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Imaging to eliminate water-bottom multiples

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ABSTRACT

On a common midpoint (CMP) gather, the water-bottom multiples are characterized by their moveout of small stacking velocities. In this paper I describe a new velocity discrimination method to eliminate these low velocity events. The new method is similar in principle to the velocity filtering method. First the CMP gather is transformed into a distorted image by using a velocity-dependent operator. The velocity of the operator is chosen in such a way that the distorted images of the water-bottom multiples are separable from the distorted images of other events. Eliminating the images of the water-bottom multiples is followed by the backward transformation, to give a CMP gather free of the interference of the water-bottom multiples. The synthetic example and field-data example show that the algorithm works to a certain extent; however, several problems remain to be solved.

INTRODUCTION

Water-bottom multiples have long been recognized as a problem in marine seismic prospecting. This problem becomes serious in an area such as the Arctic where the seafloor has much higher reflectivity than that of deeper geological horizons. In this case multiple interference is often strong enough to completely overpower primary reflections and thus makes interpretation impossible.

Water-bottom multiples are those multiples whose raypaths lie entirely within the water layer. Because water velocity is much lower than the seismic velocities of sediments, velocity discrimination methods are commonly used to suppress the water-bottom multiples. Examples of such methods are velocity filtering (Thorson and Claerbout, 1985) and f-k filtering (Yilmaz, 1987).

Here I present a new algorithm that, in theory, can accomplish the goal of eliminating the water-bottom multiples on common midpoint (CMP) gathers, provided that the differences between the stacking velocities of primaries and multiples are sufficiently large. As does velocity filtering, my algorithm consists of three steps:

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forward transformation, filtering, and backward transformation. But the transformation operators in my algorithm are completely different from the one used in velocity filtering. In the following sections, I will first explain the principles of the algorithm and then show synthetic examples and field-data examples. In the end, I will discuss several limitations of the algorithm that may arise in practice.

PRINCIPLES OF THE ALGORITHM

I will explain the algorithm in common-shot geometry under the assumption of horizontal reflectors. Later, I will show that this assumption is unnecessary when we apply the algorithm to CMP gathers.

Travel-time relations

In a homogeneous medium, the seismic energy generated by a shot propagates along the straight rays that connect the shot position x_s, reflector position (x, z), and receiver position x_r. The total travel-time of this propagation can be formulated as

$$t = \frac{1}{2}\left[\sqrt{t_0^2 + \frac{4(x-x_s)^2}{v^2}} + \sqrt{t_0^2 + \frac{4(x-x_r)^2}{v^2}}\right], \tag{1}$$

where v is the velocity of the medium, and

$$t_0 = \frac{2z}{v}$$

is the vertical two-way travel time. We can solve equation (1) for t_0,

$$t_0 = \frac{1}{t}\sqrt{t^2 - \frac{(x_r - x_s)^2}{v^2}}\sqrt{t^2 - \frac{(2x - x_r - x_s)^2}{v^2}}. \tag{2}$$

Equations (1) and (2) define the relations between two travel times and are the key equations of kinematic operators for imaging.

Kinematic operators

Let $D(t, x_r)$ be a common shot gather (CSG). Define $\mathbf{P}(v)$ to be a velocity-dependent operator that transforms the data $D(t, x_r)$ into its image $I(t_0, x)$.

$$\begin{aligned} I(t_0, x) &= \mathbf{P}(v) D(t, x_r) & (3)\\ &= \sum_\tau \sum_{x_r} W_D(\tau, x_r) D(\tau, x_r) \delta(\tau - t), & (4)\end{aligned}$$

where W_D is a weighting function and t is defined in equation (1). Notice that the operator $\mathbf{P}(v)$ is actually a Kirchhoff migration operator of constant velocity v. Similarly, the backward transformation from $I(t_0, x)$ to $D(t, x_r)$ is

$$\begin{aligned} D(t, x_r) &= \mathbf{Q}(v) I(t_0, x) & (5) \\ &= \sum_{\tau} \sum_{x} W_I(\tau, x) I(\tau, x) \delta(\tau - t_0), & (6) \end{aligned}$$

where t_0 is defined in equation (2). $\mathbf{Q}(v)$ may be the inverse or transpose of $\mathbf{P}(v)$ depending on the choice of weighting function W_I. The parameter v of the operators plays an important role in both forward and backward transformations. For the purpose of imaging, we set v to be the velocity of the medium. As a result, $I(t_0, x)$ will be the precise image of the earth. For other values of v, however, $I(t_0, x)$ will be a distorted image. Our goal is to select a special value of v such that after the forward transformation with operator $\mathbf{P}(v)$ the images of the water bottom multiples are as separable as possible from the images of other events.

Choice of v

Without loss of generality, we consider experiments with a receiver array spreading on the right side of the shot. (A split-spreading experiment can be decomposed into two one-side spreading experiments). Let us first look at a simple model that consists of a flat reflector with an overburden of a constant velocity medium. We want to investigate how the image of the reflector will be distorted as the parameter v in $\mathbf{P}(v)$ deviates from the true velocity v_t of the medium.

Goldin (1982) proved that when v is equal to $\sqrt{2}v_t$ the crossing points of the caustics of rays and the image of the reflector are located right below the shot location. In Figure 1, a sequence of distorted images are drawn with $v = 0.9v_t \sim \sqrt{2}v_t$. We see that when $v = \sqrt{2}v_t$, the image of the reflector is completely migrated to the left side of the shot. Let us demonstrate this fact by graphics.

As I mentioned earlier, $\mathbf{P}(v)$ is a Kirchhoff migration operator. Its output is the superposition of a set of migration ellipses. Therefore the image $I(t_0, x)$ is constructed by the envelopes of these ellipses. Because the principle axes of migration ellipses depend on the parameter v, so do their envelopes. In Figure 2, I show a set of migration ellipses and their envelopes. When $v = \sqrt{2}v_t$, the envelope of the migration ellipses appears entirely on the left side of the shot. In contrast, when $v = 0.9v_t$, the envelope of the migration ellipses appears entirely on the right side of the shot. In fact, it can be mathematically proved that, after the transformation with operator $\mathbf{P}(v)$, the images of the events whose velocities are equal to $v/\sqrt{2}$ will be completely separable from the images of the events whose velocities are greater than v.

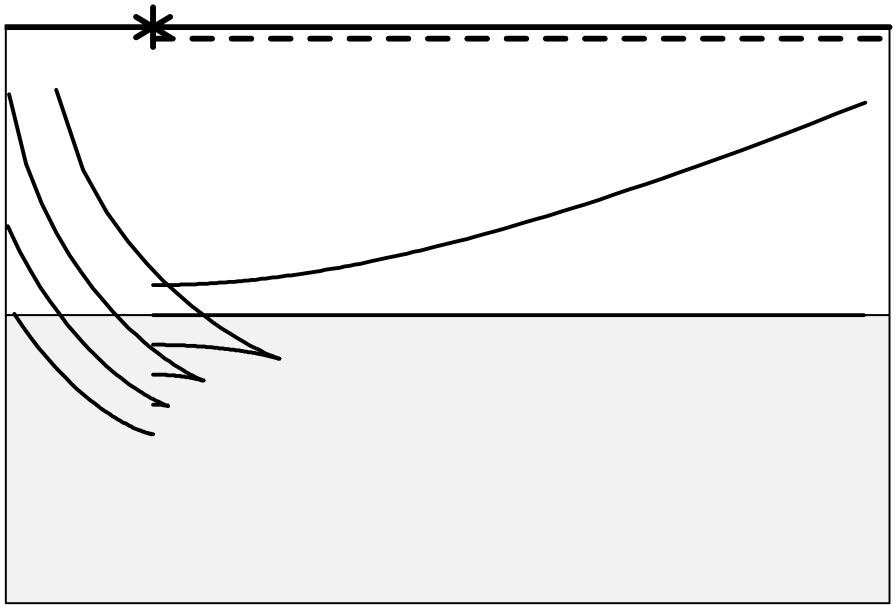

Figure 1: The distorted images of the interface between two flat layers. The star indicates the shot position and the dashed line represents the receiver array. When $v = 0.9v_t$, the distorted image bends upwards. As v increases, the distorted image gradually moves to the left. Finally when $v = \sqrt{2}v_t$, the distorted image is completely migrated to the left side of the shot. lin1-imaset [NR]

Algorithm

Let v_w be the velocity of water. Assume that the seismic velocities of sediments are greater than $\sqrt{2}v_w$. The algorithm of eliminating the water-bottom multiples consists of three steps:

1. Applying the forward transformation with velocity $\sqrt{2}v_w$ to the shot gather:

$$I(t_0, x) = \mathbf{P}(\sqrt{2}v_w)D(t, x_r). \tag{7}$$

   In the image $I(t_0, x)$, the water bottom multiples are migrated to the left side of the shot while other events stay on the right side of the shot.

2. Setting $I(t_0, x)$ to zero for all x smaller than x_s:

$$\hat{I}(t_0, x) = \begin{cases} 0 & x \le x_s \\ I(t_0, x) & x > x_s \end{cases} . \tag{8}$$

3. Applying the backward transformation with velocity $\sqrt{2}v_w$ to $\hat{I}(t_0, x)$:

$$\hat{D}(t, x_r) = \mathbf{Q}(\sqrt{2}v_w)\hat{I}(t_0, x) \tag{9}$$

   The final result $\hat{D}(t, x_r)$ will not contain events of velocity v_w.

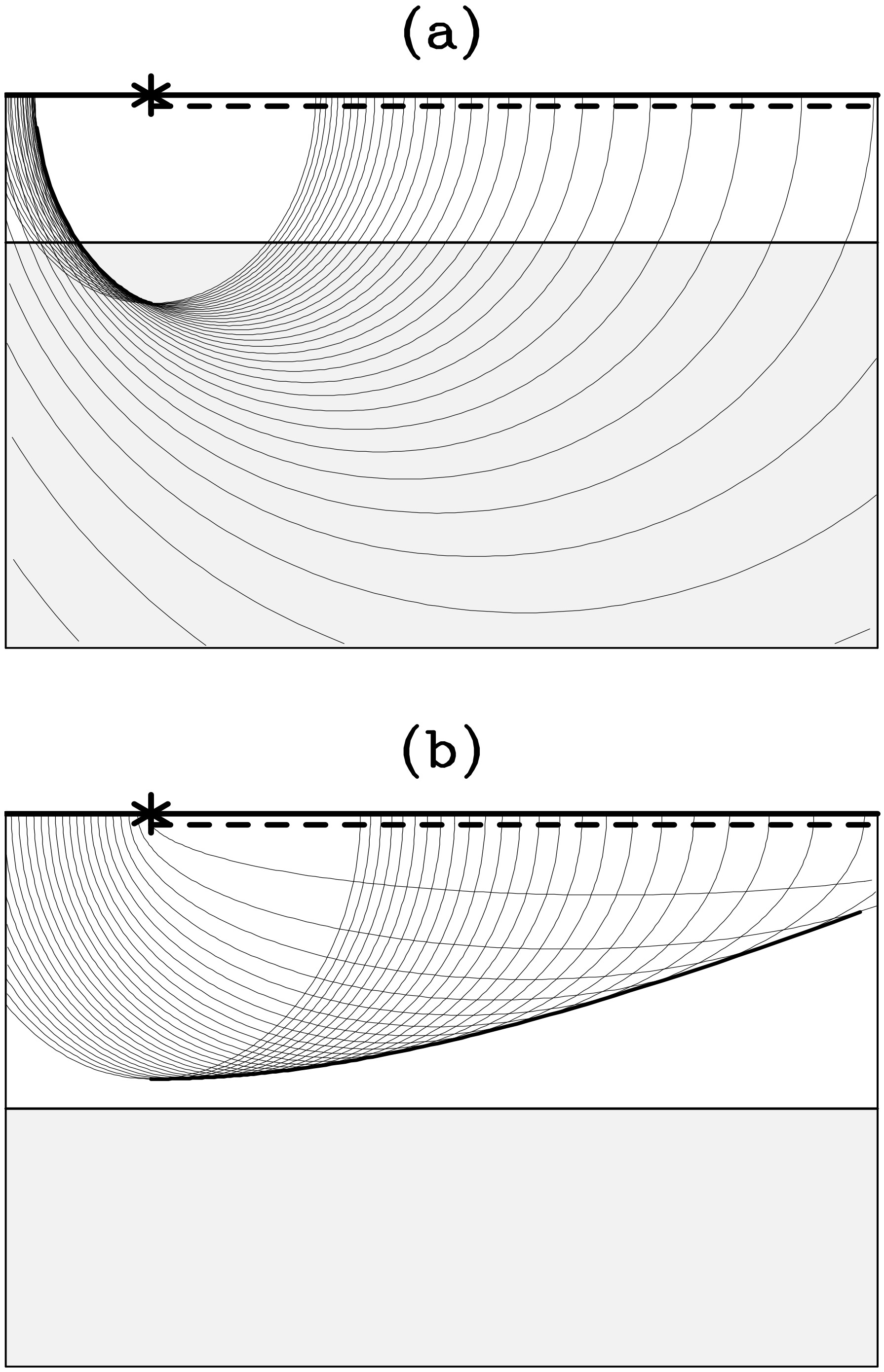

Figure 2: The distorted image of a reflector is defined by the envelopes of the migration ellipses that are v-dependent. The star indicates the position of the shot and the dashed line represents the receiver array. (a) When $v = \sqrt{2}v_t$, the envelope of the migration ellipses is entirely on the left side of the shot; (b) when $v = 0.9v_t$, the envelope of the migration ellipses is entirely on the right side of the shot. lin1-ellset [NR]

Until now, I have always assumed that $D(t, x_r)$ is a common shot gather and the subsurfaces are horizontal reflectors. Actually, the same algorithm can be applied to CMP gathers because the travel-time equations for the reflection from a horizontal reflector are identical in a CMP gather and a CSG. For a CSG, the assumption of horizontal reflectors is necessary because the receiver array on the right side of the shot can receive the reflected energy from a dipping bed that is on the left side of the shot. Therefore, after forward transformation, the image of the dipping bed could possibly appear on the left side of the shot. The situation is different when the algorithm is applied to CMP gathers. On a CMP gather, the moveout of a dipping reflector is the same as the moveout of a flat reflector of high stacking velocity. After the forward transformation, the image of a dipping reflector will stay in the positive offsets. Thus the algorithm can adapt the reflection from a dipping layer when it is applied to a CMP gather.

EXAMPLES

I will use both synthetic and field-data examples to demonstrate the effectiveness of the algorithm in eliminating the water-bottom multiples.

Synthetic data

My synthetic CMP gather is generated from a velocity model that consists of six layers. Figure 3a displays the stacking velocity function of the model on the background of the velocity spectrum of the data. Figure 3b shows the synthetic CMP gather. Several strong water-bottom multiples can be identified. The result of forward transformation is shown in Figure 3c. Clearly the water-bottom reflection and its multiples are migrated entirely to the negative offsets and are well separated from other events. Now we set the traces at negative offsets to zero and then do backward transformation. The result is shown in Figure 3d. The water-bottom reflection and its multiples are eliminated. At far offsets, some events with linear moveout are truncation artifacts. For simplicity, constant weighting functions are used in the transformations. Therefore, operator $\mathbf{Q}(v)$ is the transpose, not the inverse, of operator $\mathbf{P}(v)$. We see that the amplitudes of the events are not preserved after backward transformation.

Field data

Figure 4a displays a shot gather recorded in a marine survey in the Arctic. A strong water bottom reflection and its multiples are visible. After the processing, these low-velocity events are eliminated, as shown in Figure 4b. At far offsets we see some events with linear moveout. These are either refracted waves propagating along the water bottom with an apparent velocity equal to the velocity of the first layer of sediments, or truncation artifacts. Because the structures of the area are simply horizontal layers

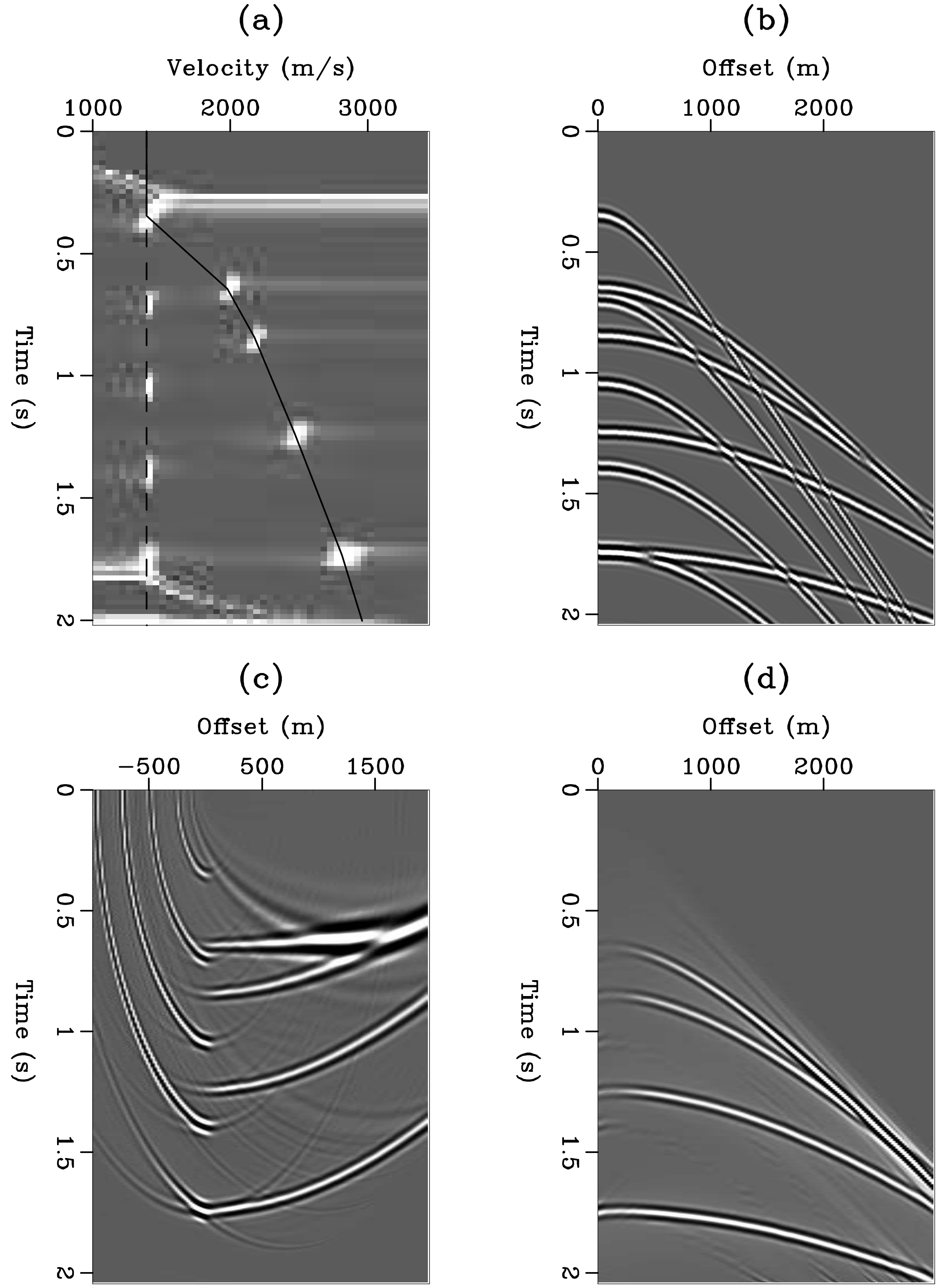

Figure 3: A synthetic example: (a) stacking velocity function of the layered model; (b) a synthetic CMP gather generated from the velocity model; (c) the result of forward transformation; (d) the output of backward transformation after the traces at negative offsets have been set to zero; the events at far offsets with linear moveout are truncation artifacts. lin1-synexa [NR]

I can compute the velocity spectrum from the shot gather. The velocity spectrum of the data before and after processing are shown in Figure 5 for comparison. It is seen that the energy of the water-bottom multiples is greatly reduced.

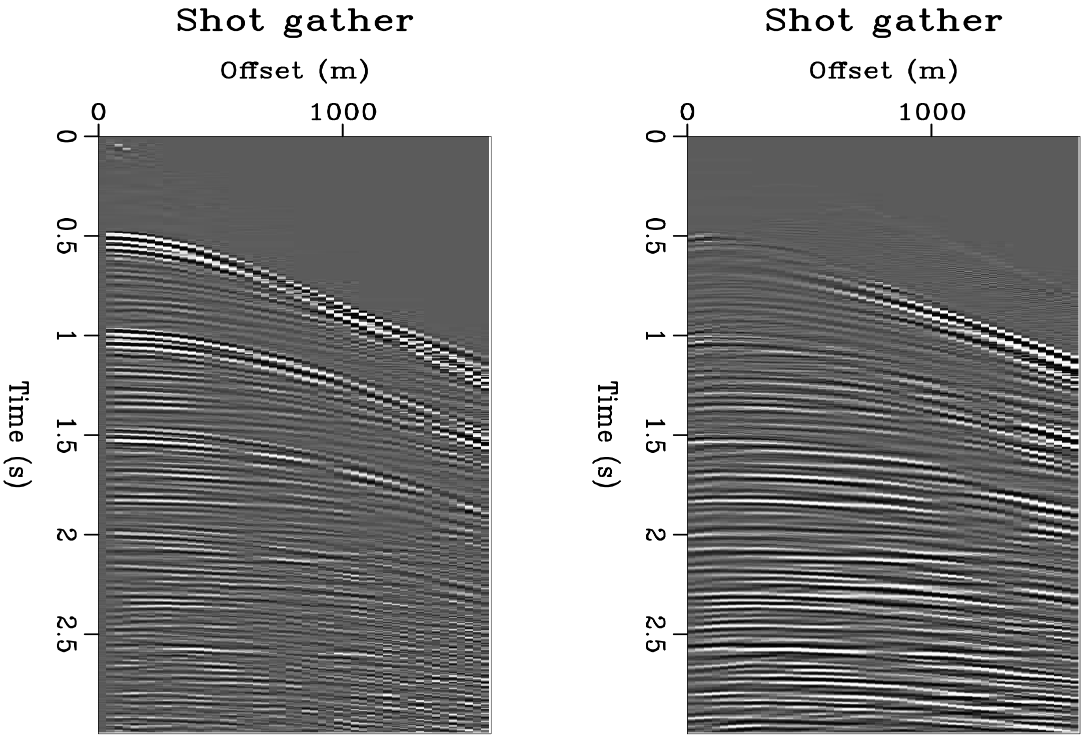

Figure 4: A field data example: (a) a shot gather recorded in a marine survey in the Arctic; (b) the same gather after the removal of the water-bottom multiples. At far offsets, refracted waves are seen. lin1-fieexa [NR]

LIMITATIONS

The algorithm presented above requires large differences between the velocity of the multiples and the velocities of the primaries. The safe criterion is that the ratio of the minimum velocity of primaries and the velocity of the multiples is larger than $\sqrt{2}$. Therefore, the proposed algorithm can be used to eliminate the simple water-bottom multiples but probably not pegleg multiples.

The forward and backward transformations in the algorithm assume that the data is sufficiently sampled. In practice, however, aliasing could happen to low-velocity events. If this does happen, then a proper interpolation has to be done before the algorithm is applied.

To preserve the correct amplitudes of the events, the backward transformation should be the inverse, instead of the transpose, of the forward transformation. But this inverse operation is very expensive. This is the major barrier for the algorithm to be used in practice.

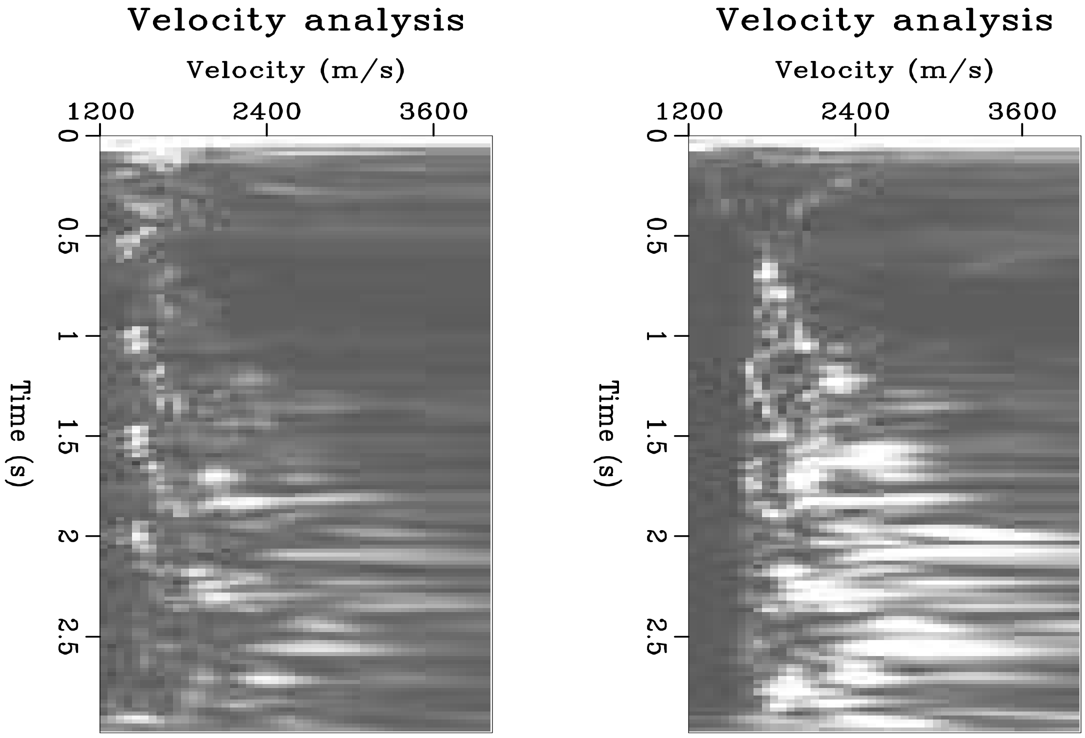

Figure 5: Velocity spectra: (a) before and (b) after the removal of low-velocity events. lin1-velspe [NR]

CONCLUSIONS

In this paper, I have presented a new method to eliminate the water-bottom multiples in a CMP gather. The proposed algorithm is different from the conventional velocity filtering in the way that multiples are separated from the primaries. Although for the purpose of multiple elimination the algorithm does not offer any improvement and even adds several restrictions, the principle of the algorithm represents a new approach to the problem. The result can be possibly improved by the selection of proper weighting functions in the transformations.

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