Stanford Exploration Project, Report 65, January 13, 1998, pages 115–124

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Snell-driven beam-stack

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keywords: *slant stack, amplitudes, velocity, modeling, synthetic data*

ABSTRACT

Combining the concepts of Snell-rays and beam-stack, I define a transform that maps prestack data from the offset-traveltime domain into the vertical-traveltime—Snell-parameter domain. The transform is defined for a common-mid-point (CMP) gather, and it requires the basic assumption that the earth can be well approximated by a stack of horizontal layers. Each output trace represents the reflections associated with the propagation of a Snell beam through the layers of the medium. Energy is summed up, inside the beam, along the reflections, and divergence effects for either a point or line source are properly compensated for by the transform.

INTRODUCTION

Several elastic inversion methods have been developed in the past decade to estimate the elastic parameters and density of the subsurface media. Among these methods, it is possible to define a subgroup, loosely called Amplitude *versus* Offset (AVO), to refer to a specific class of methods which are based on the Zoeppritz or Knot's equations (or an approximation to them). Such equations relate the reflection and transmission coefficients, of a plane wave reflected by the interface, with some of the elastic properties of the two media involved, as a function of the angle of incidence. However, the Reflectivity versus Angle (R*v*A) relation obtained by these equations is not directly available from the recorded data, which contains basically Amplitude versus Offset (AVO) information. The usual way to overcome this restriction is to replace the angular dependence of the equations by a horizontal slowness (p) dependence, since the latter is a constant of propagation in a plane layered half-space. Therefore, the transformation of the original AVO data into Reflectivity versus p (R*v*p) data is an important first step for the implementation of some of these methods.

Since this paper is mainly focused on the retrieval of amplitude information, I will, for all purposes, consider the subsurface as a horizontally layered medium, within the spatial range spanned by one record (i.e. within a CMP range), which is the same

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basic assumption of the standard CMP stacking process (Kolb and Chapel,1989). Under this assumption the horizontal slowness (or Snell parameter) is a constant of propagation. Although the conventional slant-stack transform is a commonly used method to obtain the desired plane-wave decomposition of the arriving wavefield, there are some limitations that must be considered: Conventional slant-stacks assume a line source geometry, and the proper way to deal with the actual point source data is the use of Cylindrical slant-stacks (Brysk and McCowan, 1986); a differential moveout is still required to align the response of a reflector point (with the inevitable stretch); the finite aperture of the data requires the use of weighting on the integration and some undesirable events can be amplified in this process; summing over a large interval results in an decrease of the signal/noise ratio. Local slant-stacks can be used to solve the last two problems, but then, the geometrical spreading correction (implicit in the plane-wave decomposition process) is lost.

I propose a transform, similar to the slant-stack, that maps the data into the vertical-traveltime—horizontal-slowness domain ($t_0 - p$), instead of into the usual intercept-time—horizontal-slowness domain ($\tau - p$). This transform properly handles the above mentioned limitations of the standard slant-stack, performing thus some of the required steps to convert the recorded data into the reflectivity series.

DEFINING THE TRANSFORM

The proposed transform is a variation of the slant-stack, that mixes concepts of beam-stack and Snell rays. Beam-stacks can be used to achieve a better resolution in the computation of slant-stack, as described by Kostov and Biondi (1987). Basically, beam-stacks perform a local sum over hyperbolic trajectories (in the $x - t$ domain), which are parametrized by the vertical traveltime, offset, and horizontal slowness. Therefore, although the transformed domain is one dimension higher than the original, the method has the advantage that no apriori velocity model is required.

I define the transform so that, under some usual assumptions, the transformed data would provide an averaged, relative amplitude response of the subsurface region illuminated by a Snell beam. The assumptions are, that within the range of a CMP gather the subsurface geology can be approximated by a plane-layered, isotropic elastic medium; that a previous multiple-removal process had been applied; and that transmission losses are either negligible or previously corrected. However, only the first assumption is required for the validity of the transform, which properly compensates for the effects of spherical divergence, source and receiver radiation patterns and normal-moveout.

Figure **??** shows two Snell beams illuminating a small subsurface region. Each beam is defined by the Snell parameters (horizontal slowness) of the boundary rays of the beam, and if we do not take into account transmission or absorption losses, the total energy in a wavefront inside the beam remains constant before and after the reflection. Therefore, the ratio between the energy in a wavefront recorded at the surface and the energy in a wavefront near the source will be affected only by the reflection losses.

A small region of a reflector is illuminated by two Snell beams.

The transformation is performed in the following way:

- Using an a priori velocity model, we first define a given number of Snell rays in a common midpoint gather. Thus, within the limits of ray theory, the energy propagating between any two adjacent rays (which I will term a Snell beam) is constant at any wavefront.
- Then, using the same velocity model, we define for each vertical traveltime (t_0), those curves in the $x - t$ domain that correspond to the reflected wavefronts. This stage can be accomplished either by a usual hyperbolic fit, or by raytracing (which, of course, substantially slows down the process).
- Finally, for each vertical travel time (t_0) and each beam, we perform a weighted sum of the amplitude squared, of the samples inside the beam, along the curve corresponding to that particular t_0. This sum will generate a sample in the transformed space, at time t_0 and Snell parameter given by the central horizontal slowness of the beam.

The analytic expression for the described transform is

$$\bar{R}(p, t_0) = H(p_t, p_b, t_0)\,\mathrm{sgn}(\bar{A_u^2})\sqrt{\|\int_{S_u} \mathrm{sgn}(A_u) A_u^2(x, t[x, t_0])\, x\, dx\|}, \tag{1}$$

where x is the shot-receiver offset; A_u is the amplitude of the recorded upcoming field at offset x and time t; x_t (x_b) is the offset corresponding to the crossing point between the top (bottom) ray of the beam and the reflection curve associated with time t_0; and H is a factor that depends on t_0 and the horizontal slownesses p_t and t_b of the top and bottom rays of the beam. The exact expression for H, as well as the derivation of equation (1) and the exact meaning of $\bar{A_u^2}$, are given in Appendix A.

I have tested two possibilities for the sampling rate of the horizontal slowness (i.e. the interval between adjacent output traces). In one case, the Snell parameter interval was kept constant, and in the other the interval between the square roots of two adjacent Snell parameters was fixed. I prefer this second option, because the transformed data shows a more uniform scanning of the $x - t$ domain (Figure **??**).

Snell beams corresponding to *(a)* a constant interval between adjacent values of the square root of p, and *(b)* a constant interval between adjacent values of p.

For shallow reflectors and large values of horizontal slowness, the hyperbolic approximation for the reflected wavefront loses its validity and a ray-tracing routine (controlled by the background model) must be used to determine the stacking curve.

The main advantages of this transformation are that no further moveout correction is required; geometrical spreading correction is done in an appropriate way; the effects of source directivity are directly incorporated in the transform; the transformed dataset approximates a local plane-wave response at the reflector's position; the number of undesirable events that are summed to the signal is considerably smaller here than in the conventional slant-stack, since just a local sum is involved. Although the method described uses an apriori velocity model, a data-driven Snell ray-tracing can be implemented as described by Ottolini (1987) or using of a local dip estimation process (Claerbout, 1990).

SOME SYNTHETIC AND REAL DATA EXAMPLES

In order to compare the efficiency of the Snell beam and the $\tau - p$ transforms for the retrieval of the reflectivity matrix ($R(t_0, p)$), I generated a synthetic CMP gather, using an elastic modeling program. This program assumes a *1D* geology, and is based on Haskel-Tompson propagation matrices. The layers (specified in Table 1) were chosen so that five different reflectivity functions could be obtained, with a constant density and only two different values of P-wave velocities and Poisson's ratio.

Layer	P velocity (m/s)	Poisson's Ratio	Density (g/cc)
0	1500	0.5	1.0
1	2300	0.1	2.0
2	3000	0.3	2.0
3	2300	0.1	2.0
4	2300	0.3	2.0
5	3000	0.1	2.0
6	2300	0.3	2.0

Table 1 *Elastic constants of the synthetic model*

(a) Synthetic CMP gather, with the five reflections of interest labeled (with numbers), and *(b)* its $\tau - p$ transform.

The synthetic data, along with its $\tau - p$ transform, is shown in Figure **??**. The strong reflection at 0.4 seconds corresponds to the water-bottom interface (with a large impedance contrast). To avoid the presence of multiples corresponding to the air-water boundary, the air-filled upper space was replaced by water in the model. The five primary P reflections of interest are labeled (with numbers) in the figure: reflection **1** corresponds to the interface between layers **1** and **2** on Table 1, and so on. These labels will be used later to refer to each of these reflectors.

The Snell-beam transform of the synthetic data and its normal-moveout-corrected $\tau - p$ transform are shown on Figures **??** **a** and **??** **b**. The first evident difference between the two transforms is the absence of information for small values of p, in the Snell-beam transform. Since there are some missing traces close to the source, we should expect the presence of the gap (that decreases with time). However, the $\tau - p$ transform shows a smearing effect, generating information on values of slowness not covered by the experiment. Another visible difference is the relatively greater strength of the ocean bottom multiples on the $\tau - p$ transform.

(a) Snell-beam transform of the CMP gather shown in Figure **??** **a**, and *(b)* the normal-moveout-corrected $\tau - p$ transform.

The general amplitude behavior of the five labeled reflections are very similar on both transforms. I chose reflection **5** to compare how closely the two transforms approximate the theoretical plane-wave response. Figures **??** **a** and **b** shows a small window of eighteen samples around reflection **5**, for both transforms (negative amplitudes are plotted in the up direction). Although more visible on the Snell-beam transform, the phase inversion (zero crossing) is also clear on the $\tau - p$ transform.

(a) and *(b)* Small windows around reflection **5** of Figures **??** **a** and **b** respectively, and *(c)* and *(d)* the two central samples of these windows (continuous lines) as well as the theoretical curves for the reflection coefficient (dotted line), and the reflection response with the transmission losses considered (dashed line).

The two central time samples of Figures **??** **a** and **b** are plotted in Figures **??** **c** and **d** (continuous lines), as well as the theoretical curves for the reflection coefficient (dotted lines), and the reflection response with all the transmission losses (including mode conversions) taken into account (dashed lines). Although both transforms show a good fit with the theoretical curve for values of p between the minimum recorded value and the zero-crossing, the $\tau - p$ transform seems to be more susceptible to multiples contamination. For this reason the zero crossing value of p, is better defined in the Snell-driven transform. However, for values of p beyond the zero crossing, the amplitudes of the transformed data are lower than those of the theoretical curve for both transforms.

The same kind of curves, corresponding to reflections **1**–**4** of the Snell-beam transform, are presented in Figure **??**. Continuous lines correspond to the central time sample of the Snell-beam transform. Dotted lines are the theoretical curves for the reflection coefficient, and dashed lines are the reflection responses with the transmission losses taken into account. In **a** and **d** (where the P wave velocity increases) the reflection coefficient becomes imaginary after the critical p, and the imaginary part of the two theoretical curves are also present (dot-dash and small dash).

As in Figure **??c** and **d** *(a)*, *(b)*, *(c)* and *(d)* correspond to reflections **1**, **2**, **3**, and **4** respectively. See further explanation in the text.

In all cases there is a good fit for small values of p (on the range of available information), but for large values of p the transformed amplitudes are always smaller than the theoretical values. The problem arises from the modeling scheme's failure to maintain the same frequency content for the entire range of p. A comparison of the spectra of a near-, intermediate- and far-offset traces for the synthetic data and for real data are shown in Figure **??**. While for real data the amplitude loss is about the same for all offsets, the synthetic data has a considerable decrease of the high frequency content.

(a) Spectra of three traces at near-, intermediate-, and far-offset for a real data, and *(b)* the same for the synthetic data of Figure **?? a**.

Figure **??** shows a shot gather of a marine dataset from Brazil, and its Snell-beam transform. The data was recorded with a long cable (150 receiver-groups, maximum offset of 4.3 Km), and the absence of structural features is in good agreement with the assumption of a plane-layered earth. We can notice some strong events appearing at large values of p as we should expect from reflections near the critical angle. Also, a careful examination of the sharp reflection at 1.25 seconds, on the transformed data, suggests an increase in the reflection coefficient with p. For such large values of p, the hyperbolic approximation for the reflection moveout is not appropriate any more. I used the stacking velocities (found with the use of only pre-critical offsets) and constructed a model for raytracing.

(a) A long cable marine data from Brazil, and *(b)* its Snell-beam transform. Notice the strong events associated with reflections near the critical angle.

The two central time slices of this reflection are shown in Figure **??** and, as we can notice, the general shape of the curve is very similar to the curve of reflection **1** (Figure **?? a**) of the synthetic data. We can infer, even without submitting the data to an inversion algorithm, that the reflection corresponds to an increase in P velocity [2] and an increase in the Poisson's ratio.

Relative amplitudes of the two central time slices of the reflection at 1.25 seconds of Figure **?? b**.

CONCLUSION

Using a given velocity model, the Snell-beam transform maps data from the offset—traveltime domain into the horizontal-slowness—vertical-traveltime domain. This transform can be used as an alternative to the usual $\tau - p$ transform to perform the plane wave decomposition of the data, with the difference that no further moveout correction is required. Furthermore, as opposed to local slant-stacks, geometrical corrections are performed in an appropriate way (within a 2.5 D assumption), without losing the advantages of a local stack transform.

The application of the transform to synthetic data shows a good agreement between shapes of the amplitude curves of the several different elastic interfaces, and their theoretical curves.

[2] An increase in density instead of velocity would have a different behavior at large values of p.

The results are also in accordance with the application of the $\tau - p$ transform, followed by a normal-moveout correction to the vertical traveltime. However, multiples and noises in general are weaker on the Snell-beam transform, due to the local character of the beam-stack transform.

To retrieve the reflectivity matrix ($R(t_0, p)$) several operations are necessary. The Snell-driven beam-stack transforms the recorded data into the desired $t_0 - p$ domain, with the additional compensation for divergence, and radiation pattern.

ACKNOWLEDGMENTS

I would like to thank Francis Muir for the discussions and suggestions that contributed to this work.

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APPENDIX A

If we disregard absorption and transmission losses, and consider the asymptotic approximation (where the ray theory holds), the integral of the energy flux over a wavefront inside a Snell beam, is a constant of propagation. This condition can be expressed by

$$\int_S \rho v A^2 \, ds = \text{CONSTANT} \tag{2}$$

(Aki and Richards, 1980), where S is the wavefront surface, ρ is the density of the medium, and v and A are the velocity and amplitude of the wave. equation (2) applies to both the downward and upward ring-shaped propagating wavefront of Figure **??**, but the constants are different before and after the reflection.

An azimuthly symmetric conical beam is reflected by an interface. Under the assumption of a plane-layered earth, the amplitude at any point of the ring-shaped wavefront depends only on the Snell parameter p and the offset x.

Let's first apply equation (2) for the downgoing beam, at a unit distance from the source, and just before it hits the reflector:

$$\int_{S_i} \operatorname{sgn}(A_i) A_i^2(p)\, v_z\, ds_i = \int_{S_0} \operatorname{sgn}(A_0) A_0^2(p)\, v_0\, ds_0, \tag{3}$$

where the indexes 0 and i apply to the wavefronts near the source and near the reflector, respectively. The *sgn* function introduced in equation (3) does not affect the equality. It is possible to integrate separately infinitesimal beams with positive and negative amplitudes, and the equality will still hold for both independently, since no sign change is possible inside a beam.

Applying equation (2) for the upcoming beam; just after the reflection and just before it hits the surface, we find that

$$\int_{S_r} \operatorname{sgn}(A_r) A_r^2(p)\, v_z\, ds_r = \int_{S_u} \operatorname{sgn}(A_u) A_u^2(p)\, v_0\, ds_u, \tag{4}$$

where the indexes r and u apply for the wavefronts near the reflector and near the surface, respectively.

I define now an average amplitude square over a wavefront, in such a way that it keeps the "average sign of the wavefront"

$$\bar{A}^2 = \frac{\int_S \operatorname{sgn}(A) A^2 ds}{S}. \tag{5}$$

It is possible then to have an estimation of the average reflection coefficient over the region illuminated by the beam through the following relation

$$\bar{R} = \operatorname{sgn}(\frac{\bar{A}_r^2}{\bar{A}_i^2}) \sqrt{\left\| \frac{\bar{A}_r^2}{\bar{A}_i^2} \right\| \frac{S_r}{S_i}}. \tag{6}$$

Dividing equation (4) by equation (3) and using equations (6) and (5) results in

$$\bar{R} = \operatorname{sgn}(\frac{\bar{A}_r^2}{\bar{A}_i^2}) \sqrt{\left\| \frac{\int_{S_u} \operatorname{sgn}(A_u) A_u^2\, ds_u}{2\pi \bar{A}_0^2(p)(\cos\theta_b - \cos\theta_t)} \right\|}, \tag{7}$$

where $\bar{A}_0^2(p)$ is an average (in the sense of equation (5)) estimation of the source radiation pattern, for the beam with Snell parameter p, at unit distance from the source. Indexes b and t apply for the bottom and top rays of the beam as illustrated in Figure **??**.

Vertical cross section of the upcoming beam. If we approximate the surface S by a conical surface, the area element ds is given by $ds = 2\pi x \cos\theta_t dx$.

A last necessary assumption is that

$$\mathrm{sgn}(\frac{\bar{A}_u^2}{\bar{A}_0^2}) = \mathrm{sgn}(\frac{\bar{A}_r^2}{\bar{A}_i^2}), \tag{8}$$

which is equivalent to the assumption that the signal of the average described by equation (5) does not change during propagation, unless the beam is reflected.

Finally, if the wavefront ring reaching the surface is approximated by a conical surface, the application of straightforward geometry leads to

$$ds_u = 2\pi x \cos\theta_t dx.$$

Using this result and substituting equation (8) into equation (7), a final expression for the output sample $\bar{R}(p, t_0)$ is found:

$$\bar{R}(p, t_0) = H(p_t, p_b, t_0)\mathrm{sgn}(\bar{A}_u^2)\sqrt{\| \int_{x_b}^{x_t} \mathrm{sgn}(A_u) A_u^2(x, t[x, t_0]) x dx \|}, \tag{9}$$

where

$$H(p_t, p_b, t_0) = \mathrm{sgn}(\bar{A}_0^2)\sqrt{\frac{\cos\theta_t}{(\cos\theta_b - \cos\theta_t)}}. \tag{10}$$

All the cosine factors in equation (10) can be expressed in terms of p_t or p_b, the horizontal slownesses of the boundary rays of the beam, according to

$$\cos\theta = \sqrt{1 - (v_0 p)^2}.$$

The offsets x_t and x_b that appear in the integral limits of equation (9) are determined by the interceptions (in the $x - t$ domain) of the top and bottom rays with the reflection curve (or wavefront) $t = t(x, t_0)$. The specific function that describes the wavefronts in the $x - t$ domain can be given either explicitly by

$$t = \sqrt{t_0^2 + (x/v(t_0))^2},$$

if an hyperbolic approximation is satisfactory, or implicitly with the use of raytracing.