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Prestack Partial Migration Analysis

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ABSTRACT

Prestack partial migration (PSPM) is a well-known process which transforms the prestack data to zero offset. I discuss several properties of the PSPM spreading operator and of the equivalent PSPM summation operator reflected by a transformation of coordinates from (x, t) domain to (p, τ) domain, where $p = 2dt/dx$ and τ is the NMO correction. This transformation allows for a more general representation for the PSPM operator and can explain the apparition of triplications in the DMO curve in a variable velocity medium. Then I attempt to find a partial differential equation formulation for the PSPM operator in the family of first order partial differential equations using as characteristics the curves defined by the new transformation of coordinates.

INTRODUCTION

Prestack Partial Migration (PSPM) transforms data from constant-offset to zero-offset by applying normal moveout correction (NMO) followed by dip moveout correction (DMO) in a single pass.

The impulse response of PSPM is computed by considering the PSPM process as the combination of full prestack migration followed by zero-offset modeling (Deregowski and Rocca, 1981; Deregowski, 1986). Popovici and Biondi (1989) proposed a general PSPM algorithm that transforms the prestack data to zero offset when the velocity is a general 2-D function. Raytracing is used for modeling wave propagation in a general 2-D medium. The algorithm constructs kinematically a DMO + NMO operator which has to be applied to the data as an integral (Kirchoff-style) method. The PSPM operator is identical in shape to the DMO operator, except that it contains a time shift which is the NMO. For a constant velocity medium, the PSPM operator is analytically expressed as a function of the source-receiver travel-time and dip angle.

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THE PSPM OPERATOR

Prestack partial migration is a mapping of prestack data to zero-offset data. This mapping can be divided in two steps:

- Full prestack migration to a depth model. An input data point in a constant offset section is migrated into an ellipse.
- Zero-offset modeling. Each point on the ellipse with a local dip angle α, is imaged in a zero-offset section.

As a result each sample in a constant-offset section is spread over the whole range of possible dips which could generate the given sample in a zero-offset section.

For zero-offset, the time and space coordinates are (t_0, x_0); v represents the velocity; t_h represents the source-receiver travel time for a constant offset; α represents the dip angle and h is the half-offset. The parameters V, a and b are

$$\begin{cases} V = v/2 \\ a = t_h V \\ b = \sqrt{t_h^2 V^2 - h^2} \end{cases}$$

For the constant velocity medium, the analytical formulation for the PSPM operator as expressed by Popovici and Biondi, (1989)is

$$\begin{cases} t_0 = \dfrac{b^2}{V\sqrt{a^2 \sin^2 \alpha + b^2 \cos^2 \alpha}} \\ \\ x_0 = \dfrac{h^2 \sin \alpha}{\sqrt{a^2 \sin^2 \alpha + b^2 \cos^2 \alpha}} \end{cases} \tag{1}$$

The equations (1) are the parametric equations (functions of the dip angle α) of the DMO ellipse containing the velocity-time truncation. In the constant velocity case the PSPM operator is equivalent to NMO followed by DMO. The form of the PSPM operator is identical to the form of the DMO operator, as NMO introduces merely a time shift. Equations (1) are velocity dependent, and the velocity truncation has to be determined for each medium with a given velocity.

PSPM as a spreading operator

The PSPM operator defined in equations (1) can be written as function of the half-offset h, half-velocity V, and source-receiver travel time t_h:

$$\begin{cases} t_0 = \dfrac{(t_h^2 - \frac{h^2}{V^2})}{\sqrt{(t_h^2 - \frac{h^2}{V^2}) + h^2 \frac{sin^2 \alpha}{V^2}}} \\ \\ x_0 = \dfrac{h^2 \frac{\sin\alpha}{V}}{\sqrt{(t_h^2 - \frac{h^2}{V^2}) + h^2 \frac{sin^2 \alpha}{V^2}}} \end{cases} \tag{2}$$

Defining the variables $\tau = t_h^2 - \frac{h^2}{V^2}$, $p = \dfrac{sin\alpha}{V}$, we can rewrite equations (2) as

$$\begin{cases} t_0 = \dfrac{\tau^2}{\sqrt{\tau^2 + h^2 p^2}} \\ x_0 = \dfrac{h^2 p}{\sqrt{\tau^2 + h^2 p^2}} \end{cases} \tag{3}$$

Equations (3) are parametric representations of the PSPM operator, function of two new variables: $\boldsymbol{\tau}$ and $\mathbf{p}$. The variable $\boldsymbol{\tau}$ is the NMO corrected constant-offset travel time, while the variable $\mathbf{p/2}$ is the $\boldsymbol{p_x = dt/dx}$ parameter.

In Figure 1 the PSPM operator is mapped using equations (3). The elliptic curves represent curves of constant $\boldsymbol{\tau}$ parameter, while the vertical curves are curves of constant $\mathbf{p}$ parameter. The PSPM operator is represented by $\mathbf{t_0}$ and $\mathbf{x_0}$ coordinates, which is a velocity independent characterization. The velocity cutoff is contained in the parameter $\mathbf{p}$, which is increasing toward the edges of the plot.

The cutoff is determined by the parameter $\mathbf{p}$. Higher velocities introduce the restriction in the parameter $\mathbf{p}$ which cannot be higher than $1/V$, where V is half-velocity. A medium with velocity V_1 will have a tighter cutoff than a medium with lower velocity V_2, $(V_1 > V_2)$. In the first case the $\mathbf{p}$ parameter takes values from 0 to $1/V_1$, $(p \in [0, \frac{1}{V_1}])$ while in the second case the $\mathbf{p}$ parameter ranges from 0 to $1/V_2$ $(p \in [0, \frac{1}{V_2}])$. Two different media will share the same PSPM operator for the same values of $\mathbf{p}$. On the same DMO ellipse two media with velocities V_1, V_2 migrate an impulse to the same place if the equality $p = \sin\alpha_1/V_1 = \sin\alpha_2/V_2$ is satisfied. For a higher velocity medium, a high dip is migrated in the same place as a lower dip in a lower velocity medium.

Equations (3) map the PSPM operator as a function of the variables $\mathbf{p}$ and $\boldsymbol{\tau}$. In a constant velocity medium, for each point in the ($\mathbf{x_0}$, $\mathbf{t_0}$) space there corresponds a pair of values ($\mathbf{p}$, $\boldsymbol{\tau}$) and vice-versa. For a medium with variable velocity with depth $\boldsymbol{V(z)}$, it was shown that for certain velocity models the DMO impulse response presents triplications (Popovici and Biondi, 1989). Supposing the PSPM operator is represented in a system of coordinates ($\mathbf{p}$, $\boldsymbol{\tau}$), for certain velocity models the transformation $(\mathbf{x_0}, \mathbf{t_0}) \longrightarrow (\mathbf{p}, \boldsymbol{\tau})$ is not unique, thus generating the triplications in the PSPM curve. In a medium with variable velocity, this can be determined by following the variation of the $\mathbf{p}$ parameter along the equal travel time reflector. When the value of $\mathbf{p}$ is repeated along the same reflector with a given $\boldsymbol{\tau}$, the PSPM operator will have multiple values of the pair of parameters ($\boldsymbol{x_0}$,$\boldsymbol{t_0}$) for the same value of $\mathbf{p}$. According to this scheme it can be predicted for what types of media the PSPM operator will be multivalued in the parameter $\mathbf{p}$.

PSPM as a summation operator

Spreading data along the DMO impulse response is identical with summing along a conjugate curve. For zero-offset migration in a constant velocity medium, spreading the data over circles produces the identical result with summing along hyperbolas. The DMO ellipse corresponds to the circle while the summation DMO-conjugate curve corresponds to the hyperbola. Each point along the summation curve contributes to the point situated on the top of the curve, just like summation is done along a hyperbola in zero-offset migration. The equations for the equivalent PSPM summation operator are

$$\begin{cases} t_0 = \sqrt{\dfrac{\tau^2 + \tau\sqrt{\tau^2 + 4h^2p^2}}{2}} \\ \\ x_0 = \dfrac{h^2 p}{\sqrt{t_0^2 + h^2p^2}} \end{cases} \tag{4}$$

In Figure 2 the summation curves are represented in the same coordinates as the PSPM operator ($\boldsymbol{\tau}$ and $\mathbf{p}$). The family of hyperbolic type curves corresponds to a constant $\boldsymbol{\tau}$ parameter, while the vertical curves correspond to a constant $\mathbf{p}$ parameter. The cutoff is given by the same limits imposed on the parameter $\mathbf{p}$.

In Figure 3 is plotted the PSPM summation operator in order to correlate and compare it with the full prestack migration operator. For small velocities the PSPM operator and the full migration operator are identical. The full migration operator in the figure is limited laterally to the space between the source and receiver. The convergence of the full prestack summation curve to the PSPM operator for small velocities is easy to understand, as the migration ellipse for a medium with low velocity converges toward the PSPM ellipse, with semi-axis $\boldsymbol{h}$.

Amplitudes of the PSPM operator

In Figure 1 the parameter $\mathbf{p}$ varies with a constant step. When examining the figure, the conclusion is that a higher range of dips is migrated toward the ends of the DMO operator. From a $\boldsymbol{p_x}$ parameter point of view, the distribution of the dips is more uniform, though for data closer to the surface (i.e. smaller values for the time impulse), even the constant $\mathbf{p}$ sampling shows an increased density updip.

Several methods have been proposed to calculate the amplitudes of a DMO operator (Deregowski and Rocca, 1981; Gardner and Forel, 1988; Black and Egan, 1988; Beasley and Mobley, 1988; Liner, 1989). The ultimate goal of any DMO amplitude related procedure is that the zero-offset section obtained from the constant-offset section honors the wave-equation.

PARTIAL DIFFERENTIAL EQUATIONS FOR THE PSPM OPERATOR

The PSPM operator, and any other migration operator with the same reasoning, represents a surface when the migration operator is defined in 2-D. The solutions to partial differential equations (PDE) in two variables are families of well behaved surfaces (piecewise smooth surfaces), so naturally the question arises if our migration operators cannot be represented as solutions to certain PDE. Fundamentally a migration operator is defined kinematically by the migration curves, and dynamically by an amplitude function along the migration curve. The zero-offset migration operator can be viewed as a surface. For a constant velocity you have circles and radial lines as potential characteristics. A common amplitude scheme assumes constant amplitudes on a circle (migration circle) equal to $1/t$ and the characteristics are the family of curves $t = p * x$, which are the rays. Obviously, finding just the surface without the migration curve is of little use for the geophysicist, as often the correct kinematics offer better imaging even when the amplitude function is not well defined. The characteristic curves which are encountered in the solution of a broad family of partial differential equations may offer a solution to this problem.

The amplitudes of the operator can be discussed separately, as they don't influence the form of the characteristics. The characteristics are determined by the first two terms in a linear or quasi-linear partial differential equation. While there is a consensus in the kinematics of the migration operators, there are as many opinions in the distribution of amplitudes along the operators as there are geophysical schools. However to examine the validity of the PDE, the amplitude definition can be expressed as a general function $f(x_0, t_0)$ without affecting our analysis.

The motivation for finding the PDE that describe the operator in a constant velocity medium is the assumption that the PDE in a variable velocity medium should have a similar form, or close enough, which can be found using analogous reasoning.

The PSPM operator is defined by the migration curves shown in Figure 1 and by the cutoff curves of constant parameter **p**. It is natural to think about one of these families of curves as characteristic equations and try to find the partial differential equation which can describe the PSPM operator in a constant velocity medium. The simplest family of partial differential equations is the one of linear (and quasi-linear) partial differential equations, for which the solution is found along the characteristic curves.

The equations for the curves which appear in Figure 1 are

$$t_0 = p\frac{(h^2 - x_0^2)}{x_0} \tag{5}$$

$$t_0 = \tau\frac{\sqrt{h^2 - x_0^2}}{h} \tag{6}$$

Equation (5) represents the location of the points which are DMO migrated with the same parameter **p**. Equation (6) represents the DMO migration curves, a family of ellipses with horizontal semi-axis $\boldsymbol{h}$.

The corresponding PDEs whose characteristic curves are the curves presented in Figure 1 are

$$-u_{x_0}\frac{x_0(h^2 - x_0^2)}{h^2 + x_0^2} + t_0 u_{t_0} = f_1(x_0, t_0) \tag{7}$$

$$-u_{x_0}\frac{h^2 - x_0^2}{x_0} + t_0 u_{t_0} = f_2(x_0, t_0) \tag{8}$$

where f_1 and f_2 are two arbitrary functions which are defined by the variation of the amplitudes of the operator along the characteristics. While studying the kinematics of the operators, these functions can be ignored.

The solution to each equation as defined in the Appendix, will provide two families of curves, as two parameters are used in solving the equation using the method of characteristics. One is the parameter which varies along the characteristic curve, the other varies along the initial condition curve.

In Figure 4 the characteristic curves are the curves of constant **p** parameter shown in Figure 1. The initial condition is given along an ellipse. While the characteristic curves are represented correctly in the solution, the associated curves do not follow the migration ellipses.

In Figure 5 the characteristic curves are the curves of constant τ parameter shown in Figure 1. The initial condition is given along a vertical curve of constant parameter **p**. While the characteristic curves are represented correctly in the solution, the associated curves do not follow the cutoff curves in Figure 1.

We are faced with the problem that while the amplitudes of the operator are represented correctly in the domain considered, the pair of family curves are not.

CONCLUSION

The solution to each partial differential equation, given the amplitude values along an initial curve, will generate a surface. The surface represents the correct distribution of amplitudes across the 2-D operator. The problem of finding the characteristic curves is formulated in an overdetermined form. I expected to find both families of curves as solutions to the partial differential equation. However each PDE offers a single family of curves which coincides with the required characteristic, while the other associated family of curves is different from the form presented in Figure 1. The answer to this problem could be found in either using a higher order of differential equations to have as solutions the two families of curves, or to require a single family of curves to be described by the characteristics associated with the PDE. Using a single family of curves will not generate the cutoff in the DMO ellipse. While the approach to find the partial differential equations was an inductive approach, a deductive analysis of the amplitudes of the operator could generate a better understanding of the problem.

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APPENDIX

The solutions to the partial differential equations (7) and (8) are obtained using the method of characteristics. The characteristic curves are curves on which the partial differential equation in several independent parameters (in our case the parameters $(x_0,\ t_0)$) can be transformed into a system of ordinary differential equations in a single parameter along the characteristic curve.

A partial differential equation of the form

$$u_x\, a(x,t) + u_t\, b(x,t) + u\, c(x,t) = d(x,t)$$

can be reduced to the form

$$\frac{\partial u}{\partial \tau} + u\, c(x,t) = d(x,t)$$

if we can find a parameter τ such that

$$\begin{cases} \dfrac{\partial x}{\partial \tau} = a(x,t) \\ \\ \dfrac{\partial t}{\partial \tau} = b(x,t) \end{cases}$$

so that we can express the variables $(x,\, t)$ in the form

$$\begin{cases} x = x(\tau) \\ t = t(\tau) \end{cases} .$$

As a result we have

$$\frac{\partial u(x,t)}{\partial \tau} = \frac{\partial u[x(\tau),t(\tau)]}{\partial x}\frac{\partial x}{\partial \tau} + \frac{\partial u[x(\tau),t(\tau)]}{\partial t}\frac{\partial t}{\partial \tau} = u_x\, a(x,t) + u_t\, b(x,t)$$

which we can introduce in the original equation.

For equation (7) the system of associated equations is

$$\begin{cases} \dfrac{\partial x}{\partial \tau} = \dfrac{x(x^2-h^2)}{x^2+h^2} \\ \\ \dfrac{\partial t}{\partial \tau} = t \\ \\ \dfrac{\partial u}{\partial \tau} = f_1(x,t) \end{cases} \tag{9}$$

which after integration becomes

$$\begin{cases} \dfrac{h^2-x^2}{x} c_1 = e^{\tau} \\ \\ t = c_2 e^{\tau} \\ \\ u = \displaystyle\int f_1[x(\tau),t(\tau)]\, d\tau + c_3 \end{cases} \tag{10}$$

The constants c_1, c_2, c_3 are functions of the parameter given on the initial curve, and are determined by setting $\tau = 0$ in equations (10).

The initial condition is given along an ellipse, and has the form

$$\begin{cases} t = \dfrac{\tau_0^2}{\sqrt{\tau_0^2 + h^2p^2}} \\ \\ x = \dfrac{ph^2}{\sqrt{\tau_0^2 + h^2p^2}} \\ \\ u = g_1(p) \end{cases} \tag{11}$$

As a result the solution for the function u is given in parametric form

$$\begin{cases} t = \dfrac{\tau_0^2}{\sqrt{\tau_0^2 + h^2 p^2}} e^{\tau} \\ \\ x = \dfrac{-e^{\tau}\tau_0^2 + \sqrt{e^{2\tau}\tau_0^4 + 4h^2p^2(\tau_0^2 + h^2p^2)}}{2p\sqrt{\tau_0^2 + h^2p^2}} \end{cases} \tag{12}$$

For equation 8 the system of associated equations is

$$\begin{cases} \dfrac{\partial x}{\partial \tau} = \dfrac{(x^2 - h^2)}{x} \\ \\ \dfrac{\partial t}{\partial \tau} = t \\ \\ \dfrac{\partial u}{\partial \tau} = f_2(x, t) \end{cases} \tag{13}$$

which after integration becomes

$$\begin{cases} \sqrt{h^2 - x^2} c_1 = e^{\tau} \\ \\ t = c_2 e^{\tau} \\ \\ u = \displaystyle\int f_2[x(\tau), t(\tau)]\, d\tau + c_3 \end{cases} \tag{14}$$

The constants c_1, c_2, c_3 are functions of the parameter given on the initial curve, and are determined by setting $\tau = 0$ in equations (14).

The initial condition is given along a curve of constant **p** parameter, and has the form

$$\begin{cases} t = \dfrac{p_0\sqrt{h^2 - s^2}}{s} \\ \\ x = s \\ \\ u = g_2(p) \end{cases} \tag{15}$$

As a result the solution for the function u is given in parametric form

$$\begin{cases} t = \dfrac{p_0(h^2 - s^2)}{s} e^{\tau} \\ \\ x = \sqrt{h^2 - e^{2\tau}(h^2 - s^2)} \end{cases} \tag{16}$$

The amplitudes of the function u along the characteristic curves are found by calculating the integrals in equations (9) and (14).