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Short Note

Wavefield extrapolation in heterogeneous media using the LITWEQ method

Daniel L. Mujica[1]

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INTRODUCTION

Time migration algorithms have the disadvantage of erroneously handling lateral velocity variations, even if the correct velocity is provided (Hatton *et al.*, 1981). Additionally, algorithms based on the one-way wave equation cannot handle steep dips correctly. Depth migration seems to be the answer to time migration weaknesses. This explains why it is necessary to develop depth migration algorithms. Li (1986), presents a theoretical model that he calls Linearly Transformed Wave Equation (LITWEQ) to do migration in lateraly varying media. However, it was only implemented for vertical velocity variations.

I propose a numerical method which allows us to incorporate lateral velocity variations. The method discretizes completely the LITWEQ operator. This discretization is performed with a nine point star pattern finite difference scheme. The circular shape of the impulse response shows the capability of this algorithm to migrate events with arbitrary angle of inclination. Further tests are necessary to show the stability of the scheme in the presence of strong lateral velocity variations.

REVIEW OF LITWEQ OPERATOR

LITWEQ method is based on the scalar wave equation for variable velocity media,

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} = \frac{1}{v^2(x,z)} \frac{\partial^2 P}{\partial t^2}. \tag{1}$$

The basic idea in this method is to rewrite the 2-D scalar wave equation in a new coordinate system. This allows us to produce uniformly space grids and to approximate the solution of (1) by using finite difference. First, let's define an auxiliary

[1]**email:** not available

variable τ sometimes called pseudo-depth,

$$\tau = \int_0^z \frac{1}{v(x,\theta)} d\theta. \tag{2}$$

Second, let's define a reference velocity to measure the lateral velocity variation in the media:

$$\frac{1}{v^2(z)} = \frac{1}{nx} \sum_{i=1}^{nx} \frac{1}{v^2(x_i,z)}, \tag{3}$$

where nx represents the number of points in the common mid point direction. With the definitions above it is possible to rewrite (1) in terms of (2) and (3) as

$$\frac{\partial^2 P}{\partial x^2} + \frac{1}{v^2(\tau)}(\frac{\partial^2 P}{\partial \tau^2} - \frac{\partial^2 P}{\partial t^2}) = \frac{1}{v^3(\tau)} \frac{\partial v(\tau)}{\partial \tau} \frac{\partial P}{\partial \tau} + (\frac{1}{v^2(x,\tau)} - \frac{1}{v^2(\tau)}) \frac{\partial^2 P}{\partial t^2}, \tag{4}$$

where $v(\tau)$ and $v(x,\tau)$ are the corresponding $v(z)$ and $v(x,z)$ using the definition of τ.

Equation (4) is defined on the (X,τ,t) domain which is independent of the velocity. This is because we have already integrated along it when defining τ in (2). Now, we are ready to define a linear transformation (independent of the velocity) that incorporates the solutions of (4) along their characteristic lines. The linear transformation is:

$$t_1 = \frac{1}{\sqrt{2}}(\tau + t), \tag{5a}$$

$$t_2 = \frac{1}{\sqrt{2}}(\tau - t). \tag{5b}$$

Using the chain rule for scalar fields is easy to show the following relations for this particular transformation,

$$\begin{aligned} \frac{\partial}{\partial \tau} &= \frac{1}{\sqrt{2}}(\frac{\partial}{\partial t_1} + \frac{\partial}{\partial t_2}), && (6a) \\ \frac{\partial^2}{\partial \tau^2} &= \frac{1}{2}\frac{\partial^2}{\partial t_1^2} + \frac{\partial^2}{\partial t_1 \partial t_2} + \frac{1}{2}\frac{\partial^2}{\partial t_2^2}, && (6b) \\ \frac{\partial^2}{\partial t^2} &= \frac{1}{2}\frac{\partial^2}{\partial t_1^2} - \frac{\partial^2}{\partial t_1 \partial t_2} + \frac{1}{2}\frac{\partial^2}{\partial t_2^2}. && (6c) \end{aligned}$$

Finally, using (5) and substituting (6) in (4) we get the LITWEQ operator:

$$\begin{aligned} \frac{\partial^2 P}{\partial x^2} + \frac{2}{v^2(\tau)} \frac{\partial^2 P}{\partial t_1 \partial t_2} &= \frac{1}{2v^3(\tau)} \frac{\partial v}{\partial \tau}(\frac{\partial P}{\partial t_1} + \frac{\partial P}{\partial t_2}) \\ &+ (\frac{1}{v^2(x,\tau)} - \frac{1}{v^2(\tau)})(\frac{\partial^2 P}{\partial t_1^2} - 2\frac{\partial^2 P}{\partial t_1 \partial t_2} + \frac{\partial^2 P}{\partial t_2^2}). \qquad (7) \end{aligned}$$

A detailed derivation of (7) is presented in Li (1986).

IMPLEMENTATION OF LITWEQ BY FINITE DIFFERENCE

Li (1986) presents the finite difference discretization of (7) with the diffraction and reverberation terms. He didn't include the second term at the right hand side of (7), usually called thin lens term. I am going to show a full LITWEQ discretized operator using a second order finite difference scheme. This scheme should be tested with synthetic or real data that has strong lateral velocity variations to check its stability.

The finite difference scheme uses a nine point star pattern on (t_1, t_2). Before the discretization, we need to define the linear operator L which represents a convex combination of the points in the star:

$$Lu(t_1, t_2) = \alpha_1 u(t_1, t_2) + \cdots + \alpha_9 u(t_1 + 1, t_2 + 1), \tag{8}$$

where u represents the approximate solution to the wave-field. Using L, the second and first order differential operators can be approximated as follows:

$$\frac{\partial^2 P}{\partial x^2} \approx -\frac{1}{(\Delta x)^2} T Lu(t_1, t_2), \tag{9}$$

$$\frac{\partial P}{\partial t_i} \approx \frac{\sqrt{2}}{\Delta t} F_i Lu(t_1, t_2), \tag{10}$$

where T represents the classical centered finite difference scheme to approximate a second order derivate. F_1 and F_2 represent forward schemes to approximate the first order derivate, along the t_1 and t_2 respectively. Substituting (9) and (10) in (7) and approximating the others differential operators in the conventional way we get the discretized LITWEQ operator:

$$\begin{aligned} c_1 T Lu(t_1, t_2) + c_2(F_1 F_2)u(t_1, t_2) &= c_3(F_1 + F_2)Lu(t_1, t_2) \\ &+ c_4(T_1 + T_2)u(t_1, t_2) \\ &+ c_3(F_1 F_2)u(t_1, t_2), \end{aligned} \tag{11}$$

where c_i are functions of Δt, Δx, $v(x, \tau)$ and $v(\tau)$ and T_1, T_2 represent centered schemes along t_1 and t_2 respectively.

IMPULSE RESPONSE

Figure 1 shows the impulse response of the LITWEQ operator in a constant velocity medium. I selected a velocity equal to 1500 m/sec and steps of .004 and 10 m along the time and horizontal directions, respectively.

CONCLUSION

I presented a full derivation of a finite difference scheme to LITWEQ operator using a nine point star pattern. This finite difference scheme is intended to include lateral

Figure 1: Impulse response of the LITWEQ operator in a constant velocity media.

velocity variations. The impulse response of the operator shows that the LITWEQ method can handle events with large dips. However, more tests are needed to check the stability of the proposed finite difference scheme.

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