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Anisotropic Velocity Analysis

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ABSTRACT

This paper examines the meaning of the anelliptic terms in relation to the processes of approximating the ray velocity surface of transversely isotropic media recently introduced by Muir (1985) and Byun et al. (1989). Compared with stacking velocity obtained by simple hyperbolic velocity analysis, the additional parameters estimated by the non-hyperbolic method contain more physically meaningful geologic information regarding the anisotropy of the subsurface. Synthetic qP-wave model experiments demonstrate that the non-hyperbolic moveout formulas yield an excellent fit to time-distance curves over a wide range of ray angles. Thus the measurement parameters adequately reflect the characteristics of velocity dependence on ray angle, in other words, velocity anisotropy.

INTRODUCTION

Velocity analysis based on a hyperbolic moveout model has been in wide use until now. Velocities estimated this way are routinely used to improve signal quality by stacking multifold seismic data. Also, the stacking velocity spectrum is used to obtain lithologic information about the subsurface. However, under the existence of anisotropy the hyperbolic moveout equation is no longer adequate to preserve signal resolution through stacking. Furthermore, stacking velocity estimates alone are not sufficient to distinguish among different lithologies.

In order to approximate anisotropic ray velocity, Muir (1985) introduced a practical anisotropic system as a rational form and lately another approximation as a form of truncated Fourier cosine transform was presented by Byun et al. (1989).

In this paper I examine the meaning of additional parameters, anellipticity q, in two anelliptic approximations for a transversely isotropy velocity curve, which result in non-hyperbolic moveout equations, and explore the usefulness of the two methods by applying them to the synthetic data.

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ANELLIPTIC APPROXIMATION

Among the many types of deviation from isotropy, the most probable form for normal earth sections is called transverse isotropy. Transverse isotropy assumes that physical properties are the same in all planes perpendicular to one axis of symmetry. In transverse isotropy, the group velocity curve, which is a graph of the velocity of energy propagation versus the angle of energy propagation, does not define an ellipse for qP-wave or qSV-wave, but the velocity for qSH-wave is elliptical. So the elliptical approximation as velocity equation of transverse isotropy which results in the hyperbolic moveout equation is usually inappropriate and misleading for qP- and qSV-waves. Good approximations might include the additional term which represents the deviation from the best fitting ellipse.

Assuming the velocity deviation from the ellipse has typical pattern, the approximation for the ray velocity of transverse isotropy can be represented by the form

$$\text{anelliptic velocity curve} = \text{best fitting ellipse} + f(\theta) \tag{1}$$

where the best fitting ellipse means the ellipse that has the same vertical and horizontal velocity as the anisotropic velocity curve and where $f(\theta)$ corresponds to the deviation from the best fitting ellipse.

Figure 1 shows ray velocity deviations from the best fitting ellipse in several transversely isotropic models in terms of sloth (inverse velocity squared) and shows patterns that have zero value in the vertical and horizontal directions and a maximum value of about 45 degrees.

Byun's approximation

Among the possible forms of deviation from the ellipse, one probable function is $\sin^2\theta\cos^2\theta$ which has zero value in the vertical and horizontal directions, and maximum value of 45 degrees. In this approximation, the ray velocity equation becomes

$$v^{-2}(\theta) = v_z^{-2}\cos^2\theta + v_x^{-2}\sin^2\theta + q^{-2}\sin^2\theta\cos^2\theta \tag{2}$$

where q^{-2} has four times the value of the deviated sloth from the besting fitting ellipse at 45 degrees and where the case $q = 0$ corresponds to elliptically anisotropic velocity. By using trigonometric identity, equation(2) becomes Byun's approximation :

$$v^{-2}(\theta) = a_0^2 + a_1^2\cos^2\theta - a_2^2\cos^4\theta \tag{3}$$

where

$$a_0^2 = v_x^{-2}$$
$$a_1^2 = v_z^{-2} - v_x^{-2} + q^{-2}$$
$$a_2^2 = q^{-2}$$

Muir's Approximation

From this viewpoint, the deviation term of Muir's approximation can be put into the form

$$q\frac{v_z^{-2}v_x^{-2}\cos^2\theta\sin^2\theta}{v_z^{-2}\cos^2\theta + v_x^{-2}\sin^2\theta} \tag{4}$$

This equation also has zero value in the vertical and horizontal directions and maximum value near 45 degrees because it contains the $\sin^2\theta\cos^2\theta$, but has a kind of weighting along the angle that causes a little movement of the maximum value according to the anisotropy factor, defined as the horizontal to vertical phase velocity ratio $\frac{v_x}{v_z}$. In this case the ray velocity can be represented by

$$v^{-2}(\theta) = v_z^{-2}\cos^2\theta + v_x^{-2}\sin^2\theta + q\frac{v_z^{-2}v_x^{-2}\sin^2\theta\cos^2\theta}{v_z^{-2}\sin^2\theta + v_x^{-2}\cos^2\theta} \tag{5}$$

Figure 2 and Figure 3 show how well these two approximations for deviation term fit to the deviation from the best fitting ellipse in the cases of weak and moderate anisotropy, respectively.

NON-HYPERBOLIC MOVEOUT EQUATIONS

In the case of the Byun's approximation the travel time equation was represented into the form

$$t_x^2 = t_0^2 + (\frac{z^2}{v_\gamma} + \frac{x^2}{v_x})\frac{x^2}{x^2+z^2} \tag{6}$$

where v_γ represent $(v_x^{-2} + q^{-2})^{-\frac{1}{2}}$. But in this case, v_γ can be replaced by the more physically meaningful parameter, v_q, which is the velocity at 45 degrees. By using v_q, we can formulate the moveout equation as follows:

$$t_x^2 = t_0^2(\frac{z^2 - x^2}{x^2+z^2}) + \frac{x^2}{v_x^2}(\frac{x^2 - z^2}{x^2+z^2}) + \frac{4}{v_q^2}(\frac{x^2z^2}{x^2+z^2}) \tag{7}$$

where

$$v_q = (\frac{1}{2}v_z^{-2} + \frac{1}{2}v_x^{-2} + \frac{1}{4}q^{-2})^{-\frac{1}{2}}$$

In the case of the Muir's approximation, the moveout equation has the form of

$$t_x^2 = t_0^2 + \frac{x^2}{v_x^2} + q\frac{t_0^2\frac{x^2}{v_x^2}}{t_0^2 + \frac{x^2}{v_x^2}} \tag{8}$$

In both approximations, q represents the anelliptic factor and set equal to zero gives hyperbolic moveout equation. In the case of Byun's approximation, the anelliptic factor, q or v_q, directly represents the deviated velocity from the ellipse or the velocity at 45 degrees, respectively. In the case of Muir's approach, the unit of q has no dimension, and the magnitude is controlled by the ratio of horizontal velocity to vertical velocity.

From a practical point of view, Muir's method is superior because it can be used for surface common midpoint gather data by fixing time for some event and applying semblance analysis as a function of q and v_x. Byun's approach requires one more parameter, v_z, to calculate moveout. For a given event, therefore, semblance analysis can be performed by scanning over ranges of the three parameters, v_z,v_x and v_q. If the average vertical velocity v_z is known, as may the case from VSP or check-shot data, then the semblance scan can be done over the two parameters, v_x and v_q.

SYNTHETIC EXPERIMENTS

Synthetic qP wave data experiments were performed on two radically different, homogeneous, transversely isotropy models to examine the feasibility of the two techniques for improved normal moveout correction and lithology discrimination based on the degree of estimated anisotropy of the medium and the degree of anellipticity. Table 1 shows the elastic properties of the models.

Table 1 : Elastic constants for transversely isotropic solids (ft/sec)						
transversely isotropic medium	c_{11}	c_{33}	c_{13}	c_{44}	c_{66}	density
Sandstone-Shale	234E6	228E6	90.0E6	68.9E6	71.3E6	2.34
Sandstone-Anisotropic shale	255E6	228E6	90.0E6	68.9E6	91.4E6	2.34
Limestone-Shale	364E6	287E6	113E6	86.9E6	114E6	2.44
Limestone-Anisotropic Shale	384E6	287E6	113E6	86.9E6	134E6	2.44
Anisotropic Shale	233E6	193E6	76.1E6	58.3E6	98.6E6	2.38

The first model was a sandstone-shale stratified medium with weakly transversely isotropy . The qP-wave vertical velocity of the medium was 9865 ft/s, and the anisotropy factor, defined as the horizontal to vertical phase velocity ratio $\frac{v_x}{v_z}$ was about 1.014. The second model was a limestone-shale subsurface with moderate anisotropy. This medium had a vertical velocity of 10850 ft/s and a moderate anisotropy factor of 1.125.

Synthetic common shot point gather were obtained by 2-dimensional finite difference modeling for the two models for analysis. The source was at a depth of 1850 ft and receivers were at the surface out to offsets of 3200 ft. The range of angles was from 0 to 60 degrees. The vertical component of synthetic common shot point gather for the sandstone-shale model and limestone-shale model are shown in Figures

4 and 5 ,respectively. For applying Byun's method, we can consider this common shot point gather as the offset VSP gather and for applying Muir's method this can be considered as common midpoint gather by considering the source is image source and the depth of common midpoint is a half of the depth of image source.

Figure 6 shows semblance analysis by using Byun's method for sandstone-shale model as a function of horizontal velocity v_x and velocity at 45 degrees, v_q. High semblance values appear at nearly horizontal velocity and smaller velocity at 45 degrees than horizontal velocity which represent very well weak anisotropy or anellipticity.

Figure 7 shows semblance analysis by using Muir's method for sandstone-shale model as a function of horizontal velocity v_x and anellipticity q. In this case, peak semblance appears near the horizontal velocity and the small value of q represents weak anisotropy. Figures 8 and 9 show semblance analysis for limestone-shale model by Byun's method and Muir's method, respectively. In both cases stacking velocity represents a somewhat bigger value than the actual horizontal velocity and anellipticity is bigger than in the case of sandstone-shale model.

CONCLUSION

Anisotropic velocity functions can be well-modeled with anelliptic approximation. The non-hyperbolic moveout equations which derive from the anelliptic approximation can be used to improve signal coherency over large source-geophone offsets. The estimated additional parameters represent the degree of anellipticity that characterize transversely isotropic velocity. These anelliptic parameters might be examined more deeply in order to providing a map from group velocity to phase velocity so that non-hyperbolic moveout estimates can provide non-hyperbolic migration operators.

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