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Refining the image of profile migration: residual moveout and residual migration

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ABSTRACT

In complex geologic environments, imaging of seismic reflection data requires model-driven depth migration. It is well known that such migration is sensitive to velocity errors. If the velocity model used in migration does not correctly model the travel-times of reflection events, the images of reflectors on migrated shot profiles will be distorted from the true images of the earth. By solving the Eiconal equation, I find a residual-moveout equation that can be used in estimating residual velocities, and a residual-migration operator that can revise the distortion of the images on migrated shot profiles. For a constant-velocity medium, these results are precise in the kinematic sense.

INTRODUCTION

In a multifold seismic experiment, a small portion of a subsurface is illuminated by shots at different offsets, as shown in Figure 1. The reflected energy is recorded by a receiver array on the surface. Imaging of such seismic reflection data can be accomplished by a prestack depth migration of individual shot records, followed by a stacking of the results at common surface locations. If the velocity model used for the shot-record migration does not correctly model the travel-times of reflection events, the images on migrated shot profiles at a constant surface location (CSL) will be distorted from one another. Under this circumstance, stacking of the CSL gathers can not effectively enchance the signal-noise ratio of the images. It may even damage the images.

It is observed on field data that after profile migration, the moveout trajectories on CSL gathers can curve upwards or downwards, depending on the migration velocities being higher or lower than the true velocities. With these observations, Al-Yahya (Al-Yahya, 1989) employed profile migration for velocity analysis. In his method, velocity errors are estimated by measuring the curvatures of the residual moveout in

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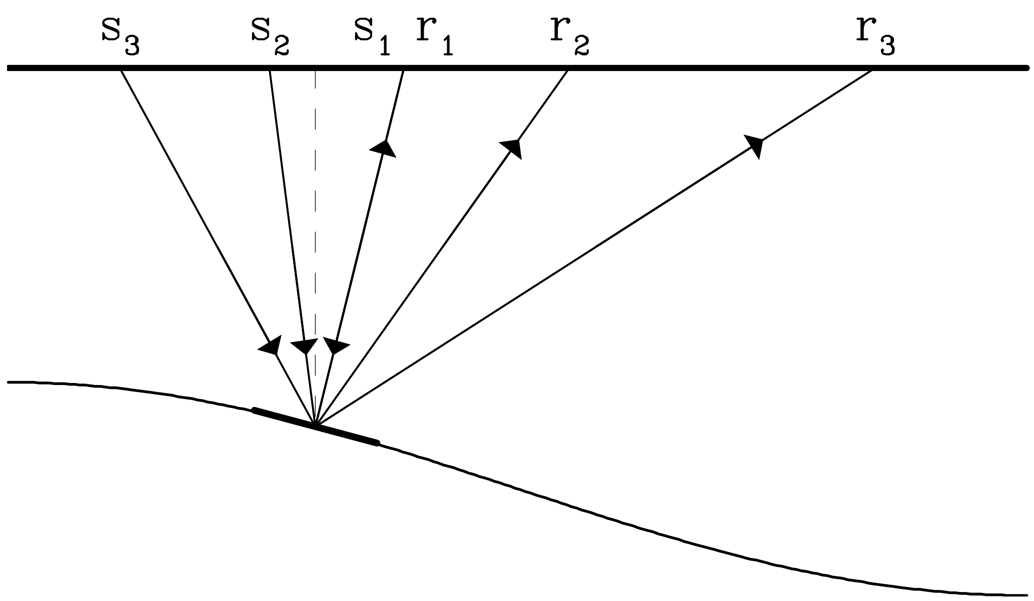

Figure 1: In a multifold seismic experiment, a small portion of the subsurface is illuminated by shots at different offsets. The reflected energy is recorded by a receiver array on the surface. lin2-multi [NR]

CSL gathers. Al-Yahya's curvature scanning is based on the residual moveout (RMO) equation with common-midpoint (CMP) approximation.

It is noticed that when the velocities used in migration are not equal to the actual velocities of the media, the images of reflectors move away both horizontally and vertically from their actual position. Using asymptotic ray theory, Lemoine (Lemoine, 1989) extended Al-Yahya's method to the common-depth-point (CDP) concept. From second-order approximations, he derived an equation that relates velocity errors and measurable phase-contour curvatures.

Al-Yahya's work and Lemoine's work mentioned above have two common restrictions. First, the RMO equations they derived are approximations for small offset, large depth, and small velocity errors. Second, the equations assume horizontal reflectors. To attack the second restriction, both authors applied the residual-velocity analysis and shot-profile migration iteratively. However, such an iterative algorithm is expensive because residual-velocity estimation and shot-profile migration both are time-consuming.

Goldin (Goldin, 1982) proposed a three-step method to find the location of the image of a reflector on migrated sections. When a homogeneous medium is considered, this algorithm can give analytical solutions. As an example, Goldin derived a pair of parametric equations that describe the image of a horizontal reflector on a migrated shot profile.

In this paper, I apply Goldin's method for the purposes of residual-velocity analysis and residual migration. First, I extend Goldin's result to adapt arbitrary reflectors.

Then, by specifying the relationship between the angles of the incident ray and the reflected ray, I find the residual-moveout equation for an arbitrary reflector and a residual-migration operator for a migrated shot-profile. I also use graphic tools to justify my solutions. In conclusion, I discuss the further work I want to investigate in the future.

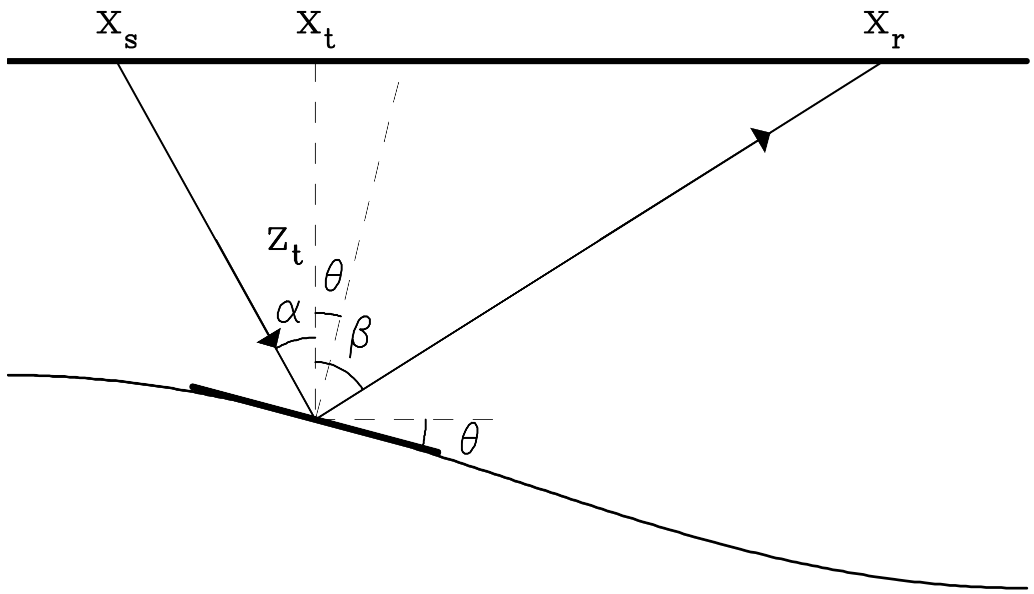

Figure 2: For a reflector of dipping angle θ, the relation of β and α satisfies Snell's law. When the offset between the shot and receiver is zero, both $-\alpha$ and β are equal to the dipping angle θ. lin2-dip [NR]

IMAGES OF ARBITRARY REFLECTORS

General reflection geometry

To understand how velocity errors influence the results of shot-profile migration, we consider the simple case of a reflector of dipping angle θ, located at (x_t, z_t) of a homogeneous medium of velocity v_t. As shown in Figure 2, a shot is at surface location x_s and a receiver is on the surface with coordinate $x = x_r$. We define α to be the angle of the incident ray to the vertical and β to be the angle of the reflected ray to the vertical. Suppose we migrate shot records with velocity v_m, and we want to determine the location of the image of the reflector on the migrated sections. For a fixed shot location, the travel time $\tau(x, z)$ of a reflection event is a function of the location of observation.

Now we apply Goldin's three-step method. The first step is to find the travel time of the reflection event observed by a receiver at $x = x_r$ on the earth's surface.

$$\tau_0(x_r) = \tau(x_r, 0) = \frac{z_t}{v_t}\left(\frac{1}{\cos\alpha} + \frac{1}{\cos\beta}\right). \tag{1}$$

where

$$\cos\alpha = \frac{z_t}{\sqrt{(x_t - x_s)^2 + z_t^2}}, \qquad \cos\beta = \frac{z_t}{\sqrt{(x_r - x_t)^2 + z_t^2}}.$$

The next step is to find $\tau(x,z)$ by solving Eiconal equation

$$\left(\frac{\partial\tau}{\partial x}\right)^2 + \left(\frac{\partial\tau}{\partial z}\right)^2 = \frac{1}{v_m^2} \tag{2}$$

with equation (1) as the boundary condition. In a case of homogeneous media, the problem can be solved analytically.

$$\tau(x,z) = \tau(x_r,0) - \frac{1}{v_m}\sqrt{(x-x_r)^2 + z^2}. \tag{3}$$

It can be shown that (Goldin, 1982)

$$\begin{aligned} x &= x_r - \lambda v_m \tau_x \\ z &= \lambda\sqrt{1 - v_m^2\tau_x^2}, \end{aligned} \tag{4}$$

where

$$\tau_x(x,z) = \frac{\sin\beta}{v_t}$$

and λ is a parameter to be determined.

The last step is imaging. Let the travel time of the reflection event observed at point (x,z) be equal to the travel time from the shot to this point.

$$\tau(x,z) = \frac{1}{v_m}\sqrt{(x-x_s)^2 + z^2}. \tag{5}$$

Combining equations (1), (3) and (5), we have relation

$$\frac{1}{v_m}\sqrt{(x-x_s)^2 + z^2} = \frac{z_t}{v_t}\left(\frac{1}{\cos\alpha} + \frac{1}{\cos\beta}\right) - \frac{1}{v_m}\sqrt{(x-x_r)^2 + z^2}. \tag{6}$$

To determine the λ in equation (4), we replace x and z in equation (6) with equation (4). In Appendix A, I show that

$$\lambda = \frac{z_t}{\gamma\cos\beta}[1 + (\gamma^2 - 1)\frac{\cos^2(\frac{1}{2}(\beta-\alpha))}{\cos\alpha\cos\beta}] \tag{7}$$

and

$$\begin{aligned} x &= x_t + z_t\tan\beta - \lambda\gamma\sin\beta \\ z &= \lambda\sqrt{1 - \gamma^2\sin^2\beta} \end{aligned} \tag{8}$$

where

$$\gamma = \frac{v_m}{v_t}.$$

Thus, for given location (x_t, z_t) of a reflector and the relationship between α and β, we can compute the location (x, z) of the migrated image.

We consider, as an example, a plane reflector of dipping angle θ,

$$z_t = z_0 + x_t \tan\theta, \tag{9}$$

that is illuminated by a shot at $x = x_s$. The relation between α and β follows Snell's law,

$$\phi = \beta - \theta = \alpha + \theta \tag{10}$$

where ϕ is the open angle between the incident (or reflected) ray and the normal of the reflector. Substituting this relation and equation (9) into equations (7) and (8) offers us formulas to compute the coordinates of the migrated image. In Figure 3 are plotted the migrated images for two different γ values. When $\gamma < 1$, the migrated image is stretched out and curves upwards; when $\gamma > 1$, the migrated image shrinks and curves downwards. In Figure 3 is also drawn a set of migration ellipses. As expected, the migrated images are the envelopes of those migration ellipses.

Residual-moveout equation

To perform residual-velocity analysis, we need to find RMO equations that describe how the image of a reflector moves as a function of the offset between the surface locations of the reflector and the shot. By substituting relation (10) and equation (7) into (8), we have

$$\begin{aligned} x &= x_t - (\gamma^2 - 1)\frac{\cos^2\theta}{\cos\alpha\cos(\alpha+2\theta)}\tan(\alpha+2\theta) z_t, \\ z &= [1 + (\gamma^2 - 1)\frac{\cos^2\theta}{\cos\alpha\cos(\alpha+2\theta)}]\frac{\sqrt{1-\gamma^2\sin^2(\alpha+2\theta)}}{\gamma\cos(\alpha+2\theta)} z_t. \end{aligned} \tag{11}$$

where

$$\alpha = \arctan\frac{x_t - x_s}{z_t}.$$

This pair of parametric equations defines the RMO equation for a common depth point (x_t, z_t) where the dipping angle of the reflector is θ. As we mentioned before, if the migration velocities are not equal to the actual velocities of the media, the image of the reflector moves away both horizontally and vertically from the actual position of the reflector. Therefore the residual-moveout curve for the CDP is three-dimensional. Figure 4 shows the residual moveout curves for a given reflector location (x_t, z_t), with dipping angle θ and velocity ratio γ.

The application of equation (11) requires the random access of whole 3-D dataset. This process is usually expensive and perhaps unnecessary for mildly varying structures. Let us assume that the structures are so smooth that, within the range of

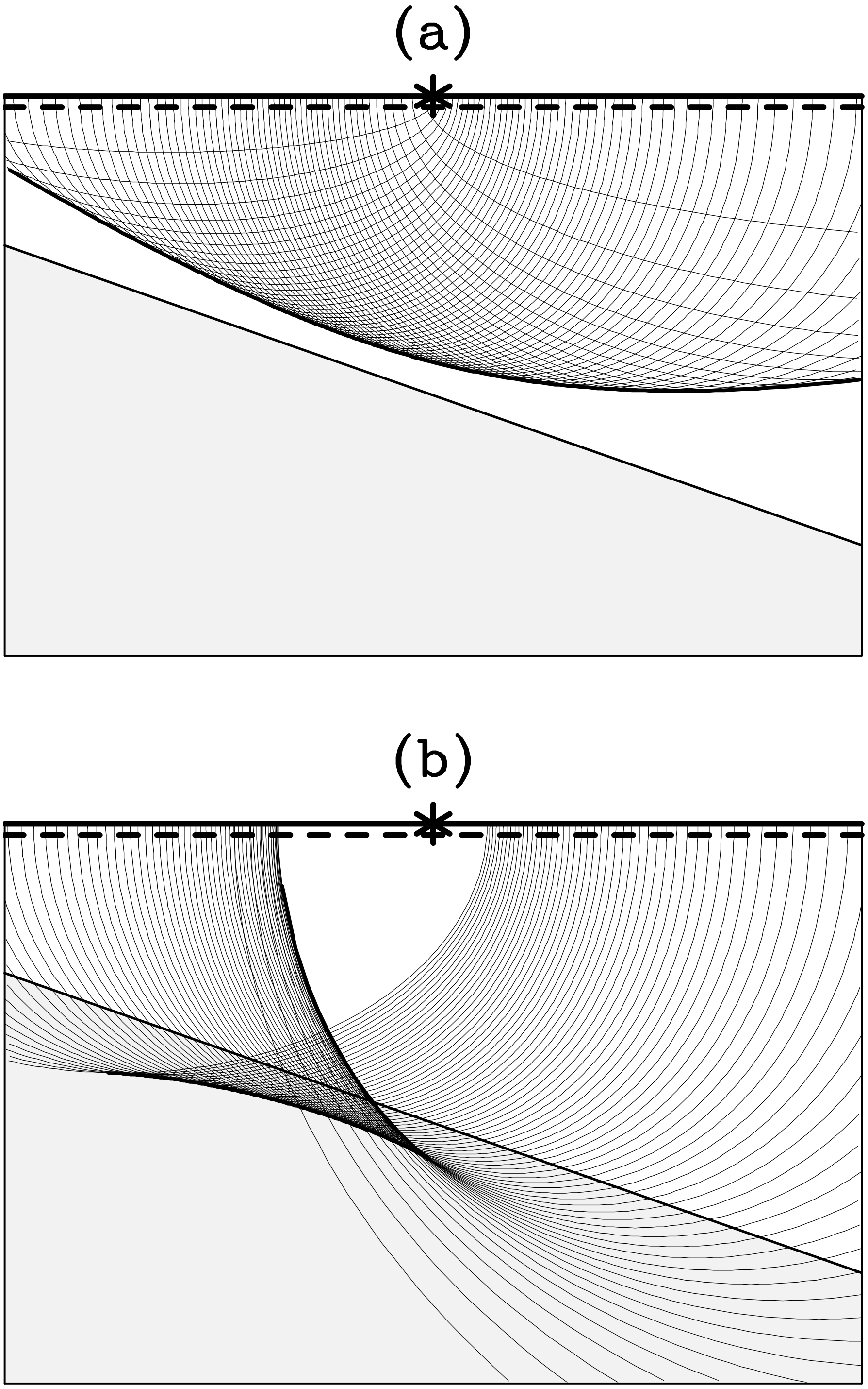

Figure 3: When the reflected energy from a plane reflector in a homogeneous medium is migrated with a velocity of (a) −10% error; (b) +10% error, the image of the reflector, defined by the envelope of the migration ellipses, is distorted from the actual image of the reflector. The dipping angle of the reflector is 15°. lin2-envbed [NR]

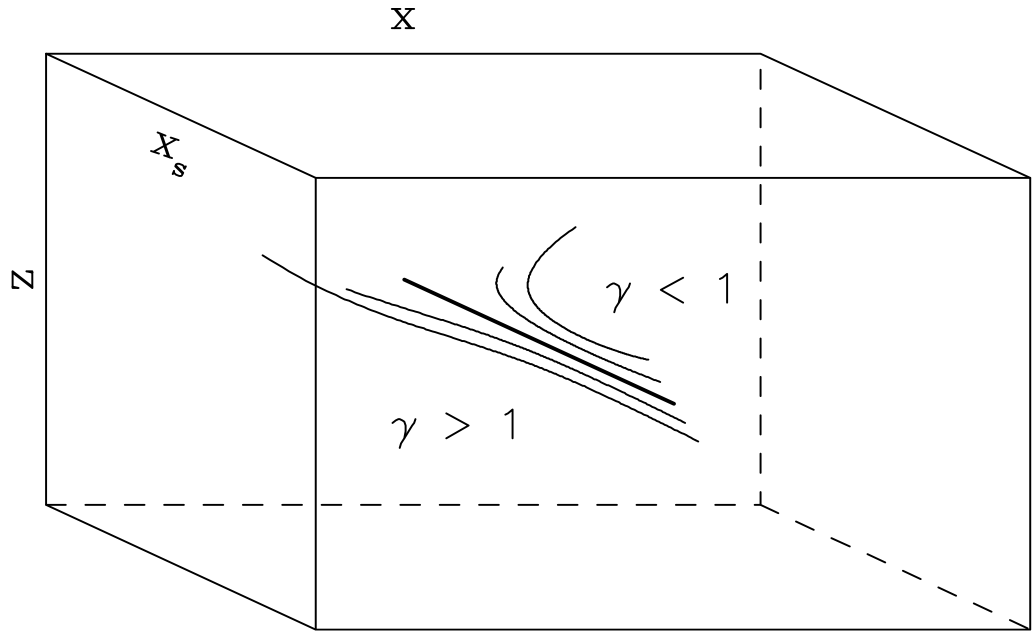

Figure 4: The residual moveout of a common depth point is a 3-D curve in the coordinates of surface location x, shot location x_s, and depth z. This figure shows the residual moveout of a reflector of dipping angle $\theta = 10°$ with γ ranging from 0.9 to 1.1. The thick curve corresponds to a velocity ratio $\gamma = 1$. lin2-rmo3d [NR]

horizontal movement of the image (due to velocity errors), the lateral variation of the dipping angle of the reflector can be neglected. Under this assumption, we can consider the RMO in common-surface-location gathers. In Appendix B, we show that for a given surface location x, depth z_t, dipping angle θ and velocity ratio γ, the RMO equation for a CSL is

$$\begin{aligned} x_s &= x - \frac{\sin(\phi-\theta)\cos^2(\phi+\theta) - (\gamma^2-1)\cos^2\theta\sin(\phi+\theta)}{\cos(\phi-\theta)\cos^2(\phi+\theta) - (\gamma^2-1)\sin\theta\cos\theta\sin(\phi+\theta)} z_t, \\ z &= \frac{\cos(\phi-\theta)\cos^2(\phi+\theta) + (\gamma^2-1)\cos^2\theta\cos(\phi+\theta)}{\cos(\phi-\theta)\cos^2(\phi+\theta) - (\gamma^2-1)\sin\theta\cos\theta\sin(\phi+\theta)} \frac{\sqrt{1-\gamma^2\sin^2(\phi+\theta)}}{\gamma\cos(\phi+\theta)} z_t. \end{aligned} \tag{12}$$

We can compute the RMO depth z as a function of the offset $x - x_s$ with open angle ϕ as a parametric variable. Figure 5 shows the CSL residual-moveout curves for several dipping angles.

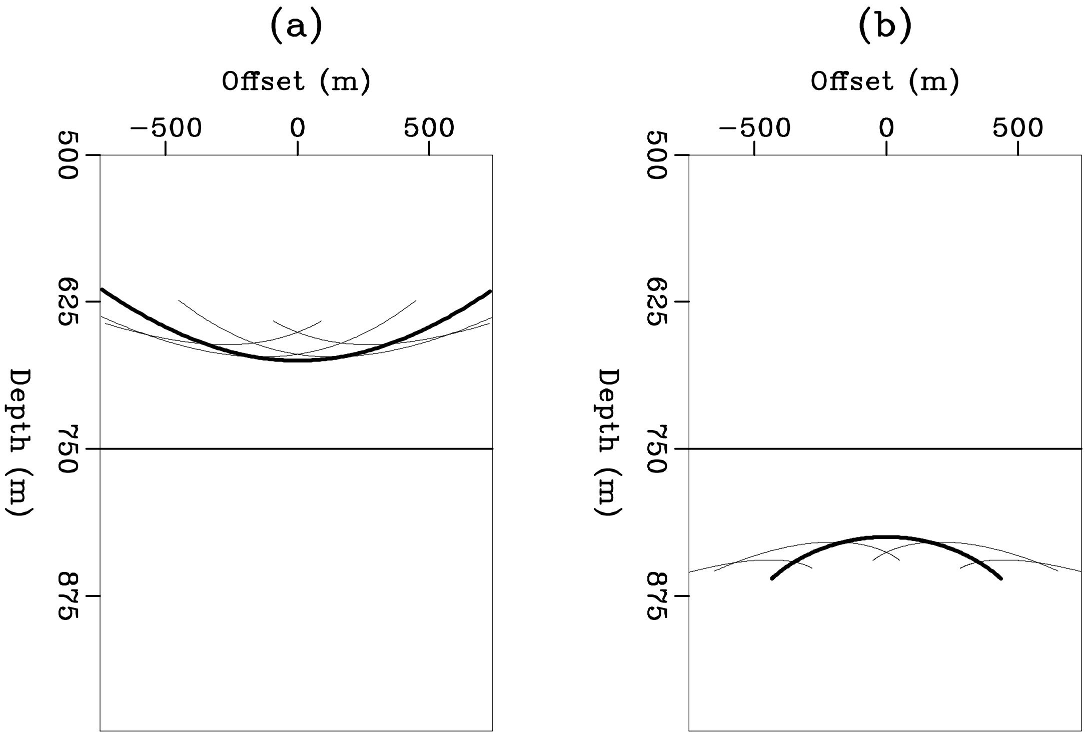

Figure 5: The residual moveout of a common surface location is a 2-D curve in the coordinates of the offset $x - x_s$ and depth z. This figure shows the residual moveout of a reflector of dipping angle ranging from $-25°$ to $+25°$: (a) $\gamma = 0.9$; (b) $\gamma = 1.1$. The thick curves correspond to horizontal reflectors. lin2-rmo2d [NR]

Residual-migration operator

Now we consider the image of a scatter in a homogeneous medium. Let the scatter have coordinates (x_t, z_t). Then, the angle of incident ray to the vertical is fixed.

$$\alpha = \arctan \frac{x_t - x_s}{z_t}. \tag{13}$$

The angle of the reflected ray can be any value between $-\pi/2$ and $\pi/2$. Therefore, equations (8) and (13) define the image of the scatter in the migrated shot profile. Figure 6 shows the two examples of the image of a scatter. We see that the image of the scatter is stretched out in both cases. When $\gamma < 1$, the image moves upwards and curves downwards; when $\gamma > 1$, the image moves downwards and curves upwards. Figure 6 also shows graphically that the curves defined by equations (8) and (13) are indeed the envelopes of migration ellipses.

For constant-velocity media, equations (8) and (13) actually define the kinematics of the summation operator that does residual migration for the migrated shot profiles. Figure 7 shows two sets of curves computed from these two equations. To revise distorted images on migrated shot profiles, we can sum the data samples along the curves defined by equations (8) and (13).

CONCLUSIONS

I have applied Goldin's method to determine the locations of the images of reflectors on migrated shot profiles. Two examples I discussed are RMO equations and a residual-profile-migration operator. The results of these two examples are essential to the purpose of refining the images of profile migration.

To find the residual velocity, two methods can be used. One is to measure the curvatures of the residual moveout. The other is to do many residual migrations and find the velocity that gives maximum stacked energy. The second method is possible because the size of the residual-migration operator is generally much smaller than that of full-migration operator. My future work will include 1) experiments of estimating the curvatures of the residual moveout; 2) dynamic analysis of the residual-migration operator of shot-profile migration; and 3) extension of the results to inhomogeneous media.

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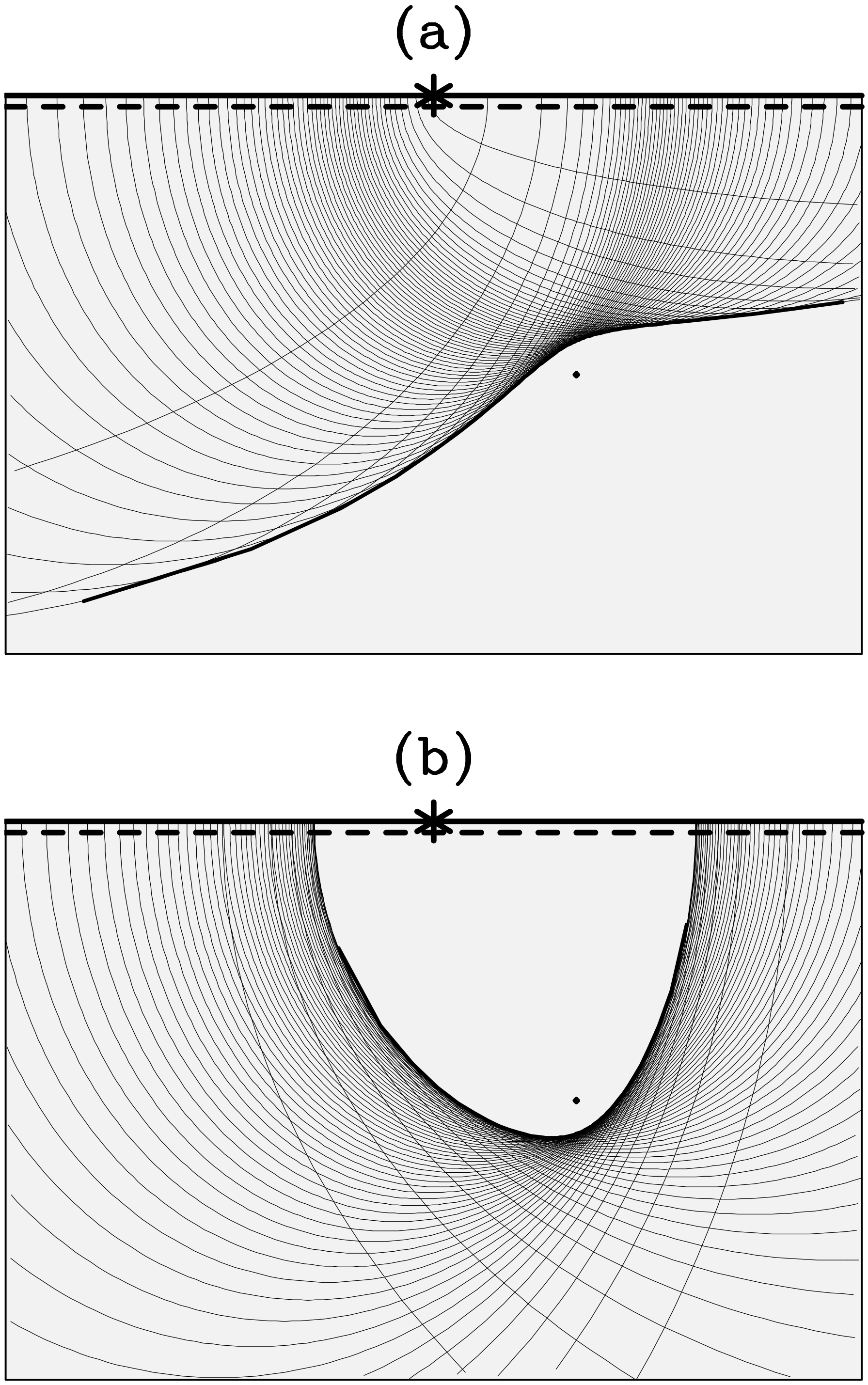

Figure 6: When the reflected energy from a scatter in a homogeneous medium is migrated with a velocity of (a) -10% error; (b) $+10\%$ error, the image of the scatter, defined by the envelope of the migration ellipses, is stretched out and (a) curves downwards; (b) curves upwards. lin2-envsca [NR]

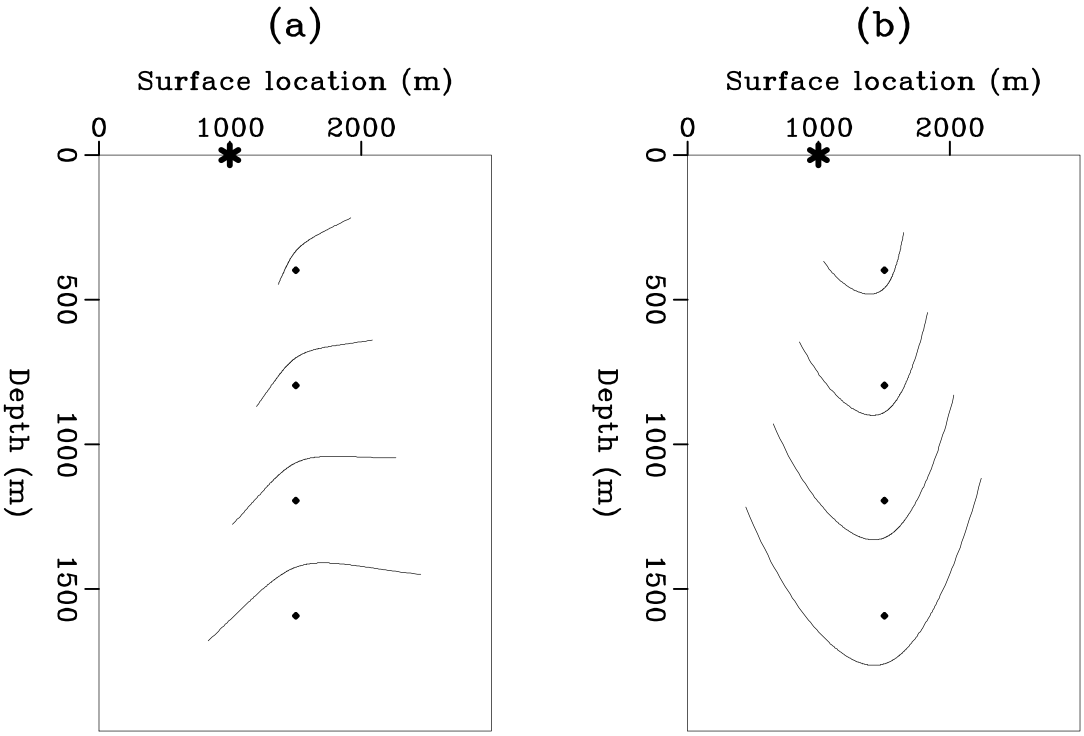

Figure 7: Examples of the kinematics of the summation operator that does residual migration for the migrated shot profiles. The star indicates the shot position. The dots indicate the actual location of the scatters: (a) $\gamma = 0.9$; (b) $\gamma = 1.1$. lin2-impres [NR]

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APPENDIX A

Let

$$\tau_0 = \tau(x_r, 0) = \frac{z_t}{v_t}\left(\frac{1}{\cos\alpha} + \frac{1}{\cos\beta}\right). \tag{A.1}$$

Substituting x and z from equation (4) into equation (6), we have

$$\sqrt{(x_r - x_s - \lambda\gamma\sin\beta)^2 + \lambda^2(1-\gamma^2\sin^2\beta)} = v_m\tau_0 - \sqrt{\lambda^2\gamma^2\sin^2\beta + \lambda^2(1-\gamma^2\sin^2\beta)}. \tag{A.2}$$

Solving this equation for λ yields:

$$\lambda = \frac{v_m^2\tau_0^2 - (x_r - x_s)^2}{2[v_m\tau_0 - (x_r - x_s)\gamma\sin\beta]}. \tag{A.3}$$

According to the geometry shown in Figure 2,

$$x_r = x_s + z_t(\tan\alpha + \tan\beta). \tag{A.4}$$

Substituting equation (A.4) into equations (A.3) and (4), we obtain equations (7) and (8):

$$\lambda = \frac{z_t}{\gamma\cos\beta}[1 + (\gamma^2 - 1)\frac{\cos^2(\frac{1}{2}(\beta - \alpha))}{\cos\alpha\cos\beta}]$$

and

$$\begin{aligned} x &= x_t + z_t\tan\beta - \lambda\gamma\sin\beta \\ z &= \lambda\sqrt{1 - \gamma^2\sin^2\beta}. \end{aligned}$$

APPENDIX B

We want to find the moveout equation of a CSL at surface location x. Suppose a reflector of dipping angle θ is located at (x_t, z_t); then equation (11) gives the coordinates, (x, z), to which the reflector is mapped after migration. According to our assumption, the lateral change of the dipping angle around the surface location x can be neglected. Let $\hat{z}$ be the true depth of the reflector at surface location x, then we have following relations:

$$\begin{aligned} &z_t - x_t\tan\theta = \hat{z} - x\tan\theta, \\ &x_t - x_s = z_t\tan\theta. \end{aligned} \tag{B.1}$$

Solving x_t and z_t from this pair of equations,

$$
\begin{aligned}
x_t &= x_s + \frac{\sin\alpha}{\cos(\alpha+\theta)}[\hat{z}\cos\theta + (x_s - x)\sin\theta], \\
z_t &= \frac{\cos\alpha}{\cos(\alpha+\theta)}[\hat{z}\cos\theta + (x_s - x)\sin\theta].
\end{aligned}
\qquad (B.2)
$$

Replacing x_t and z_t in equation (11) with equation (B.2) and reorganizing the expressions in (11), we obtain equation (12), in which the variable α is replaced by ϕ, as defined in equation (10):

$$
\begin{aligned}
x_s &= x - \frac{\sin(\phi-\theta)\cos^2(\phi+\theta) - (\gamma^2-1)\cos^2\theta\sin(\phi+\theta)}{\cos(\phi-\theta)\cos^2(\phi+\theta) - (\gamma^2-1)\sin\theta\cos\theta\sin(\phi+\theta)} z_t, \\
z &= \frac{\cos(\phi-\theta)\cos^2(\phi+\theta) + (\gamma^2-1)\cos^2\theta\cos(\phi+\theta)}{\cos(\phi-\theta)\cos^2(\phi+\theta) - (\gamma^2-1)\sin\theta\cos\theta\sin(\phi+\theta)} \frac{\sqrt{1-\gamma^2\sin^2(\phi+\theta)}}{\gamma\cos(\phi+\theta)} z_t.
\end{aligned}
$$