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A perturbation method for elastic-isotropic inversion—a proposal

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ABSTRACT

Several methods for elastic inversion have been applied in recent years, based on the optimization of either the fitting between the predicted and recorded data, or the fitting between the model-predicted and data-retrieved reflectivity series. In both cases, the model is usually described by a stack of layers that controls the kinematics and the dynamics of the physical process. I propose a method in which geometrical and physical effects are decoupled, so that the part of the model responsible for the dynamics can be modified without introducing any change on the kinematically-related part of the model. The model is characterized by a background medium, with smoothly-varying elastic properties, and an arbitrary number of perturbation layers. Each perturbation layer is responsible for the introduction of two reflections, whose amplitudes depend only on the contrast between the properties of the layer and the local properties of the background model. In addition to the kinematics-dynamics uncoupling, the number of unknown parameters necessary to describe the two reflections of a layer (top and bottom) is reduced to less than a half of the number required by usual model-description methods.

INTRODUCTION

The spatial structure (wavefront curvatures) of the recorded wavefield, or kinematics, as well as its low-wavenumber amplitude behavior (geometrical spreading, absorption, etc) are basically controlled by the average properties of the overburden medium, while the high-frequency amplitude behavior, as well as the low-wavenumber amplitude behavior along the wavefronts (dynamics), depend upon its local properties. Furthermore, average properties of the overburden can be related to the low-frequency content of the model space, while local properties can be related to its high-frequency components.

Since the wavefield is differently affected by the low- and high-frequency compo-

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nents of the model, we would expect that the estimation of one can be made independent of the other. In fact, some standard techniques employed to estimate the average properties (solve the kinematics) are based on the assumption of a dynamically passive system. For instance, conventional methods, for estimating the optimum NMO velocity field assume that the reflectivity should ideally be a constant function of the offset instead of satisfying an specific angle dependence. These methods have proved to be completely adequate to almost all practical applications.

On the other hand, most methods for acoustic and elastic inversion (solve the dynamics) try to solve both problems at the same time (Hindlet and Kolb, 1988; Mora, 1987; Snieder et al., 1988). The coupling between the kinematically and dynamically related parts of the system is responsible for both an unnecessary increase in the time needed for the inversion and an increase in the instability of the problem (Demirbag and Coruh, 1989). To separate the two problems, we must look at the crucial point, that is, the characterization of the model.

The usual way is to model the earth as a stack of layers with fixed elastic properties. As a result of this structure, a change in the model affects not only amplitudes, but also traveltimes. Therefore, both average and local properties of the subsurface medium (and consequently, kinematics and dynamics) will be coupled from the beginning—in the model itself.

In an attempt to separate the two problems, de Haas and Berkhout (1988), propose the introduction of a macro model (responsible for the kinematics) and a target zone, composed of a stack of layers (responsible for dynamics). This characterization of the model space results in a reduction of the cost and an increase in the stability of the inversion. The target zone, however, remains characterized by the same stack-of-layers scheme.

A more consistent method for solving the dynamics problem should be able to completely separate the local from the average properties of the subsurface media. One way of doing that is to impose constraints on the possible changes of the model, so that the kinematics remains unaffected. I propose a method in which the uncoupling is performed in the model itself. One part of the model (which I will refer as background) is an apriori estimation of the smoothly varying elastic properties of the subsurface. Of course, a pure kinematic method should be used to construct this part of the model, which remains unchanged during the inversion process. The other part consists of sparse perturbation layers, which affect only the dynamically-related features of the wavefield.

DYNAMICS AS PERTURBATION OF THE KINEMATICS

Before going ahead, it is important to answer the following question: is the reflection response of an interface separating two layers equivalent to the sum of the responses of the two interfaces that separate each layer from the background medium, as the separation between the layers goes to zero? This question is illustrated in Figure **??**. The answer is yes, but only if we take into account all the multiple conversions and internal reflections inside the background sandwiched layer.

I found however, after several numerical simulations (see Figure **??**), that under some conditions (on the deviation of the perturbations from the background) a good approximation can be achieved by the consideration of only the primary P energy. A theoretical consideration of this approximation is painful for the elastic case, but its possible to have a good feeling of the limitations involved by the analysis of the simple case of normal incidence. The reflection coefficient when the two layers are in direct contact is

$$R = \frac{I_2 - I_1}{I_2 + I_1},$$

where I_x is the impedance of medium x (P velocity $\times$ density).

When the two layers are separated by the background layer whose thickness approaches zero, the reflection coefficient will be (if we consider only primaries)

$$R' = \frac{I - I_1}{I + I_1} + \frac{I_2 - I}{I_2 + I},$$

where I is the impedance of background medium. This last expression can be put into the following form:

$$R' = F(\tfrac{I_1}{I},\tfrac{I_2}{I})\, R,$$

where

$$F(\tfrac{I_1}{I},\tfrac{I_2}{I}) = \frac{2\,[\frac{I_1}{I} + \frac{I_2}{I}]}{[1 + \frac{I_1}{I}][1 + \frac{I_2}{I}]}.$$

The elastic response from an interface separating two layers (I_1 and I_2) is modeled as the superposition of the elastic responses from the two interfaces that separate each layer from an hypothetical layer (I).

The ratio between modeled and actual normal incidence reflection coefficient, as a function of the background impedance, for fixed impedance between the two layers.

Figure **??** shows how the ratio (F) between the two reflection coefficients behaves for fixed I_1 and I_2, and variable I. If the impedance of the background medium lies between the impedance of the two media, the ratio is very close to unity for any practical case (for instance, $F_{max} = 1.03$ for $I_2/I_1 = 2$).

The differences between the reflectivities for non-normal incidence were computed for some specific cases corresponding to a background P velocity of 2500 m/s and a maximum variation of 500 m/s around it for the perturbation layers. The results are shown in Figure **??**. All graphs start at $p^{1/2} = 0.012$ (which corresponds to an angle of 21 degrees in the background layer) because the errors are too small for smaller values. For figures *(a)*, *(b)*, *(e)* and *(f)* (in which the velocity of the slowest medium (1) is held constant), the error starts to grow near the critical angle of medium 2 (when its velocity is larger than the background) or near the critical angle of the background (when medium 2 is slower than the background). On the other hand, when the velocity of the fastest medium (2) is fixed (figures *(c)*, *(d)*, *(g)* and *(h)*), the error begins to increase near the critical angle of medium 2. We conclude then that the approximation is valid, up to the critical angle of the faster of the three media involved.

All figures correspond to $v_{min} = 2000$m/s, $v_{max} = 3000$m/s, $v_{back} = 2500$m/s, $\rho_1 = \rho_2 = \rho_{back} = 1.0$g/cc ($v$ refers to P velocities, ρ to density, and indexes 1, 2 and *back*, to media 1, 2 and background respectively). Different curves correspond to increasing P velocity of medium 1 or 2, according to the following convention: continuous=2167, dot=2333, dot-dash=2500, fine-dash=2667, large-dash=2833. *(a)* $v_1 = v_{min} = const.$, $\sigma_1 = 0.1$, $\sigma_2 = 0.3$, $\sigma_{back} = 0.22$ (σ represents Poisson's ratios), and v_2 has a different value for each curve (according to previous convention). *(b)* Same as a, except that $\sigma_{back} = 0.3$. *(c)* Same as a, except that $v_2 = v_{max} = const.$ and v_1 varying. *(d)* Same as c, except that $\sigma_{back} = 0.3$. *(e)*, *(f)*, *(g)*, and *(h)*, are equivalent to the previous four, with the only difference of $\sigma_1 = 0.3$, $\sigma_2 = 0.1$.

FORWARD MODELING

A beam generated by a monochromatic point source (with angular frequency w and ray parameter p), and traveling through an horizontally homogeneous medium is described in cylindrical coordinates by

$$\chi(\omega, p; r, z, t) = G(p, r) e^{i\omega(pr + \int_0^z s(z)dz - t)}, \tag{1}$$

where

$$s(z) = \sqrt{1/v^2(z) - p^2},$$

is the vertical slowness of the medium at depth z, p is the horizontal slowness, r is the horizontal distance from the origin, and $G(p, r) = \sqrt{A_0/A(r)}$ is a geometrical factor corresponding to the ratio between the cross-sectional areas of the beam, at unit distance from the source and at point (r, z).

If we introduce a thin layer into the medium, the overall response of the reflected P wavefield beam (in the far-field approximation) at the surface, can be represented by the following expansion:

$$\begin{aligned} \chi_L^P(\omega, p; r, t) \;=\;& G(p, r)\, e^{i\omega(2pr + 2\int_0^{z_L} s(z)dz - t)} \{\grave{R}_B^{PP} + \\ & \grave{T}_B^{PP} \grave{R}_L^{PP} \acute{T}_L^{PP} e^{i\omega 2\Delta z_L s_L^P} + \grave{T}_B^{PS} \grave{R}_L^{SP} \acute{T}_L^{PP} e^{i\omega \Delta z_L (s_L^S + s_L^P)} + \\ & \grave{T}_B^{PP} (\grave{R}_L^{PP})^2 \acute{R}_L^{PP} \acute{T}_L^{PP} e^{i\omega 4\Delta z_L s_L^P} + ...\}. \end{aligned} \tag{2}$$

In the above equation, indexes P and S refer to P and S waves, while indexes B and L correspond to the medium of the incident wave (background or perturbation layer respectively). R and T are the reflection and transmission coefficients, while the accents (úp and dòwn) refer to the sense of propagation of the incident wave. z_L is the depth of the top of the layer and Δz_L its thickness. Finally, s_L^P and s_L^S are the vertical slownesses of P and S waves inside the layer. For instance, $\acute{R}_L^{PS}$ would correspond to a P wave that is traveling upward inside the layer and is reflected back at the top interface as an S wave.

Figure **??** shows the rays corresponding to the contributions of the two leading terms of the expansion in equation (2); they represent the P wave reflected by the top of the layer and the mode that was transmitted to the layer, reflected in the bottom and transmitted back again, always as a compressional wave.

The first two terms of the expansion in equation (2) correspond to the two rays on the figure. The first is a P wave that is reflected as a P wave by the top of the layer, and the other corresponds to the same wave, transmitted into the layer as a P wave, reflect by the bottom still as P and then transmitted back to the background medium once again as a P wave.

Beneath the P wave's critical angle (and for geologically feasible elastic contrasts), the major contribution for the recorded P wave comes from the two first terms of the expansion in equation (2), and for angles close to the critical angle, the first-order conversion terms would be enough to suitably approximate the layer's reflection response. The approximation corresponding to the first two terms of the expansion is

$$\chi_L^P(\omega, p; r, t_p) \approx G(p, r)\, e^{i\omega t_p}\, \{\mathcal{R}_L(\omega, p)\}, \tag{3}$$

where $t_p = 2pr - t,$

and $$\mathcal{R}_L(\omega, p) = e^{i\omega(2\int_0^{z_L} s(z)dz)}\, \{\grave{R}_B^{PP} + \grave{T}_B^{PP}\grave{R}_L^{PP}\acute{T}_L^{PP} e^{i\omega 2\Delta z_L s_L^P}\}.$$

If we add other thin layers into the model, the P wave response of each layer can be expressed in the same way except for a multiplicative factor, which accounts for the background transmission losses up to that layer. It is important to emphasize that in the same way as the perturbation layer does not change traveltimes (which are kinematically-related), nor does it influence average transmission losses (because of the combined effect of all the overburden above any layer). For this reason, I assume that the smoothly varying transmission (and absorption) function $\mathcal{T}(z_L, p)$ must also be estimated by an independent method.

The overall contribution from all layers is

$$\chi^P(\omega, p; r, t_p) \approx G(p, r)\, e^{i\omega t_p} \sum_L \mathcal{T}(z_L, p)\mathcal{R}_L(\omega, p) = e^{i\omega t_p}\mathcal{R}(\omega, p) \tag{4}$$

where $\mathcal{R}(\omega, p) = \sum_L \mathcal{T}(z_L, p)\mathcal{R}_L(\omega, p)$.

Summing over time gives the P wave impulsive-source response of the perturbed medium, as the time Fourier-transform of $\mathcal{R}$:

$$\chi^P(p; r, pr - t) = G(p, r) \int_{-\infty}^{+\infty} e^{i\omega t_p} \mathcal{R}(\omega, p) d\omega dp. \tag{5}$$

For a given beam (p) the smooth background model can be used to find $r(t)$, and the final result is achieved by summing the contributions from all beams. In my modeling program I used a superposition of one-half between adjacent beams and I smoothed around the central p of each beam with a triangle function.

A simple model (Figure **??**) was used to generate the synthetic data shown in Figure **??**. It is important to notice that the seismic response from the two adjacent perturbation layers does not correspond to adjacent layers. The reason for this discrepancy is that the kinematics inside the layer is different from the kinematics of the background. To simulate the response of two adjacent layers in the real world, the perturbation model should have an overlap between layers (if the top layer is faster than the background) or a gap between them (top layer slower than background). On the other hand, to transform the model space into the real world model, a variable depth-shift (given by the difference between the depth integrals of the background and perturbed model) must be applied.

Model used to generate the synthetic data in Figure **??**. The first 400 meters correspond to water. The model is composed of a smoothly varying background and some sparsely distributed layers.

Synthetic data generated with the described method, corresponding to the model of Figure **??**. *(a)* $p^{1/2} - t_0$ domain, and *(b)* $x - t$ domain.

CONCLUSION

The usual model characterization, as a stack of layers, is replaced by a model composed of a fixed, apriori smooth medium and sparse perturbation layers; this replacement can provide an alternative basis for elastic inversion. The validity of the method is limited to cases in which the fluctuations in the elastic parameters (mainly the P-wave velocity) are small compared to the low-frequency trend.

The uncoupling between kinematics and dynamics obtained by this different model structure has the advantage of enabling the implementation of a faster and more stable inversion. The sparse characterization of the dynamics part of the model brings also the capability to perform a target-oriented and/or sparsely-distributed-events inversion.

Finally, the number of parameters involved in the inversion is about half of the ones involved in the usual model-description methods. For the “stack of layers” model, 8 unknown parameters are necessary to describe two adjacent reflections, while on the perturbation model, only 4 unknown parameters are involved (see Figure **??**).

To describe the reflectivities of interfaces 1-2 and 2-3 as a function of p, 8 elastic parameters are necessary. In contrast, to describe the reflectivities of interfaces b-2 and 2-b only 3 parameters are required, since the elastic constants of medium b (background) are known.

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