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Short Note

Wave equation resampling of unevenly spaced traces

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INTRODUCTION

Many seismic data processing algorithms require that input data have uniform sampling intervals both in time and in space. However, in reality, difficulties in positioning the sources and receivers may cause the spatial sampling intervals to vary from place to place. If the variations of the sampling intervals are too large to be acceptable for data processing, seismic traces have to be resampled.

Because seismic data are almost always sampled uniformly in time, conventional methods perform the spatial resampling along each constant time slice or each constant frequency slice. The problem of two dimensional resampling is thus reduced to one dimension. In the 1950s, Yen (1956 and 1957) developed an algorithm for resampling irregularly sampled, one-dimensional data. In his algorithm, the irregularly sampled data are modeled as the interpolation of unknown uniformly spaced data. By solving a linear system analytically, Yen derived an exact, explicit formula for evaluating data at arbitrary points. Hale (1980) tested Yen's algorithm and pointed out that, although the method is exact for noise-free data, it may give erroneous results for noisy data. With Yen's method, a single noisy sample, even one far away from the correct sampling positions, may have an undesirable influence on the values of the resampled data. Moreover, Yen's algorithm is computationally expensive. To overcome these two problems, Hale uses a truncated, tapered sinc function as the basis function for interpolation. Although this procedure sacrifices accuracy at high frequencies, it performs better and at much less computational cost than Yen's method for data contaminated with truncations, aliasing data or noisy data. In the previous SEP report, Zhang (1991) described another one dimensional resampling scheme in which unevenly spaced data samples are modeled by a Fourier series. The coefficients of this Fourier series are found by minimizing the errors between the given

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data and the modeled data. Once these coefficients are determined, the data can be resampled arbitrarily.

One of the important characteristics of seismic data is the spatial coherence along the events corresponding to reflection or diffraction. These events may have varying slopes and curvatures. All the one-dimensional resampling methods mentioned above ignore this dominant feature of seismic data, and are therefore not optimal for resampling seismic data.

In studying the finite-aperture slant-stack transform, Kostov (1990) developed a two-dimensional trace-resampling method, in which unevenly spaced seismic traces are modeled as the superposition of many plane waves of various dips. The weighting coefficients of these plane waves are determined by minimizing the errors between the given unevenly spaced traces and the modeled traces. Kostov demonstrated his algorithm by filling in the missing traces in a common shot gather.

Two problems may potentially affect the results of resampling data by plane wave decomposition. First, the finite-aperture slant-stack transform is not a local operator. The aperture of this operator is usually equal to the spatial dimensions of input data. For filling large gaps of missing traces, a large aperture operator is needed. However, if the variations of sampling intervals are small perturbations, the large aperture operator becomes unnecessary. The large aperture also implies that a bad trace in the original data may have an undesirable influence on resampled traces far away from it. Second, for data containing many diffracted events or other curved events, plane wave decomposition is not the best way to model the data.

In this note, we describe a wave equation method for resampling unevenly spaced traces. We model the seismic traces recorded by an array of unevenly spaced receivers at the surface as the upward continuation of the wave field evenly sampled at a selected depth level. By solving a least squares inverse problem, we find the evenly sampled data at the selected depth level. Upward continuing the evenly sampled data allows us to resample the data at the surface. This algorithm is generally applicable to both 2-D and 3-D data, and to both poststack and prestack data. Because the wave equation modeling is used, the algorithm is robust and stable. The aperture of the upward continuation operator is determined by the distance from the selected depth level to the surface and the maximum time-dip of input data. Therefore, we can make the algorithm more efficient by reducing this distance or accept large perturbations of sampling spaces by increasing the distance. Furthermore, because the upward continuation operator is time-invariant, the computation can be done for each frequency independently. Therefore, the algorithm can be well implemented on a parallel machine. The algorithm can also be used to downward continue the wave field recorded by a finite aperture receiver array. An example with synthetic data shows that the algorithm produces a promising result.

In the following sections, we first briefly describe the theory of the new trace resampling method. We then give a synthetic example of resampling 2-D unevenly space traces.

THEORY

If an upward-propagating wave field is sampled uniformly at a given depth level, it can be accurately continued upwards to an arbitrary point on the earth's surface by a Kirchhoff summation operator. Let $P(t, x_i, y_j, 0)$ be the wave field recorded by an array of unevenly spaced receivers on the surface. We want to resample the wave field with an uniform sampling interval. We consider the wave field recorded on the earth's surface to be the upward continuation of a wave field $P(t, x_k, y_l, z_0)$ that is recorded by an array of uniformly spaced receivers at the depth z_0. This relation is described mathematically by

$$P(t, x_i, y_j, 0) = \mathbf{C}P(t, x_k, y_l, z_0), \tag{1}$$

where $\mathbf{C}$ is a Kirchhoff upward continuation operator. For a given $P(t, x_i, y_j, 0)$, we can estimate $P(t, x_k, y_l, z_0)$ by minimizing the errors between the given input data and the data modeled by upward continuation, which is as follows:

$$E = \|P(t, x_i, y_j, 0) - \mathbf{C}P(t, x_k, y_l, z_0)\|^2. \tag{2}$$

This is a linear least squares inverse problem. It can be efficiently solved by using the conjugate gradient method. Once $P(t, x_k, y_l, z_0)$ is known, we can upward continue the wave field to the surface and resample it, as follows:

$$P(t, x_k, y_l, 0) = \hat{\mathbf{C}}P(t, x_k, y_l, z_0) \tag{3}$$

where $\hat{\mathbf{C}}$ is another upward continuation operator.

The choice of z_0 is determined from two aspects. The aperture of the upward continuation operator for a given maximum time-dip of input data is inversely proportional to z_0. Therefore, to achieve high efficiency, we want to use small z_0. However, if the trace sampling intervals of the original data are large, using small z_0 causes the conjugate operator of the upward continuation to be aliased.

An assumption made by this algorithm is that the velocities above the depth level z_0 are known, which is much weaker than the assumption of the full knowledge of the velocity structure made by the least squares migration (Cole, 1992; Ji, 1992). The formulation presented above is well suited for resampling marine data because the depth level z_0 can be set within the water layer that has a constant velocity and does not generate any useful events. It should also work for land data recorded in an area of simple near-surface velocity structure. For land data with complicated near surface structures, a similar scheme can be formulated, in which the unevenly sampled traces are modeled as the reverse time propagation of the wave field sampled uniformly at a level above the earth's surface.

EXAMPLES

For the sake of simplicity, we demonstrate the method by resampling a 2-D, post stack section. Figure 1 shows a synthetic reflectivity model. A zero reflectivity layer at the top of the model represents a water layer. To test our resampling algorithm, we generate an unevenly sampled dataset from this model. Figure 2 shows the surface positions of the receivers. The receiver spacing is randomly chosen from an uniform distribution. The average error in receiver position is 0.02 km, and the variance is 0.0195 km. Figure 3 shows the zero-offset section generated with a constant velocity of 2 km/s. The influence of the errors in receiver position can be clearly identified. We solve the equation (2) using the conjugate gradient method. The implementation of the Kirchhoff upward-continuation operator is similar to the one described by Bevc (1992). After 10 iterations, we obtain the uniformly sampled wave field at the depth level of 0.1 km, which is shown in Figure 3. It is clear that the effect of irregular spacing has been removed by the inversion process. Figure 4 is a plot of the total energy of the residual as a function the iterations of the conjugate gradient method. Convergence is reached after 10 iterations, when the total energy of the residual drops close to zero. Figure 5 displays the synthetic data resampled at surface and the synthetic data modeled directly from the reflectivity model for comparison. The differences are imperceptible.

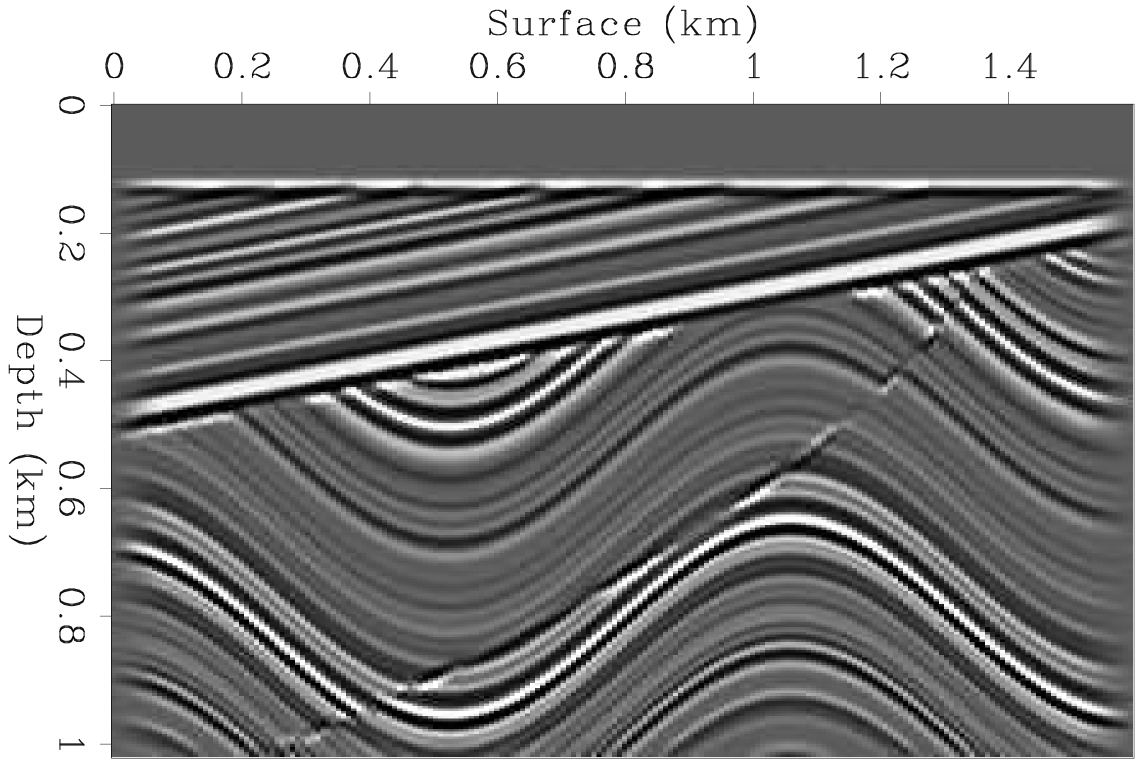

Figure 1: A synthetic reflectivity model. lin2-sigmoidm [ER]

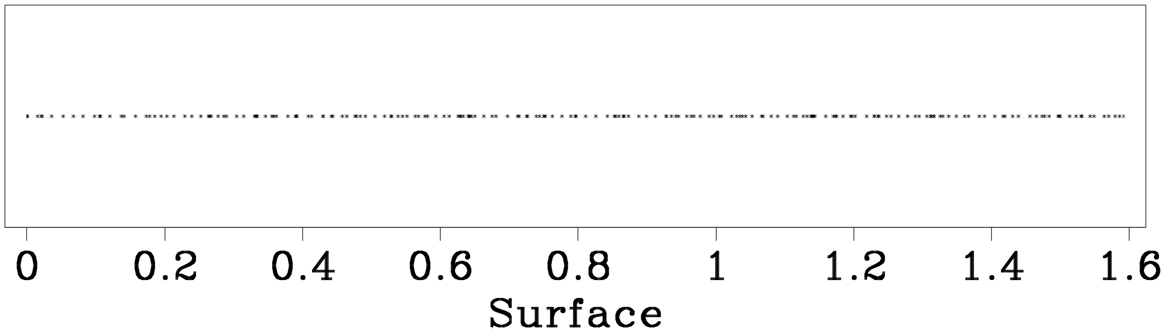

Figure 2: Surface positions of receiver array with random spacing. lin2-randxy [ER]

CONCLUSIONS

We have presented an inversion algorithm for resampling unevenly spaced seismic traces. As demonstrated with the synthetic example, the inversion process is well constrained by using the wave equation modeling. The next step is to study the effects of spatially aliased data on the inversion process. It is expected that additional constraints are needed for resampling data that are severely aliased in space.

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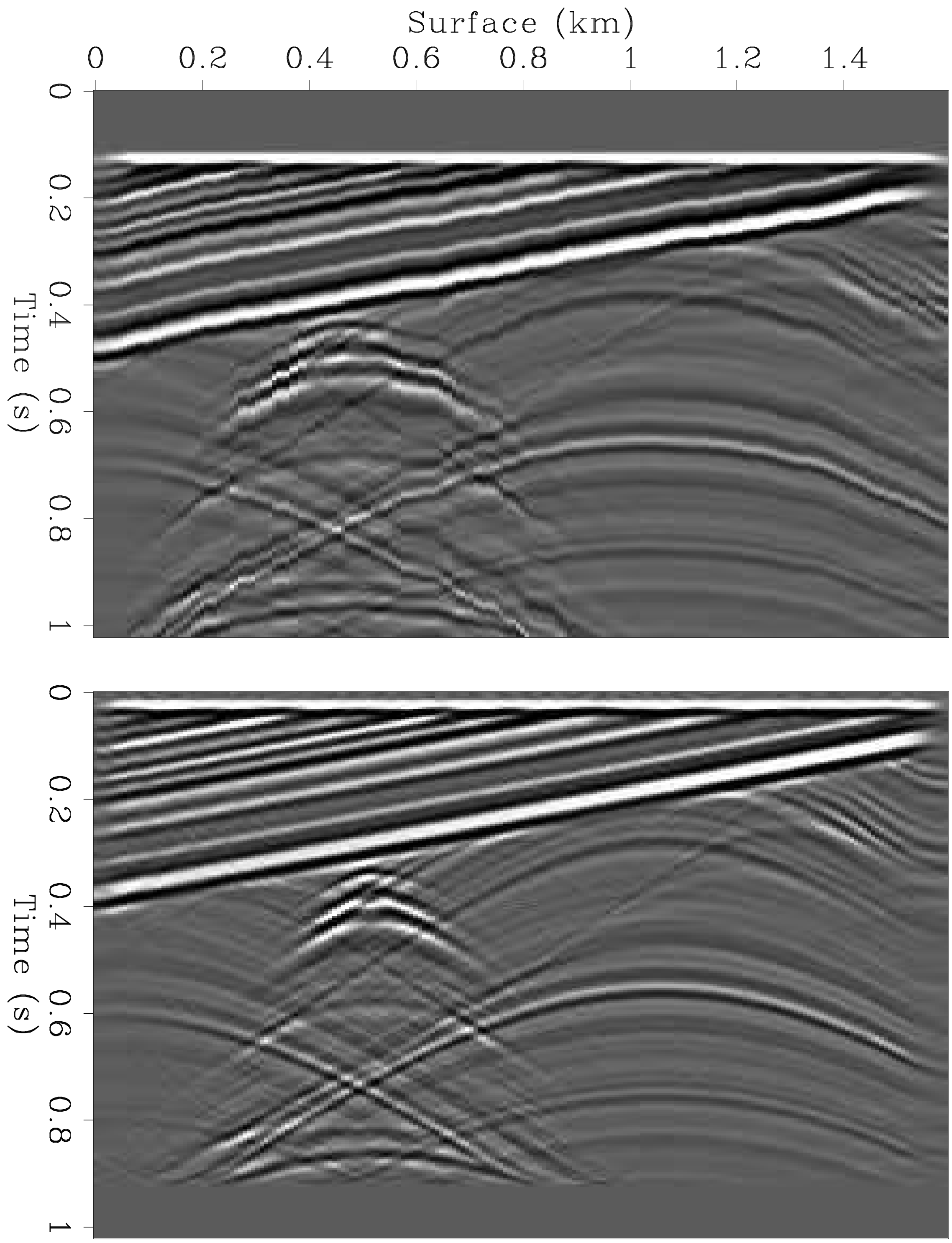

Figure 3: The top panel shows the data sampled irregularly at the surface. The bottom panel shows the data after being downward continued to 0.1 km below the surface using the least squares inverse of the upward continuation operator. lin2-irr0rsp [ER]

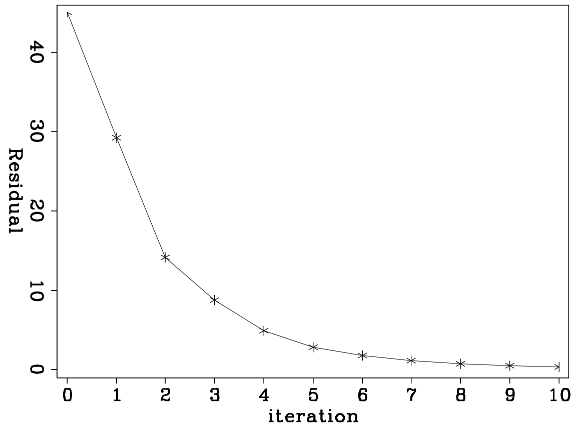

Figure 4: The total energy of the residuals of the inversion process versus the iterations of the conjugate gradient method. lin2-convg [ER]

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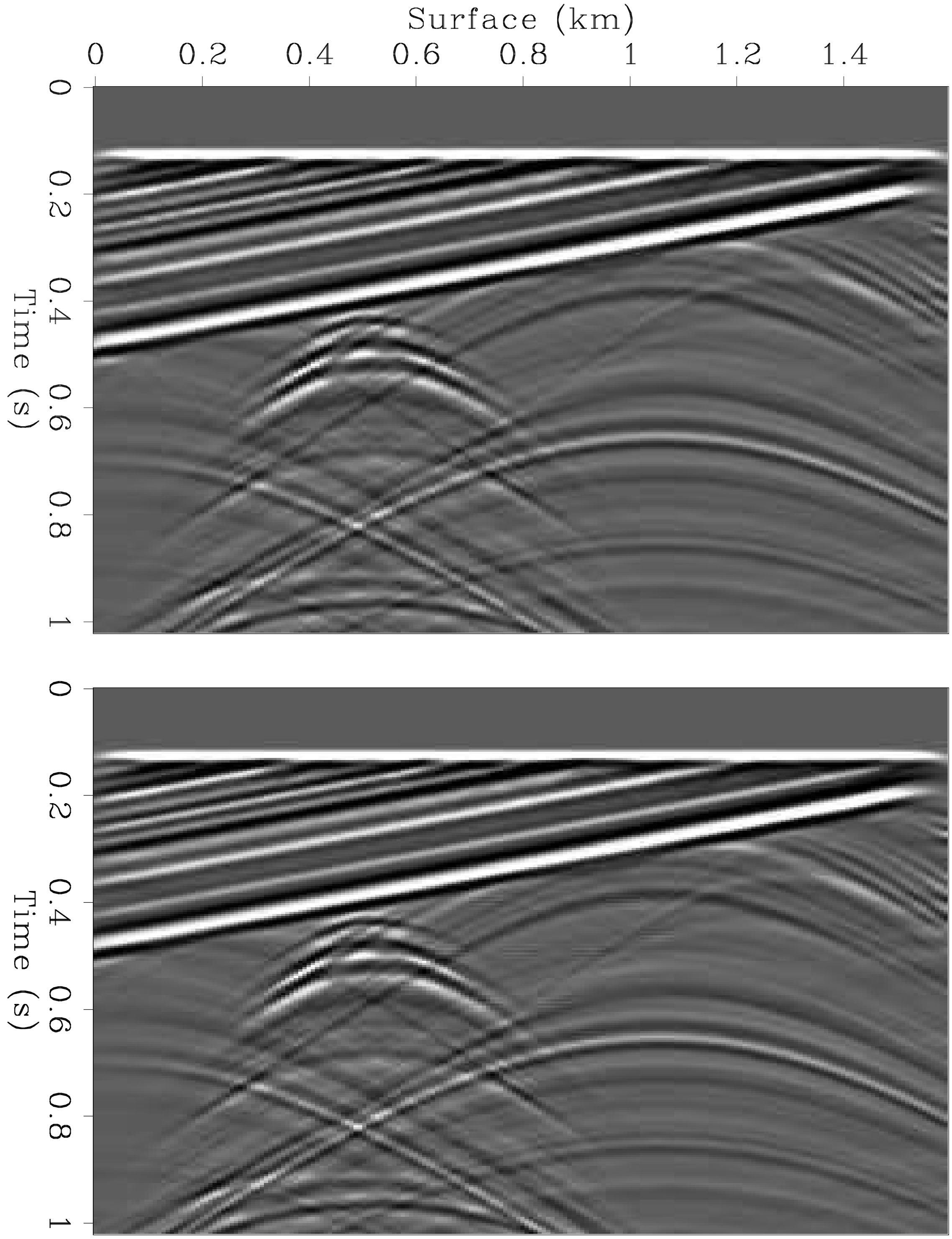

Figure 5: The top panel shows the data resampled at the surface. The bottom panel shows the data directly model from the reflectivity model shown in Figure 1. lin2-rspreg0 [ER]