Stanford Exploration Project, Report 75, November 17, 1997, pages 459–479

Stanford Exploration Project, Report 75, November 17, 1997, pages 459–479

Detecting gravitational waves using seismic data

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keywords: *Fourier transform, noise, earthquake, amplitudes*

ABSTRACT

Several years of seismic data acquired by the international deployment of accelerometers (IDA) are used in an attempt to detect gravitational waves radiated from low-frequency astronomical binary sources. These data were preconditioned with various combinations of earthquake removal, gain, and tapering. Directionally weighted spectra were calculated for 0.012 to 0.556 milli-Hz (one day to one-half hour periods) in 84 directions, one year at a time, with correction for the earth's rotation. While patterns similar to the synthetic response from a gravitational-wave source were observed, these patterns showed no consistency when compared between years. This inconsistency suggests that the patterns are noise and not the result of gravitational wave excitation by binary sources.

INTRODUCTION

Gravitational waves are time-dependent waves in the curvature of space caused by moving masses. Boughn and Kuhn (1984, 1990) have searched for a stochastic background of this gravitational radiation by examining the earth's resonant frequencies for excitations. The vibrations caused by gravitational waves are briefly discussed by Misner, Thorne, and Wheeler (1973) and in more detail by Wagoner and Paik (1976). Tuman (1982, page 129) found that the earth's even harmonics show more excitation than the odd harmonics, as would be expected if the earth was being excited by gravitational radiation.

While the previous studies examined the frequencies around the earth's resonant frequencies, I examined a broad range of frequencies corresponding to periods of one day to one-half hour. Also, where the previous studies have considered mainly stochastic or impulsive gravitational radiation, I considered only radiation produced from binary sources. Signals from these pairs of orbiting masses allow examinations of attributes other than signal strength and frequency shape, such as directional, phase, and polarity consistency. Since many stars exist as binaries, larger objects might also

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be expected to be found in binary systems. Long-period records of the earth's oscillations measured by the International Deployment of Accelerometers (IDA) should provide a sensitive detector of the gravitational-wave excitations from these binary sources. Detecting these waves with the earth's vibrations would provide information about the earth's deep interior, since the waves act on the earth uniformly with depth, and might also provide new astrophysical information.

To be detected, a gravitational-wave response must be extracted from the background noise in the IDA data. Orbital periods of most astronomical objects are expected to vary at astronomical time scales, so the frequency of a gravitational wave is expected to be sharply defined within a spectrum. The polarity, phase, and direction of the gravitational wave will also be constant for these sources and allow a further discrimination between the noise and signal at a given frequency. Frequency shape and directional consistency are considered as the criteria for detecting a binary source. Polarity and phase measures are not considered here, because they are likely to be useful only after the other measurement criteria are satisfied. To produce these measures, earthquake events were removed from the IDA data, and were then are gained to attenuate noise and to strengthen the weak signals, and finally, directional spectra are produced by weighting by direction and Fourier transforming.

Patterns similar to a synthetic response to a gravitational-wave source were observed in a directional spectrum, but these patterns showed no consistency when the same frequency was compared from year to year, and little consistency was observed even when the preprocessing was changed within the same year. These inconsistencies between years suggest that the patterns are noise caused by earthquakes and muting patterns for each station.

While no signals were detected from these data, more data, better noise attenuation algorithms, and a more sophisticated model of the earth's response might allow weaker signals to be extracted from the noise.

MOTIVATION

This attempt to detect strong gravitational waves was made possible by the availability of several years of high-quality seismic data from a global seismic array. The data from the IDA network provides long-term low-frequency measurements of the earth's oscillations. The data had been cleaned of the instrument problems and other noise for a project to examine slow and quiet earthquakes, leaving only the strong earthquake noise to be removed.

The possibility of having large objects (Jayawardhana,1992) that might produce detectable waves if present as small-period binary systems gave at least a small possibility of detection. While Boughn and Kuhn(1984) and Tuman(1982) have examined seismic data for excitation from gravitational waves, they concentrated on the earth's resonant frequencies. Large amplitude waves away from the resonant frequencies might not be recognizable, although Tuman found that the even harmonics show more excitation than the odd harmonics as would be expected if they were being stimulated (Tuman, 1982, page 129).

This project helped prepare the Stanford Exploration Project for processing 3-D datasets. While the imaging done was crude and many complicating factors were ignored, the data preparation and imaging still required a large amount of computation. The IDA data is similar in size to a small 3-D dataset, and processing these data did show some weaknesses in the SEP's data-handling capabilities. This project did show that analyzing large data volumes was practical.

If gravitational waves had been detected, the earth's response might have provided useful information about the deep interior of the earth, since the waves excite the earth uniformly. Most of the information now available about the deep interior is derived from near-surface earthquakes sources. Detection might also have provided interesting astronomical information.

GRAVITATIONAL WAVES

Gravitational waves are waves in the curvature of space caused by rapidly moving masses. These waves are predicted by the theory of general relativity but have not yet been directly observed, although small changes in the orbital timings of the close pulsar PSR 1913+16 provide indirect evidence of their existence (Thorne, 1987). The gravitational waves considered here are those caused by binary systems similar to, but larger than PSR 1913+16, where two large masses orbit each other.

Gravitational waves are transverse waves; the displacements caused are perpendicular to the direction of motion. The pattern of displacements for the two polarities are shown in Figure 1. These displacements could not be detected by accelerometers placed around the test rings in Figure 1 because space itself is distorted, but a displacement will be detectable across the ring. These displacements distort the earth as a gravitational wave passes, and the response of the earth to these displacements will be detectable if the gravitational wave is large enough.

THE INTERNATIONAL DEPLOYMENT OF ACCELEROMETERS

Project IDA (International Deployment of Accelerometers) is a global array of accelerometers that collects low-frequency seismic data (Agnew, et. al., 1986). This network has been in place since 1975, and has had as many as 18 stations in operation between 1978 to 1987. The original purpose of the network was to collect data for earth structure and earthquake mechanisms, and it has been used to detect slow and silent earthquakes (Beroza and Jordan, 1990). The data from 1978 to 1987 were available, although only 1984 to 1987 were considered carefully because of the sparse global coverage in the earlier years. The stations now have an almost uniform distribution around the globe and respond to frequencies from about one cycle per

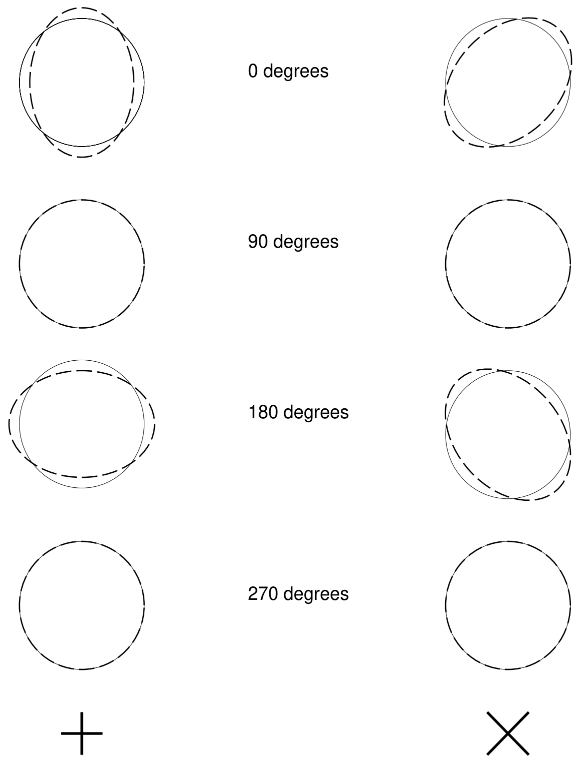

Figure 1: The distortion caused by the + and × polarities at various phases of gravitational wave cycle. The dotted line is the distorted ring and the solid line is the reference. The direction of propagation is into the page. ray1-polarities [NR]

minute to below one cycle per day. An example of a month of the data is shown in Figure 7 later in this paper. Figure 2 shows the locations of the accelerometers.

The amplitude and phase response of each station is plotted in Figures 3 and 4. Note that the amplitudes and the phases of the stations are similar enough not to require removing the response before imaging, except for the two stations with reversed polarities. These were corrected in the processing. The station showing the high response in Figure 3 did not exist in the data processed here.

TARGETED OBJECTS

Low-frequency gravitational waves produced by two orbiting masses are the targets of this effort. The advantage of considering these binary sources is that they will be continuous and reproducible as opposed to the impulses targeted by other gravitational wave detection efforts. The disadvantage is that the signal strength is likely to be weak unless the target is a binary system with an extremely large mass, or a system with a moderately-sized mass that is close to earth.

The period of the generated gravity waves will be half the orbital period of the binary system if the orbits are circular. If the orbits have significant eccentricity, much of the generated gravitational wave energy will be emitted at the higher frequency harmonics of the orbital periods. Thus, a system with an eccentric orbit might produce significant energy in the frequency range being analyzed here, where a system

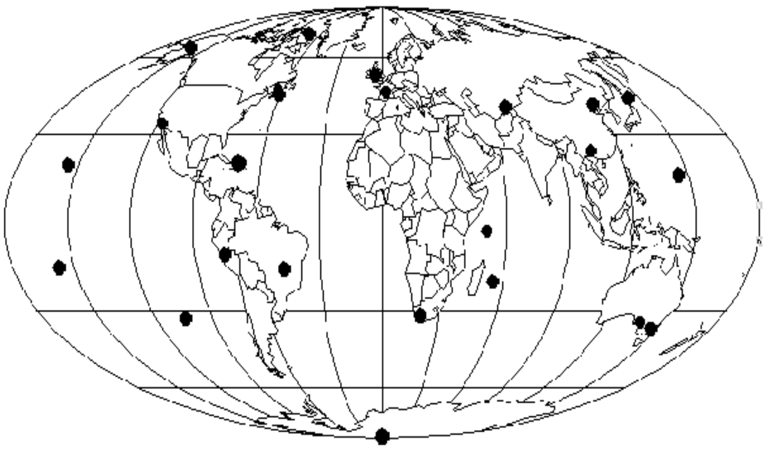

Figure 2: Accelerometer positions. ray1-world [NR]

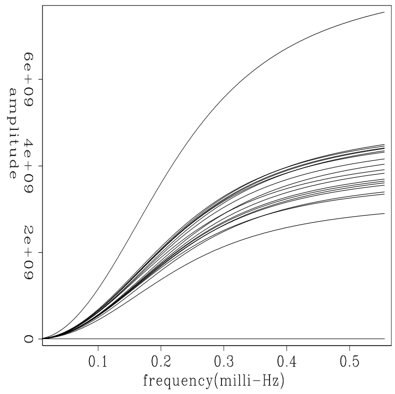

Figure 3: The amplitude responses of the IDA stations. The unusual amplitude response is from station 14, MCM, which did not exist in our dataset. ray1-amp [ER]

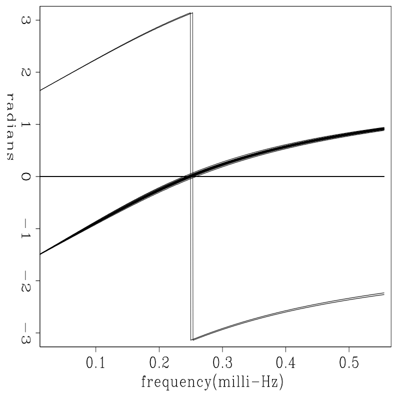

Figure 4: The phases of the IDA stations. The two unusual phase responses are from stations 10 and 20, GUA and SPA, i.e. Guam and the South Pole, indicating the polarities are reversed there. ray1-phase [ER]

with the same mass and a circular orbit with the same period would not (Peters and Mathews, 1963, Barone, et. al., 1987).

Previous attempts to detect the effects of low-frequency gravitational waves on the earth have focused on the excitation of the earth's normal modes by frequencies close to the resonant frequencies. The events targeted by these attempts would be either impulses or a stochastic background. Only a narrow frequency range around the resonant frequencies was covered. In this paper, I am considering a larger range of frequencies corresponding to periods of one-half hour to 24 hours. While the sensitivity of the earth to gravitational waves at the frequencies away from the resonant frequencies will be low, low-frequency undertones (Aki and Richards, 1980, Perkeris and Accad, 1972) may increase the expected sensitivity of the earth at frequencies lower than that of the 54-minute resonance. While no large or close binary systems that would produce detectable motions are known to exist, they cannot be ruled out.

Other attempts to detect gravitational waves have targeted catastrophic events such as supernovae and stellar collisions and have concentrated on high-frequency, short-duration impulsive signals that may be detected by the vibrations of large cylinders isolated from the earth and cooled to reduce thermal vibrations (Douglas and Braginsky, 1979) With these detectors, it was assumed that an impulsive arrival would excite the detector at its resonant frequency. Since impulsive events tend to be rare and generally are not reproducible, confirming any detected event would be a problem unless multiple detectors were used. Using the earth's response to these short-term events would be difficult since the integration over time is not possible as it is with constant-frequency waves, and short-term events would be generally indistinguishable from seismic noise.

The orbital frequency ω of a pair of masses is

$$\omega = \sqrt{\frac{G(m_1 + m_2)}{a^3}} \tag{1}$$

where a is the radius of the orbit, m_1 and m_2 are the masses of the bodies, and G is the gravitational constant. From Schutz (1988), the amplitude of the waves will go as

$$A \propto \frac{ma^2\omega^2}{r} \tag{2}$$

or

$$A \propto \frac{m^{5/3}\omega^{2/3}}{r} \tag{3}$$

or

$$A \propto \frac{m^2}{ra} \tag{4}$$

where r is the distance from the binary system, and $m = m_1 + m_2$. The wave amplitude gets larger with smaller radius and gets larger quickly with increasing mass.

For example, a pair of pulsars, PSR 1913+16, appears to be showing changes in its orbits from the emitted gravitational radiation. While the radiation from this system is too weak to be detected with the IDA network, larger objects with smaller orbital periods might be detectable. While the pulsars in PSR 1913+16 have masses of about 1.4 solar masses, larger objects, such as large black holes are better targets. Where $M_\odot$ is one solar mass, a billion $M_\odot$ binary system at the center of our galaxy would produce gravitational waves with an amplitude of 9×10^{-8} if it had an orbital period of 8 hours. This compares to an amplitude of about 10^{-20} for PSR 1913+16. There is some evidence that black holes exist in the center of certain galaxies, and other smaller and closer black holes have been strongly suspected (Jayawardhana, 1992). With objects having masses from $1M_\odot$ to $10^{12}M_\odot$ available to produce gravitational waves, a weak response of the earth to these waves may be enough for detection. It has been suggested that a 3 $\times 10^6 M_\odot$ object, probably a black hole, is at the center of our galaxy (Blandford, 1987). If this is a binary object with a short period, it might be detectable using the IDA network data.

While the direct detection of binary systems with periods greater than 24 hours is difficult, systems with orbits having significant eccentricities will generate waves with shorter periods. With greater eccentricity, more power will be output from the system (Peters and Mathews, 1963) and much of this power will be radiated at the higher harmonics of the orbital frequency (Barone, et. al. 1987). As an example, an object with an orbital period of 48 hours and an eccentricity of 0.75 will have the maximum energy radiated at a period of about 2 hours.

RESPONSE OF THE EARTH TO GRAVITATIONAL WAVES

Following Schutz(1988), the response of two masses connected by a spring to a gravitational wave with amplitude $h^{tt}_{xx,00}$ is controlled by

$$\xi_{,00} + 2\gamma\xi_{,0} + \omega_0\xi = \frac{1}{2} l_0 h^{tt}_{xx,00} \tag{5}$$

where ξ is the displacement, γ is the damping factor, ω_0 is the resonance frequency, and l_0 is the length of the spring. $\xi_{,00}$ is the second derivative with respect to time, and $\xi_{,0}$ is the first derivative with respect to time. $h^{tt}_{xx,00}$ is the amplitude of the wave as referred to by Schutz(1988).

The solution to this equation when $h^{tt}_{xx,00} = A\cos\Omega t$ is

$$\xi = R\cos(\Omega t - \phi) \tag{6}$$

where

$$R = \frac{\frac{1}{2} l_0 \Omega^2 A}{\sqrt{(\omega_0 - \Omega)^2 + 4\Omega^2\gamma^2}} \tag{7}$$

and

$$\tan\phi = \frac{2\gamma\Omega}{\omega_0^2 - \Omega^2} \tag{8}$$

Ω is the wave's frequency.

If $\omega_0^2 - \Omega^2$ is large with respect to $2\gamma\Omega$, ϕ will approach zero and ξ will be proportional to h_{xx}^{tt}.

I make a crude assumption that a column in the earth will react like a spring. Away from resonance, the response of the earth to a gravitational wave will be proportional to the strain $\cos(2\phi)\sin^2(\theta)\cos\Omega t$, where ϕ is colatitude and θ is longitude as given by Hellings (1992). This strain on a sphere for $t = 0$ is shown at an exaggerated scale in Figure **??**.

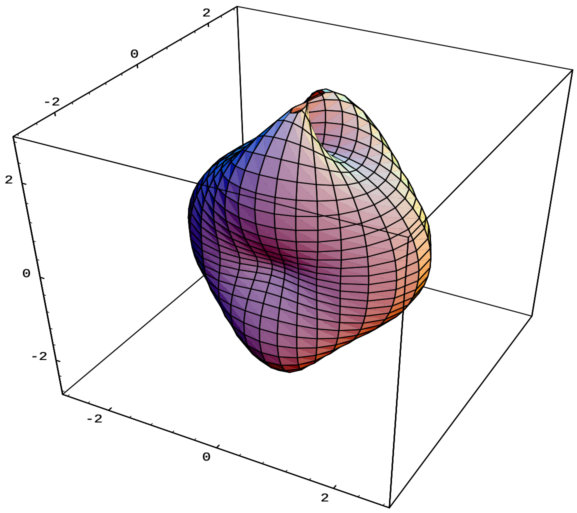

Figure 5: An exaggerated view of the strain on the earth caused by a gravitational wave. The function plotted is $(\cos(2\phi)\sin^2(\theta)) + 2$. ray1-GWresp2 [NR]

If this response is close to the correct response, the strain of the earth recorded by the IDA network can be predicted. Figure 6 shows the response of the earth as it would appear to the available stations to a source at position 43 of the grid in Figure 11, using these approximations.

PROCESSING SEQUENCE

The data were first converted from the original format into a format suitable for processing with the available seismic software. Next, it was sorted into time and station order by groups of one year. A sample of the sorted data shown in Figure

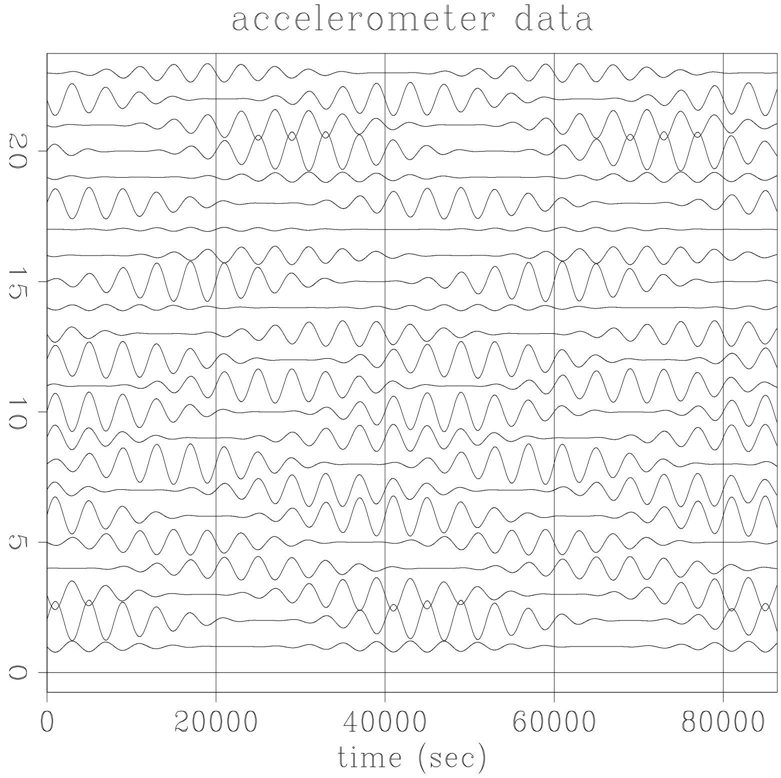

Figure 6: The response of the IDA stations to a gravitational wave source at position 43 of the grid in Figure 11. Each trace corresponds to a station in the IDA array. ray1-daysrc43 [NR]

7 shows earthquakes, noisy traces, and dead zones. The noisy traces are probably caused by water-wave action at stations located on small islands. For example, trace 16 in Figure 7 is from the station located at Rarotonga, on the Cook Islands. The missing data is caused by stations not in operation, data lost at tape changes, or data removed in the original cleaning process.

To separate the response of the earth to gravitational waves from the seismic background, high amplitude earthquake events need to be removed from the data. I was fortunate to have data with instrument spikes and much other non-seismic noise manually removed. To remove the earthquake noise, a program scanned each station's data for the maximum value, then if this maximum value was above a given threshold, the samples recorded one hour before and five hours after were zeroed. After zeroing, the station's samples were once again searched for the maximum value, and if that maximum value was again above the threshold, the zeroing and searching process was repeated until the maximum amplitude was below the threshold. Figure 8 shows that the large earthquakes were removed, but smaller events are still present. Notice that the noisy traces also had some zeroed zones. If the maximum value considered to be an earthquake event is lowered, more data are removed, and much of the signal may be lost.

This earthquake removal process could be improved by changing the mute time according to the strength of the earthquake, or by removing the earthquakes according to the tables of earthquake events that are available. The process used has the advantage of simplicity. Many small spikes still remain in the records. These spikes

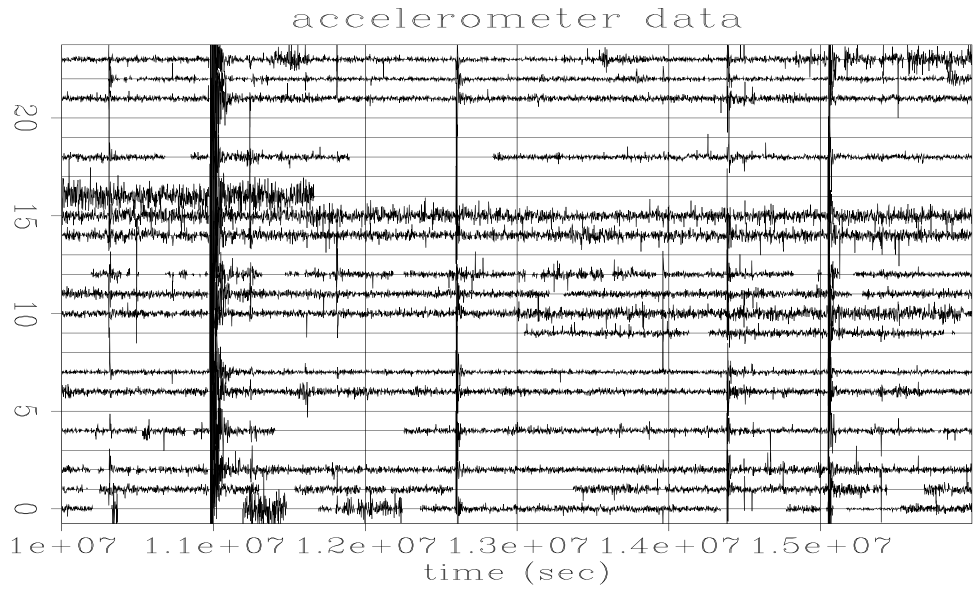

Figure 7: A one month sample of the IDA data. ray1-sort [ER]

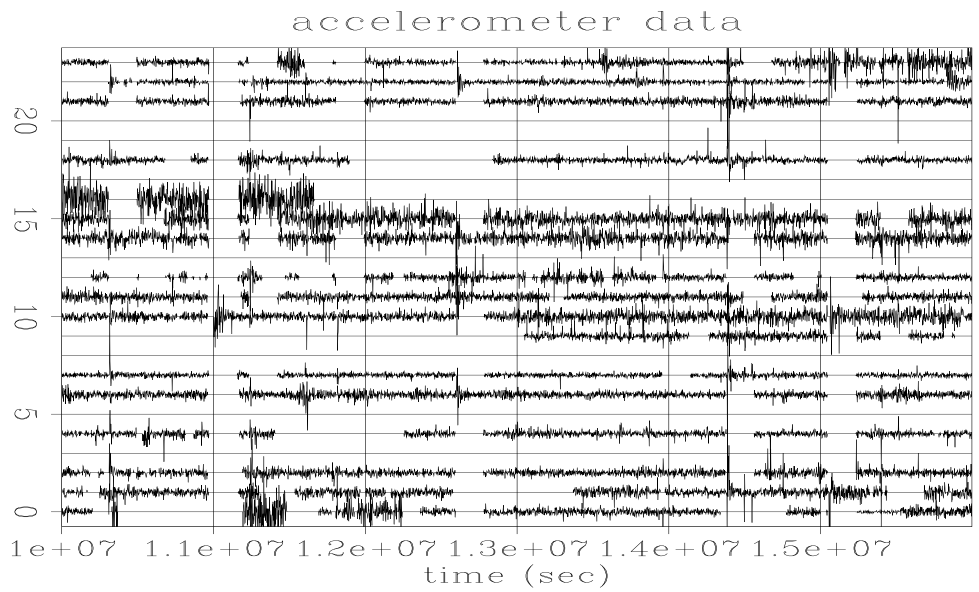

Figure 8: The sample in the previous figure with the earthquake noise removed. ray1-zero [ER]

could have been removed by a median filter.

To attempt to produce a sharp response in frequency to any signals in the data and to further suppress noise and enhance weak signal, the data were gained. In the gained examples, a 160 minute, or 480 sample sliding gain window was used. Zeroed data were ignored by this gain to avoid undesirable effects near the ends of zeroed zones. As seen in Figure 9, the traces are better equalized, but spikes remain.

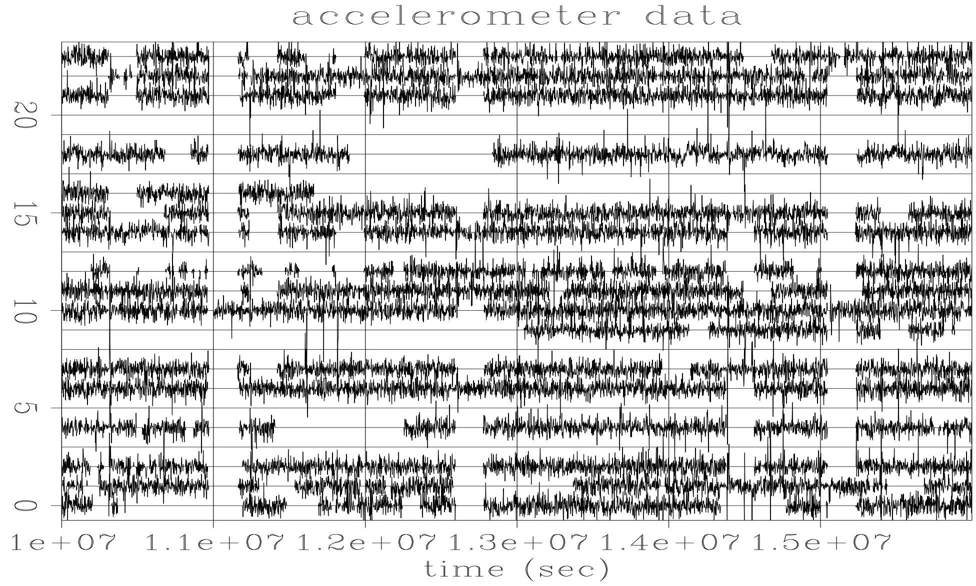

Figure 9: One month of recording with earthquake noise removed, then a gain applied. $\boxed{\text{ray1-gain}}$ [ER]

The data for each station were tapered around the zones zeroed by the previous processes with simple cosine tapers. The length of the taper was 25 hours, or 4500 samples. This taper length was used to keep the effects of the truncation away from the frequency range of interest. Before tapering, small groups of non-zero samples were removed, and during tapering, small groups of zeros were ignored to prevent excessive overlapping of the tapers. While this process eliminates much of the data, it also should avoid creating noise in the spectra. Figure 10 shows an example of this process.

Two different attempts at detecting gravitational wave excitations were done. The first attempt was to correct the data for the rotation of the earth with respect to fixed directions in space, and then produce three spectra corresponding to three perpendicular directions in space. The second attempt was to correct for the rotation of the earth, then calculate the spectra for both polarities in many directions.

The first method would have provided some indication of direction, but the signal strength would have had to be high to detect an event. This method had the advantage of being fast. The second method, that of examining many directions, would allow a more reliable measure of directional consistency. The results shown here are

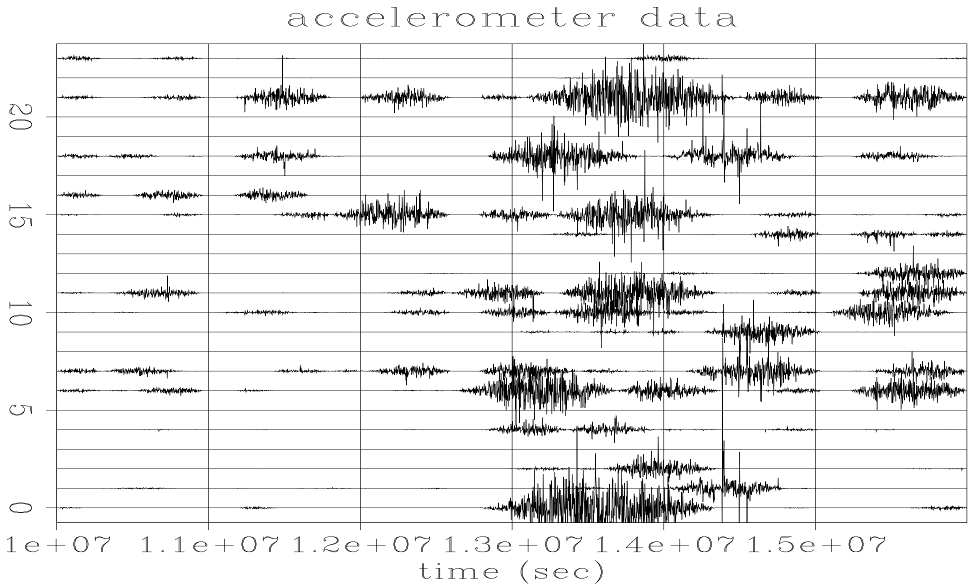

Figure 10: One month of recording with earthquake noise removed, gain applied, and tapered. ray1-taper [ER]

from the second method. The second method is slow since it calculates spectra for 84 directions and two polarities for each direction. The polarization could have been used to determine the consistency in direction of a signal in time. For binary systems, a constant direction and polarization is expected.

The directions scanned for possible gravitational waves are shown in the grid displayed in Figure 11. Only one hemisphere was examined, since the excitation caused by a gravitational wave in one direction is the same as the excitation caused by a wave moving in the opposite direction. The numbers shown in Figure 11 correspond to the trace numbers shown in Figure 12 and the other directional spectra. The center of the figure is north.

The instrument response was not removed from the data since it is expected to be constant between stations at the frequencies of interest. Removing the response also increases the energy at low frequencies making the data unbalanced and difficult to display.

Ideally, the response of a gravitational wave would be a spike in frequency on the spectra. This spike will be smoothed somewhat by the earth's orbit about the sun, but it will be a small change at the low frequencies considered here. The frequency increment corresponding to the Fourier transform of a data sequence of a year is 3.17×10^{-8} Hz. For signals with one-hour periods, the frequency change from the earth's orbital velocity is 2.28×10^{-8} Hz. For 10 hour periods, the change is 2.28×10^{-9} Hz. This smear of the response is likely to be small compared to the effects of the earth's response and to the muting, gaining, and tapering of the time series to remove noise.

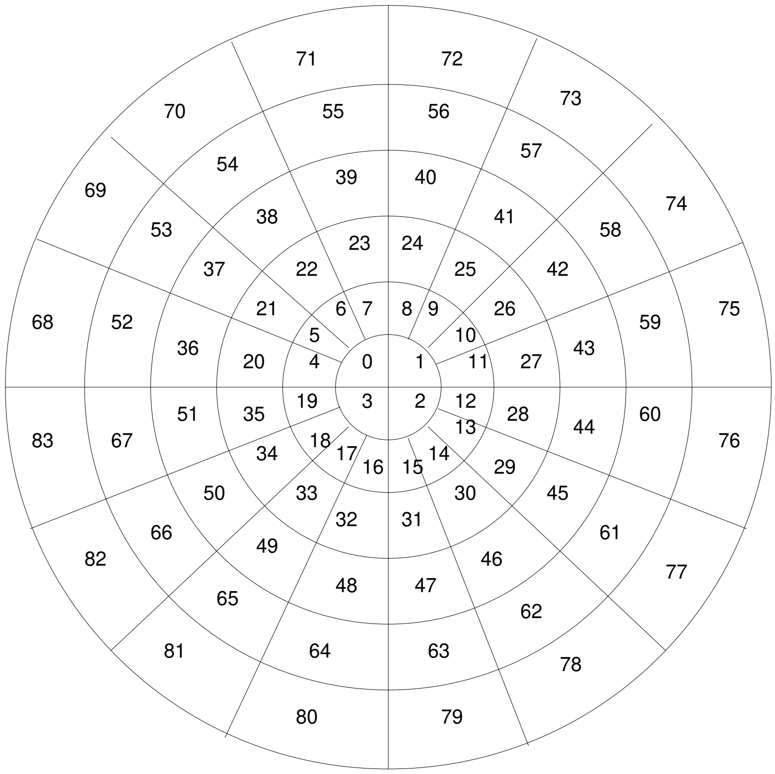

Figure 11: Imaging positions. ray1-dartboard [NR]

RESULTS

The synthetic response to a gravitational wave source in position 43 of Figure 11 is shown in Figure 12. This figure shows a set of spectra weighted to the directions shown in Figure 11 by match filtering against the predicted gravitational wave response. Each trace number corresponds to a direction shown on the grid in Figure 11. The spectra peak is at 0.25 milli-Hz, and the maximum amplitude is at trace 43 as expected, but the definition of the direction is poor.

The directionally weighted spectra for the raw data without gaining, earthquake removal or tapering is shown in Figure 13. The spectra are generally noisy, and no strong response similar to that in Figure 12 is seen, but there is a promising weak event near 0.244 milli-Hz.

The original data then had a taper applied to the nonzeroed portions to remove the effects of the sharp truncations. This result is shown in Figure 14. Notice that the spectra are still noisy, indicating that the earthquake and other noise are still dominating. The event at 0.244 milli-Hz is clearer, but still not convincing. When the 1986 spectra in Figure 14 is compared to the 1985 spectra in Figure 15, there is no similarity in the spectra at 0.244 milli-Hz. The event is then rejected as a possible gravitational wave signal. Similar events were found in other years and at other frequency ranges.

To reduce the effect of the noise caused by the high amplitude earthquakes and

Figure 12: The directionally weighted spectra of a synthetic gravitational wave source at position 43 on the grid in Figure 10. The signal peak is at 0.25 milli-Hz. Notice the weak ghost at the left caused by the corrections for the earth's rotation. ray1-specsrc [NR]

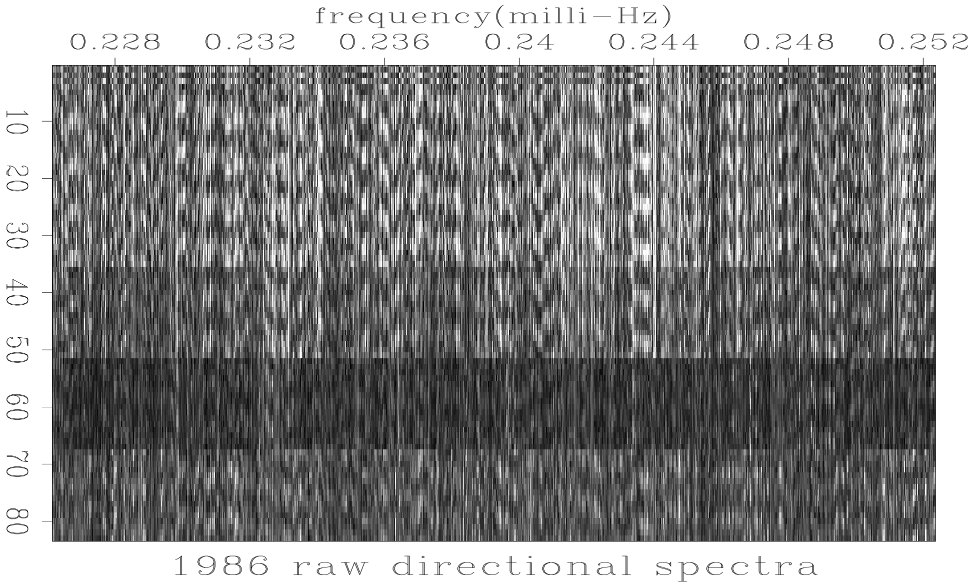

Figure 13: A sample of the directionally weighted spectra for 1986 for the raw input. ray1-rawspec [NR]

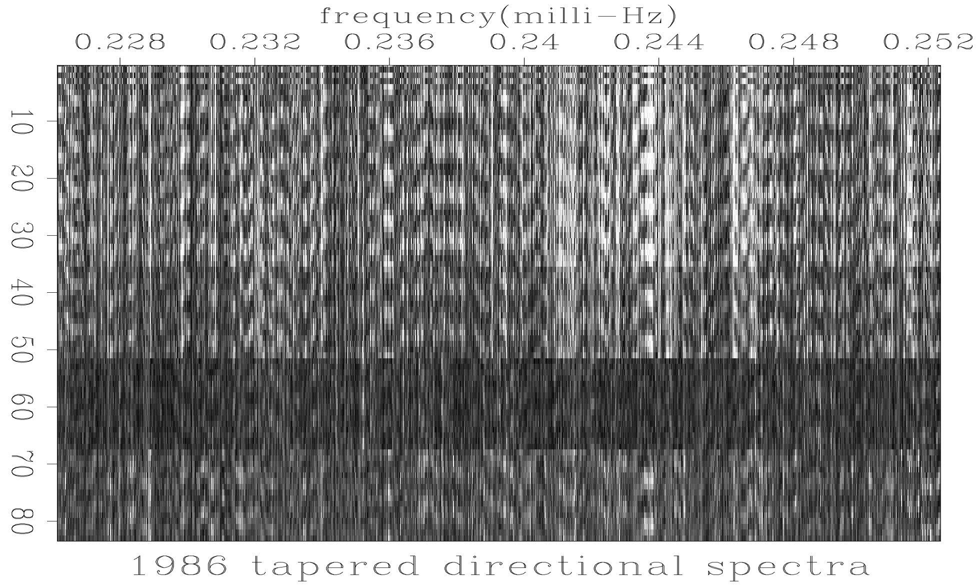

Figure 14: A sample of the directionally weighted spectra for 1986 for the tapered input. ray1-tprspec [NR]

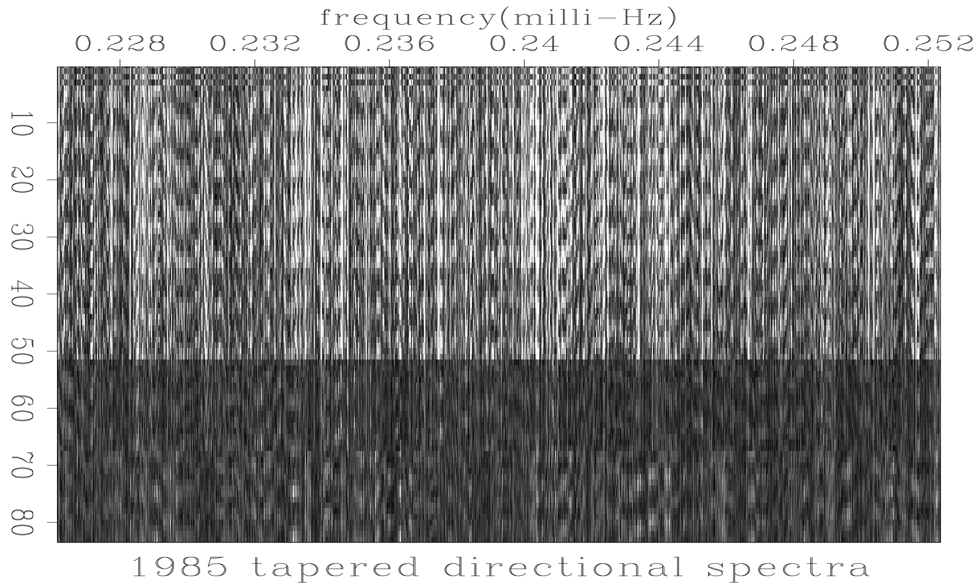

Figure 15: A sample of the directionally weighted spectra for 1985 for the tapered input. ray1-tprspec85 [NR]

ocean noise, a gain was applied to the original data. This equalized each station's amplitude, reduce the ocean noise, and attenuated the earthquake events. The directional spectra produced from these data are shown in Figure 16. Notice that the event at 0.244 milli-Hz is now much weaker. The amplitude differences between the rings of Figure 11 are much less pronounced.

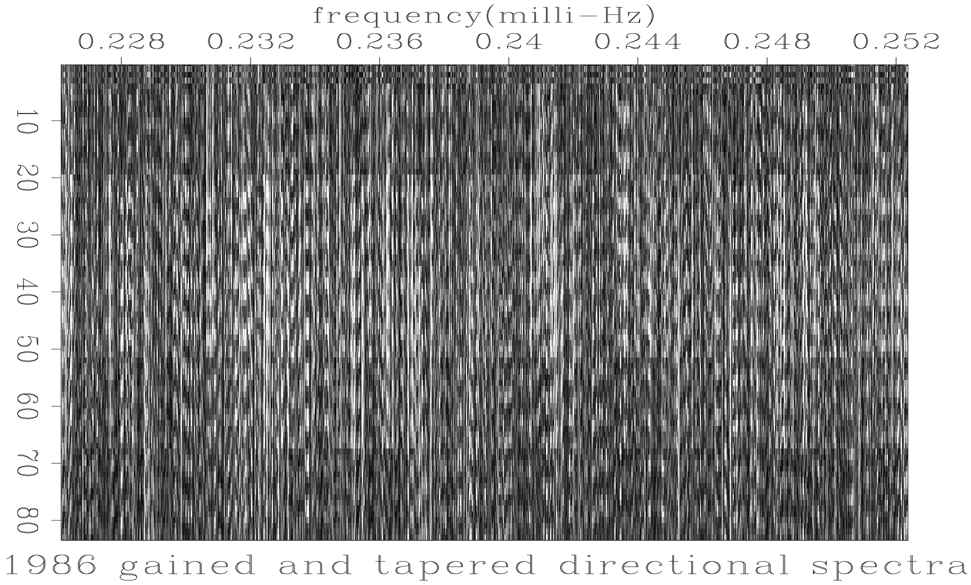

Figure 16: A sample of the directionally weighted spectra for 1986 for gained and tapered input. ray1-gtprspec [NR]

Next, the earthquake noise was removed by the process described previously, followed by the gain and taper processes. The spectra are now flatter and the event at 0.244 milli-Hz is even weaker. For comparison, the same frequency range is displayed in Figure 18 for 1985.

Finally, an interesting effect is seen in the low frequency part of the directionally-weighted spectra. In Figure 19 a series of events are seen spaced at 0.012 milli-Hz spacings. These correspond to even divisors of a 24 hour period.

POSSIBLE IMPROVEMENTS

The most obvious improvement to the techniques used here is to weight the data to account for the missing stations and for the missing data within each station. The resulting spectra show an imbalance in the imaging directions, and each set of spectra show noise that is probably caused by the muting and tapering.

The response of the earth to gravitational waves of various frequencies could be better defined than that of the crude assumptions used here by modelling using the known structure of the earth. The effects of the earth's rotation should also be included in the response.

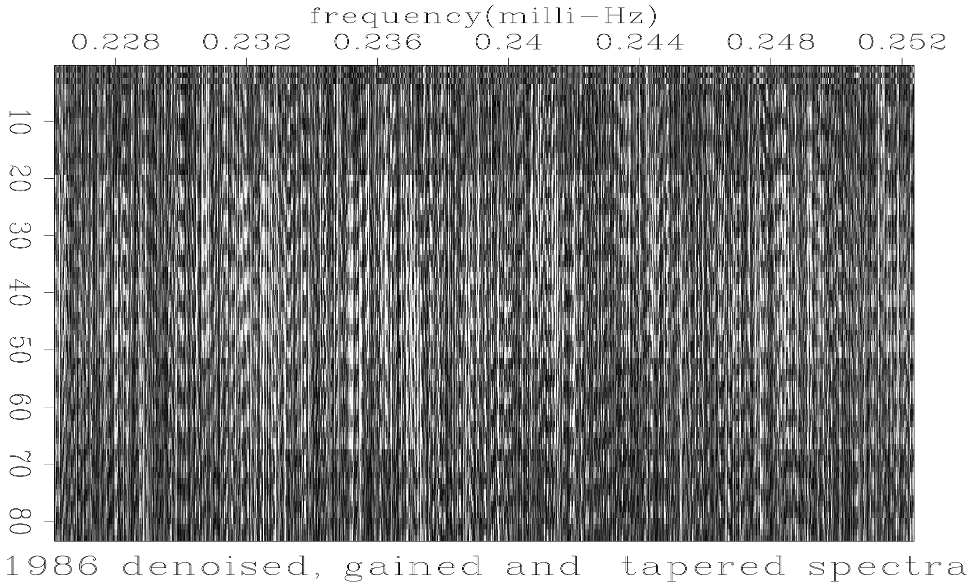

Figure 17: A sample of the directionally weighted spectra for 1986 with the earthquake noise removed, then gain and tapering applied. In the electronic version of this paper, pushing the button will review the full range of the spectrum. ray1-zgtspec [NR]

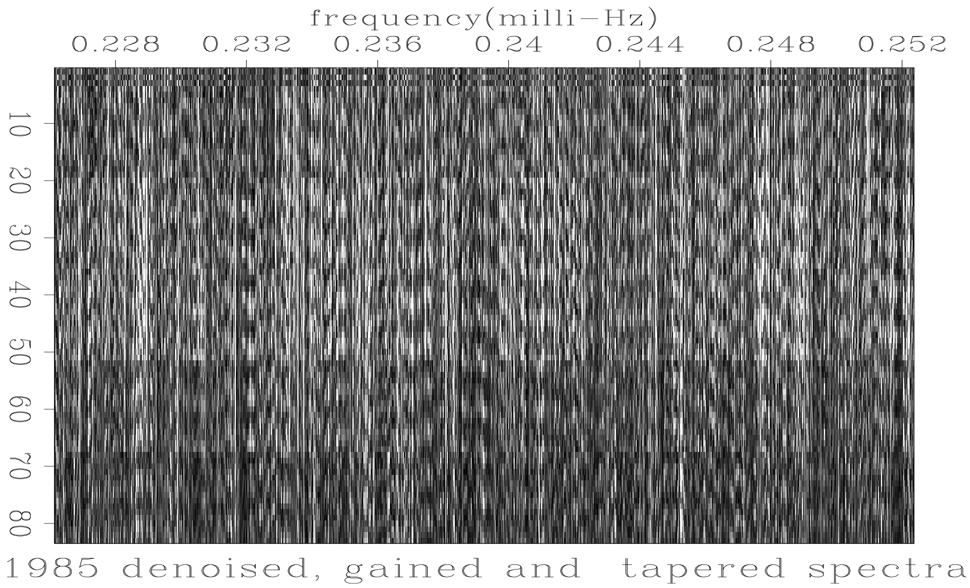

Figure 18: A sample of the directionally weighted spectra for 1985 with the earthquake noise removed, then gain and tapering applied. In the electronic version of this paper, pushing the button will review the full range of the spectrum. ray1-zgtspec85 [NR]

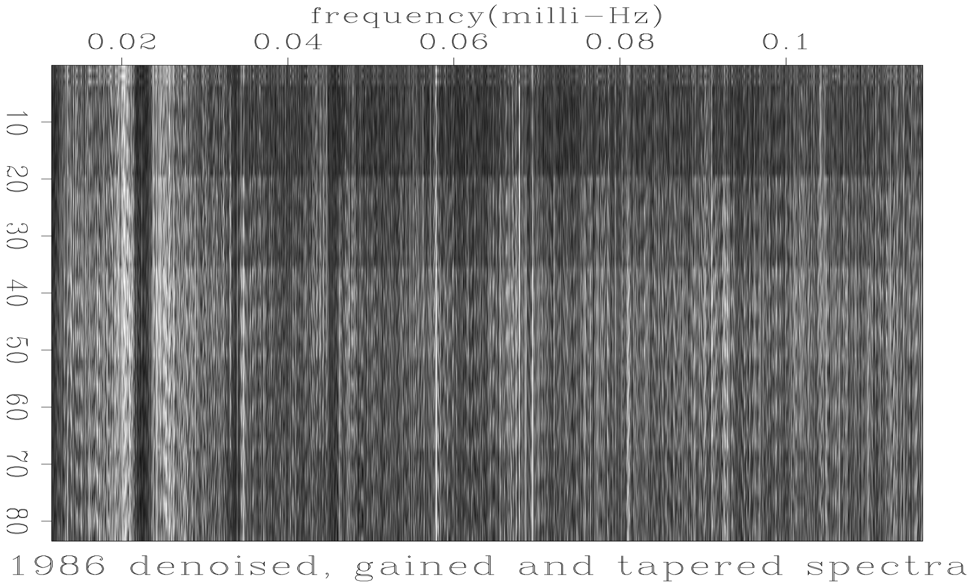

Figure 19: The low-frequency part of the 1986 directionally-weighted spectra. Notice the events spaces at 0.012 milli-Hz intervals. ray1-lowfreq [NR]

The earthquake removal and muting of noise-dominated stations could be improved. Smaller earthquakes might be ignored. Large earthquakes might have better time constraints on the mutes. The small spikes remaining after the earthquake removal should be removed, perhaps with a median filter. The effects of the taper and gain should be examined with more care.

To provide some expected responses, a numerical connection between a given binary system and the earth's response should be made. It appears the the most difficult part of this problem is defining the details of the earth's response.

The output spectra might be better displayed as an amplitude chart similar to the dartboard form of Figure 11. These results might also be deconvolved to produce better resolution of the source position, making identification of weak sources easier over several years.

CONCLUSIONS

No indications of the earth's response to gravitational waves were found. The background noise of ocean waves and earthquakes limits the sensitivity of the earth as a gravitational wave detector. Removing the earthquake and ocean-wave noise removes a significant amount of data and reduces the sensitivity of the earth as a detector. It is possible that this noise sets a high upper limit to the strength of the gravitational waves that can be detected, even with long records of the earth's vibrations using sensitive detectors. A system of detectors like the international deployment of accelerometers placed in a quieter environment, such as the moon, could lower this detection limit.

If the response of the earth to gravitational radiation were available, an upper limit to the strength of these waves could be determined for the range of frequencies examined here. While the response of a homogenous sphere is known, the response of the earth is complicated by its structure. While calculating these limits is beyond the scope of this paper, these limits might produce interesting limits on the types of systems that can exist nearby.

ACKNOWLEDGEMENTS

I am especially grateful to Greg Beroza for allowing me to use his cleaned version of the IDA datasets, for providing the utility software, and for answering my many questions about the data. He and his students spent several man-months doing the manual cleaning of the datasets. I also owe thanks to Martin Karrenbach and Carlos Cunha Filho for helping me with my Connection Machine codes. Everyone at the SEP has been patient when I needed large amounts of disk storage, when both the tape drives were tied up with the large datasets, when the network became clogged with my data, and when I used the Connection Machine for hours at a time. I am most grateful to Jon Claerbout for allowing the work on this somewhat unconventional project.

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