagation methods to compute the traveltimes and geometrical amplitudes of seismic waves in a varying velocity field. These traveltimes and amplitudes are then used to construct the Kirchhoff depth migration operator. In the second approach, I formulate the concept of residual migration for depth migration to reduce the number of arithmetic operations involved in iteratively applying migration operators associated with velocity perturbations.

The conventional methods

Kirchhoff depth migration is based on an integral solution of the scalar wave equation (French, 1975; Schneider, 1978; Deregowski, 1985). In principle, this method can correctly migrate data with a generally varying velocity model. The implementation of Kirchhoff depth migration usually consists of two steps. The first step is to compute Green's functions of the scalar wave equation for a given velocity model and source-receiver configuration. The second step is to integrate (line integral in 2-D, or surface integral in 3-D) surface data weighted by the Green's functions. The total computational cost of Kirchhoff depth migration is the sum of the costs of the two steps. Because the techniques of implementing an integration are well developed, and the number of arithmetic operations involved in the integration are more or less fixed, conventional studies of Kirchhoff depth migration mainly focus on accurate and efficient calculation of the Green's functions.

With the zeroth-order asymptotic ray approximation, a Green's function of the scalar wave equation is characterized by a traveltime function and a geometrical amplitude function (Červený et al., 1977). The traveltime function describes the time delay for seismic energy to propagate from an initial point to a destination point, and the amplitude function describes the amplitude decay of seismic energy caused by geometrical wavefront divergence. The implementation of Kirchhoff depth migration usually uses tables that contain the traveltimes and amplitudes from each survey point on the surface to a set of regularly spaced subsurface points. Conventionally, these two tables are computed by either tracing rays or propagating wavefronts.

For a given velocity model, a ray-tracing method finds the trajectory of a ray by solving the ray equations (Červený et al., 1977). The integration of the slowness function (the reciprocal of the velocity function) along the ray path gives the traveltimes on the ray. Similarly, the amplitudes along the ray are computed by solving the dynamic ray equations (Červený, 1987). To construct the traveltime and amplitude tables needed for Kirchhoff depth migration, one can trace rays connecting each survey point to each subsurface point, which is usually referred as *two-point ray tracing.* To obtain a 2-D image of $N_x \times N_z$ samples by migrating a 2-D dataset with N_s survey points, a total of $N_x \times N_z \times N_s$ rays must be traced. Because $N_x \times N_z \times N_s$ is usually at least on the order of a million, the total amount of computation is tremendous, which makes this method impractical.

A more practical approach is to perform the so-called *shooting method* (Nolet, 1987; Van Trier, 1988). From each survey point, a fan of rays is traced through the model. The traveltimes and amplitudes computed along these rays are subsequently interpolated onto a regular grid (Beydoun and Keho, 1987; Etgen, 1987). However, for models with strong velocity variation, rays may cross each other, or not penetrate "shadow" zones. Figure 1 shows two simple examples. In such cases, the interpolation process may be theoretically invalid or computationally expensive.

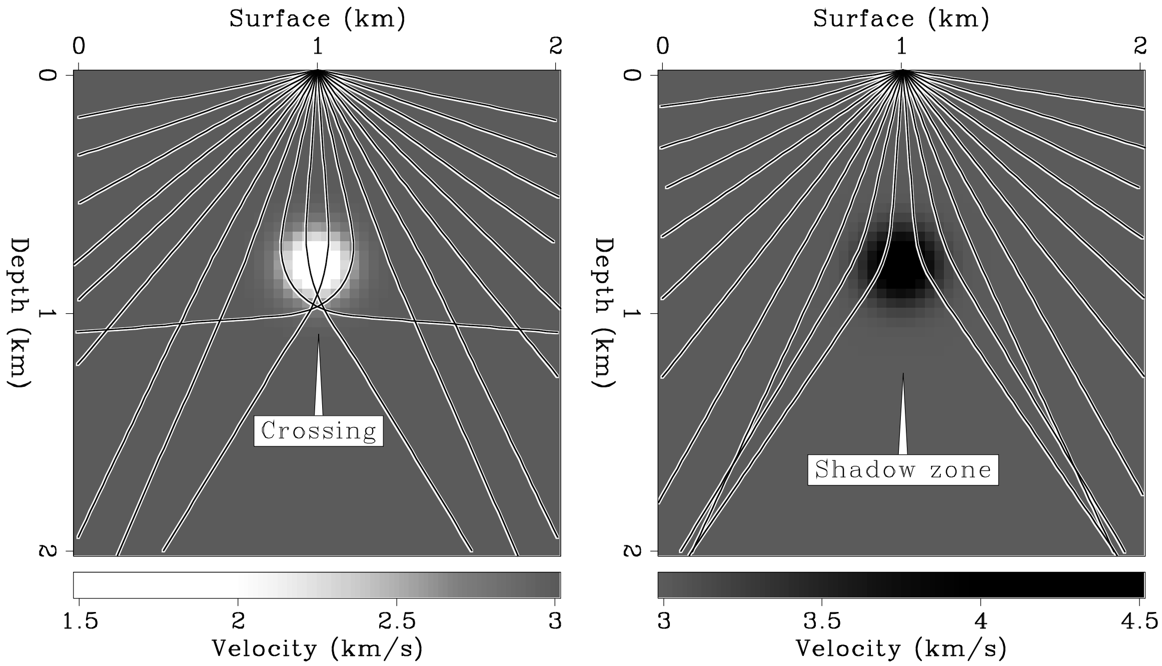

Figure 1: Ray tracing through velocity anomalies. Left: a negative anomaly causes rays to cross. Right: a positive anomaly creates a "shadow" zone. intro1-raynegpos [ER]

Wavefront propagation by the finite difference method

Vidale (1988) introduced a method that computes the traveltimes directly on a regular grid in Cartesian coordinates. Instead of tracing rays, he uses a first-order finite-difference approximation to the eikonal equation, and solves directly for traveltimes by propagating a local planar or circular wavefront. The computation starts from the source position and proceeds on an expanding square ring. Van Trier and Symes (1991) presented an alternative method in which the traveltime gradients, instead of the traveltimes themselves, are propagated. The traveltimes are obtained by simple integration of the gradients. With their method, the calculations are done in polar coordinates and the results are mapped into Cartesian coordinates afterwards. Because traveltimes and traveltime gradients are attributes of wavefronts, these two methods can be viewed as a process of wavefront propagation. Van Trier and Symes showed that the implementation of their method on a vector machine often achieves better efficiency than that of Vidale's method (Van Trier and Symes, 1991). However, experience shows that these two methods are limited to models with a smooth velocity field, and the limit on smoothness is not quantitatively defined. The results of many numerical experiments show that both methods encounter serious instability problems when applied to models containing large velocity discontinuities.

Podvin and Lecomte (1991) reported several significant improvements on Vidale's method. Their method systematically applies Huygens' principle in the finite-difference approximation so that the method can calculate accurate traveltimes in the presence of extremely severe, arbitrarily shaped velocity contrasts. They developed a massively parallel implementation of the algorithm as well as a serial implementation. In their parallel implementation, each grid point of a velocity model is associated with a processor. All the processors simultaneously update the traveltimes at the associated grid points, which is like a relaxation process. Such an updating scheme guarantees that the results are the traveltimes of first arrivals. The updating scheme of the serial implementation is similar to the one in Vidale's method. To capture the first arrivals that propagate back into the expanding square ring, they introduced backward propagation. This procedure ensures that the results of the serial implementation are minimum traveltimes.

Finite-difference calculation of geometrical amplitudes has been explored to a lesser extent than the calculation of traveltimes. Vidale and Houston (1990) published a method for calculating geometrical amplitudes. Their method can be based on any algorithm for calculating traveltimes. The geometric spreading factor that appears in the formula for amplitudes is estimated by a using first-order finite-difference approximation to gradient calculations. This method can efficiently and accurately compute the amplitudes in the case of a smoothly varying velocity model.

Among these finite-difference methods, Podvin and Lecomte's approach seems the most promising. However, there are several weaknesses in their implementations, for which better solutions can be explored. The massively parallel implementation does not effectively use computational power because, during the computation, most of the processors away from the first-arrival wavefront perform useless operations. Such waste may be unacceptable, especially for three-dimensional traveltime calculations. The computational cost of this implementation are close to that of full wave-equation modeling. As with Vidale's method, the serial implementation must start the computation at local minimum traveltime points, and thus does not run efficiently on a vector machine. Finally, because the backward propagation introduces extra cost it should be avoided whenever possible.

In Chaper **??** of this thesis, I develop an improved finite-difference method for wavefront propagation that combines the advantages of several existing algorithms. I use a simplified finite-difference representation of Podvin and Lecomte's scheme in order to handle large velocity contrasts. I choose to conduct the computations in polar coordinates, and, as a result, the algorithm is fully vectorizable along the circular computational front at each step of wavefront propagation. Using polar coordinates also reduces the cases for which backward propagation has to be performed. The stability of the algorithm is ensured by adapting the step size of the wavefront propagation to the propagating directions of the local plane waves. The accuracy of the algorithm is as good as that of other first-order finite-difference methods. Also in Chapter **??**, I derive two new partial differential equations for the geometrical amplitude calculation, and solve them using a first-order finite-difference approximation. The computational cost after incorporating amplitude calculation into the algorithm of traveltime calculation is about twice the cost of calculating traveltimes only.

Wavefront propagation by local ray tracing

Propagating wavefronts by finite-difference methods has its limitations. Although these methods can compute traveltimes accurately, they may produce large errors in traveltime gradients at interfaces of velocity contrasts. Besides the errors due to the first-order finite-difference approximations, substantial errors may result from the discrete representation of a velocity model that a finite-difference scheme assumes. For example, Podvin and Lecomte's method approximates the true velocity model by a virtual model in which space is paved by constant velocity cells of a rectangular shape. Figure 2 illustrates a simple example in which a dipping interface that separates two constant velocity regions is approximated by a stair function formed by the boundaries of rectangular cells. It is known that at an interface between two isotropic media, traveltime gradients should obey Snell's law: the tangential component of the traveltime gradient to the interface is continuous across the interface. Because the dips of the stair function are either 0 or 90 degrees, they are a poor approximation to the true dip of the interface. Consequently, as shown in Figure 2, the computed traveltime gradient points to a wrong direction after crossing the interface. This problem cannot be solved by simply reducing the grid size, because the dips of the stair function are either 0 or 90 degrees, independent of the grid size.

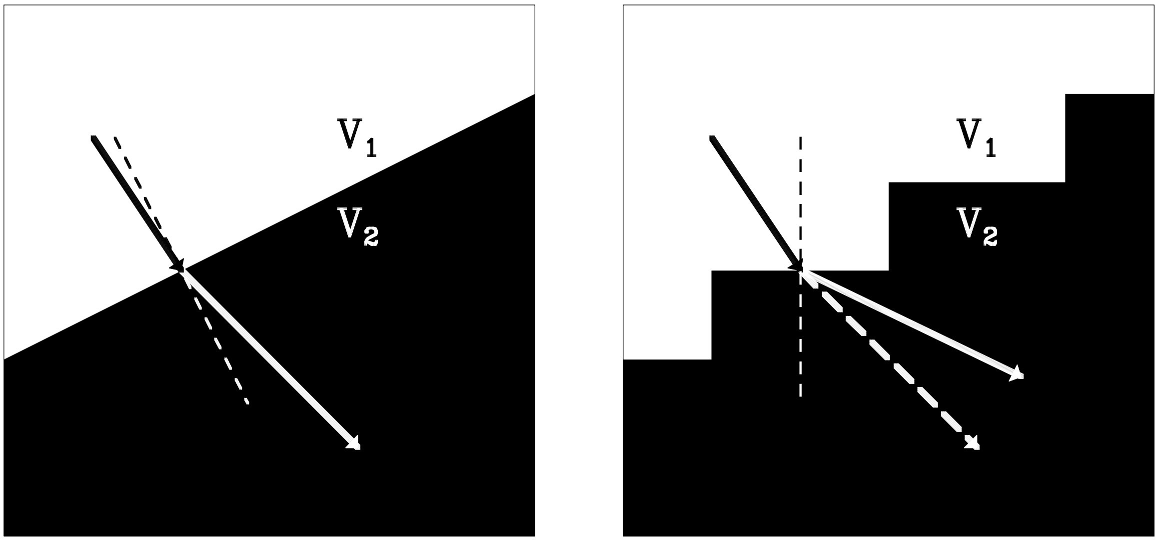

Figure 2: Traveltime gradients point to the propagation directions of wavefronts: (a) a correct change in the direction of the gradient, (b) a wrong change induced by replacing "ramp" by stairs. intro1-vpsnell [ER]

Finite-difference methods compute only the traveltimes corresponding to first arrivals, which always have continuous wavefronts. These methods are well suited for tomographic inversion of first arrivals. However, first arrivals may not be what is needed for imaging seismic reflection data. Figure 3 illustrates an example of such a situation. The background shows a snapshot of the wave field at 1.4 seconds in a two-layer medium with a velocity contrast of 1.5 to 4.5 km/s. The wave field is a high-order finite-difference solution to the scalar wave equation. Overlain on the background is the 1.4 second contour of the traveltime field computed using the finite-difference method to be described in Chapter **??**. The contour line shows the trajectory of the wavefront at the traveltime associated with that contour value. It is clear that the computed traveltime field follows the propagation of the first arrivals. Some of these first arrivals later become refracted waves, which are usually not useful for imaging reflection data; whereas the traveltimes of the energetic arrivals important for imaging are not computed when they become the second arrivals.

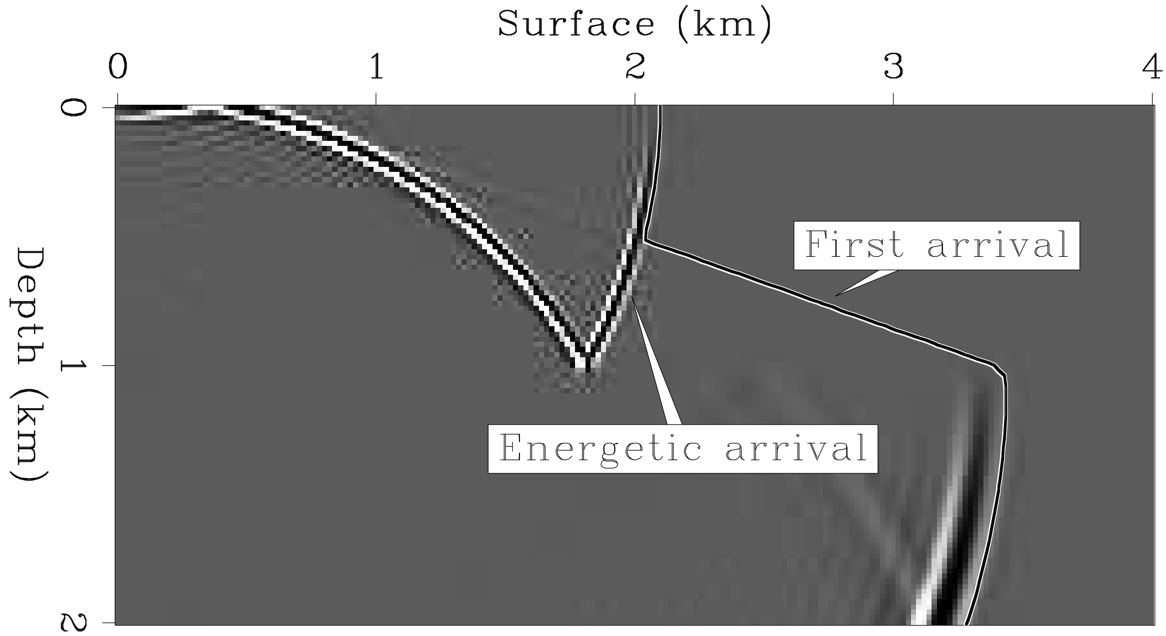

Figure 3: A snapshot of the wave field at 1.4 seconds. The shot is located at the upper left corner. The wave field is simulated by the finite difference solution to the scalar wave equation. Overlain is the 1.4 second contour of the traveltime field calculated by the finite-difference method. intro1-wavelay [ER]

Accurate traveltime gradients are necessary for the computation of geometric amplitudes and residual depth-migration operators (described in Chapter **??**). In Chapter **??**, I introduce a wavefront-propagation method that is more accurate than finite-difference methods. As illustrated in Figure 2, large errors in traveltime gradients occur at the interfaces of high velocity contrasts. Therefore, smoothing velocity functions helps reduce the errors. The wavefront-propagation method described in Chapter **??** uses a continuous velocity representation. The true velocity model is mapped onto a grid formed by regular triangle cells. Within each triangular cell in the grid, velocity function has a constant gradient. No discontinuities exist at the boundaries of adjacent triangles. Velocity contrasts at interfaces are approximated by triangular cells of large velocity gradients. In a constant gradient velocity medium, seismic waves propagate along circular ray paths; thus the method uses local ray tracing to propagate local circular wavefronts. I refer to this method as wavefront propagation by local ray tracing. Compared with finite-difference methods, this method computes accurate traveltimes and amplitudes of Green's functions not only for first arrivals but also for other arrivals. The price paid for accuracy and flexibility is high computational cost. Nevertheless, to obtain comparable results, this method is still more efficient than traditional ray-tracing methods.

Residual depth migration

In each iteration of depth-migration velocity analysis, the recorded reflection data need to be remigrated with the updated velocity model. I refer to the traditional process of completely remigrating with the updated velocity model as full depth migration. Residual depth migration is an alternative process that converts the depth image migrated with the previous velocity model into a new depth image migrated with the updated velocity model.

When perturbations of a velocity model are small, residual migration may be considerably more efficient than full migration. Rothman et al. (1985) describe residual time migration of zero-offset sections. Their analysis shows that a Kirchhoff integral implementation of residual time migration is efficient for two reasons: (1) residual time migration operators can be economically computed because they are identical to full migration operators with residual migration velocities; (2) since residual time-migration operators have smaller apertures than full time-migration operators, their application needs fewer arithmetic operations. Al-Yahya and Fowler (1986) made an effort to extend these results to prestack migration. They concluded that it is possible to construct a residual prestack migration operator, but that the operator is not equivalent to a full prestack migration operator with some residual migration velocity. Their conclusion seems to imply that residual prestack-migration operators are difficult to compute even when migration velocities are constant.

Using a kinematic approach, Etgen (1989) formulated residual prestack migration of constant offset sections in constant velocity media. In his method, the residual migration operators are calculated by solving a pair of nonlinear equations numerically. Examples with synthetic and field data show that the operators work despite troubles near their triplications. I show that the kinematic operators of residual profile migration can be efficiently computed with analytical formulas (Zhang, 1990), provided the migration velocities are constant.

Residual depth migration with constant velocities is rarely useful because depth migration is employed only when velocity models vary laterally. Benson (1989) developed a residual depth-migration algorithm. But the application of the algorithm is limited to zero-offset sections migrated initially with a constant velocity, which is not useful to iterative depth-migration velocity analysis. In Chapter **??**, I present a method for calculating residual depth-migration operators that transform an image migrated with one velocity model to an image migrated with another velocity model. The method uses an approach similar to wavefront propagation, in which the operators are extrapolated from the vicinity of a source where the operators are known. Because the extrapolation is accomplished by using finite-difference techniques, the method is efficient. The method is generally applicable to both post-stack and prestack migration, and to both common-shot and constant-offset geometries. It allows models with "reasonably" smooth velocity functions. A numerical experiment with synthetic data shows that residual depth migration can transform a distorted image migrated with an incorrect velocity model into an image corresponding to the correct velocity model.

Applications to field data

In Chapter **??**, I apply the theories and numerical methods developed in this thesis to a marine dataset recorded offshore in southern California. The dataset is sorted into constant offset sections, and a dephasing filter is used to remove the phase delay of the water-gun wavelet. I estimate the interval velocity model for depth migration by iteratively migrating data and updating the interval velocities until the offset coherence is optimized. In each iteration, the Green's functions used in Kirchhoff prestack depth migration are computed by my wavefront propagation methods. I then use the estimated interval velocity model to demonstrate the equivalence between the application of residual depth migration and full depth migration.

Assumptions

Throughout this thesis, I assume that seismic wave fields recorded on the earth surface propagate through a 2-D, isotropic, elastic medium. The wave fields in such a medium are vector fields consisting of the components of compressional waves (P-waves) and shear waves (S-waves). Each components honors a scalar wave equation. Therefore, I describe the imaging methods for a scalar wave field. The results can be applied to both P-waves and S-waves.

The zeroth-order asymptotic ray approximation to the solution of the scalar wave equation has its validity conditions. The details of these validity conditions are discussed in Červený et al. (1977). In this thesis, I assume that all the validity conditions hold so that the traveltimes and amplitudes can be calculated by solving the eikonal equation and the zeroth-order transport equation (described in Chapter **??**), respectively.