Seafloor-consistent multiple suppression

Stewart A. Levin

ABSTRACT

Seabottom multiples and peglegs are predicted in a process that combines wave propagation and seafloor-consistent reverberation reflection filters. Key features of the method are: truly seafloor-consistent filters; incorporation of the pure seabottom multiple; fitting error minimized at the surface; and simultaneous design using all the recorded data. I use this process to suppress strong seafloor peglegs from dipping beds on a line from the Barents sea for which conventional multiple attenuation was ineffective.

INTRODUCTION

Multiple reflections are often a problem in marine seismic exploration. Each shot, of unknown signature, sets up reverberations within the water layer that produce the seafloor multiple and a downgoing, reverberatory waveform below the seafloor. The downgoing wave then reflects from the subsurface, travels up through the seafloor, and again reverberates in the water layer to produce primaries and their seafloor pegleg trains. These multiple trains are a serious problem in areas where the water bottom has a high impedance contrast; the reverberations are slow to decay and correspondingly less source energy is transmitted through to illuminate the subsurface.

Such water reverberations usually have two features that help us differentiate them from primary reflections. These are their moveout velocity and their periodicity. CDP stack and moveout filters are two standard tools for multiple attenuation that rely on velocity differences between primary and multiple reflections. Gapped deconvolution is the standard tool that exploits the periodicity of multiples.

The standard tools are often ineffective for attenuating pegleg multiples, i.e. multiples with one primary subsurface reflection and one or more seafloor reflections. Figure 1 shows a sample pegleg raypath. Because their arrival times are delayed and a good portion of their travel paths lie in the subsurface, pegleg stacking velocities are often close to primary stacking velocities. Moveout discrimination is poor, making velocity filtering and CDP stack ineffective. Gapped deconvolution is also unreliable because it assumes a flat water bottom and faithful amplitude preservation. Both are rare to find in practice. Furthermore, even when these conditions do hold, the strong, pure water-bottom multiple decays at a different rate than pegleg multiples; pegleg attenuation

filters estimated from the data will be degraded.

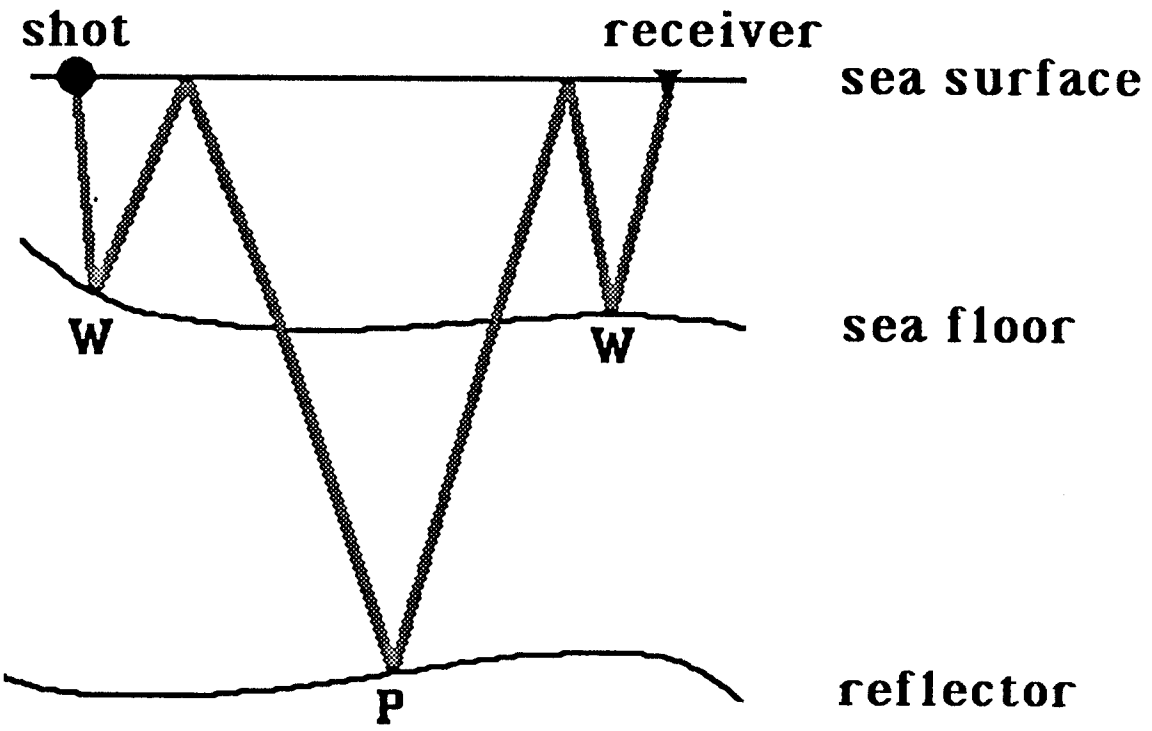


FIG. 1. A sample pegleg raypath. Here, the travel path bounces once off the seafloor before illuminating the subsurface reflector and again off the seafloor after the subsurface reflection. This **WPW** pegleg arrives at the receiver at nearly the same time as its cousins **WWP** and **PWW**, thereby tripling the recorded reflection amplitude.

Three ways exist to improve discrimination against pegleg multiples. First, we can change field acquisition parameters to better resolve small velocity differences. This can be done by increasing the source bandwidth, shortening the recording sampling interval, lengthening the cable, and/or decreasing the group interval. Second, we can use more precise models of primary and pegleg moveout and more sophisticated moveout filters. Examples of this approach are found in Schneider, Prince, and Giles (1965), Ozdemir (1981) or Hampson (1986). Third, we can use better models of multiple generation, namely those based on wave extrapolation, to improve timing and amplitude estimates of these multiples. This last approach is the one I use here.

WAVE-EQUATION PREDICTION ERROR FILTERING

The use of wave extrapolation to predict multiples, and thereby suppress them, dates back to Loewenthal, Lu, Roberson, and Sherwood (1974). They remark that at least some marine multiples can be modeled by propagating the recorded wavefield one more bounce through the water layer. The papers of Riley and Clærbout (1976) and Estevez (1977) seem to be the first published applications to seismic field datasets To make computations manageable, they restrict their applications to plane-wave stacks. Morley (1982) makes two significant advances. He first clarifies which multiples are predicted by an additional bounce through the water layer. Second, he demonstrates the importance of working with unstacked data and developed a "seafloor-consistent" theoretical model for wave-equation pegleg prediction. Using this model, he then develops gather-by-gather multiple suppression algorithms for several limiting cases.

Morley's model for pegleg multiple suppression is encapsulated in his equation (3.3.9)

$$\Delta_{W.E.} = (1 + \uparrow_s c_s \downarrow_s \nu_s)(1 + \uparrow_g c_g \downarrow_g \nu_g) \quad , \tag{3.3.9}$$

where $\nu = 1$ for a free surface, c_s, c_g are reflection operators associated with the seafloor, and $\uparrow$ and $\downarrow$ are operators that extrapolate waves from the seafloor to the surface or from the surface to the seafloor.

In recent years, several other people have applied wave extrapolation to field data. Bernth and Sonneland (1983) use a two stage adaptive process to tackle both peglegs and pure water bottom multiples. Wiggins (1985) applies Morley's model to field data from the eastern Grand Banks, with careful attention to geometric and statistical detail. Berryhill and Kim (1986) apply Kirchhoff wave-equation datuming to propagate to and from the seafloor in a hybrid approach.

In a related setting, Clærbout (1986) uses wave extrapolation for his simultaneous t-τ deconvolution. Clærbout's model, leaving out complications of spherical divergence and other weightings, is given by

$$\Delta_{W.E.} = (1 + c_g \uparrow_g \downarrow_g)(1 + c_s) \quad . \tag{1}$$

He uses to tackle both seafloor multiples and shot signature.

IMPROVING THE MODEL

In this paper I extend Morley's model in two significant ways. First, I incorporate the pure seabottom multiples as well as pegleg multiples. Second, I make the seafloor reflection filters truly seafloor-consistent rather than surface-consistent filters projected to the seafloor.

The difference in amplitude behavior between the pure seabottom multiples and pegleg multiples is described by Backus (1959). He shows that the amplitude of the pure seabottom multiple is proportional to R^n where R is the seafloor reflection coefficient

and n is the number of bounces off the seafloor. At the same time, the amplitude of the pegleg multiples arising from a primary subsurface reflection with reflection coefficient R_1 is proportional to $(1-R^2)(n+1)R_1R^n$.

To appreciate the relative strengths of the two types of multiples, take the seafloor reflection coefficient to be $R=0.25$ and the subsurface reflection coefficient to be $R_1=0.05$. Then the seafloor reflection emerges over five times stronger than the primary reflection. The first seafloor multiple is one third larger than the primary reflection. Clearly the seafloor multiple will strongly influence any estimates of seafloor reflection strengths from the recorded data.

Morley's model does not explicitly include the pure seabottom multiples and so implicitly tries to fit their amplitudes with a sequence decaying proportional to $(n+1)R^n$. This contributes an estimate of R that is about half the correct magnitude. Since the seabottom multiple is so strong, this should significantly bias the overall estimate of R towards zero.

There are two ways to deal with this problem: One can suppress or downweight the seabottom multiples in the data during pegleg processing, or, one can include them explicitly in the underlying model for multiples. I will be doing the latter. Berryhill and Kim attempt the former by adjusting the start times of their processing windows to just after the arrival of a seafloor multiple. Bernth and Sonneland do the latter in their formulation, but do not follow through in their application. Instead they do two passes over the data. The first pass tries to suppress pure seabottom multiples, trusting that pegleg amplitudes do not strongly bias their reflection coefficient estimates. (Morley also assumed this in his applications.) Then Bernth and Sonneland mute the seafloor reflection and do a second pass to predict the peglegs remaining on the data. Wiggins also does a two step procedure, relying on L^1 norm minimization to further reduce the influence of peglegs during the first pass.

The other way I improve on Morley's model is by my definition of seafloor-consistency. His equation (3.3.9) above contains two descriptions of the seafloor: c_s when the source is above it and c_g when the receiver is above it. These reflection operators are supposed to describe how waves are reflected from the seafloor; this is a physical response that is independent of the location or even existence of any equipment for seismic exploration. I will therefore constrain c_s and c_g to be identical functions of seafloor position in my model. This is the analogue of the midpoint-consistent, or structure, term in the surface-consistent statics model.

I will now detail my process for suppression of water reverberations. First scale the field records by $t^{1/2}$ to convert (approximately) from 3D to 2D amplitude divergence. Then extrapolate the shots forward down to the seafloor and up again to predict the multiples due to shot reverberations. This is Morley's $(1 + \uparrow_s c_s \downarrow_s \nu_s)$ dereverberation operator. When the right set of seafloor reflection coefficients is used, the seafloor multiple and its (direct) illumination of subsurface reflectors should vanish. Finally, predict

and remove peglegs from primaries that emerge after subsurface reflection and then reverberate in the water layer before arriving at the geophone. These are predicted by propagating each partially deconvolved common shot gather, with the seafloor primary reflection deleted by muting, one more bounce off the seafloor reflection coefficients, and removing this from the unpropagated gather. By muting I include the seafloor multiple in the process instead of trying to attenuate it separately by velocity filtering.

Assuming a free surface, i.e. ν=1, this sequence lead to the prediction error model

$$0 \approx (1 + \uparrow_g c \downarrow_g Mute\,)\,(1 + \uparrow_s c \downarrow_s)\,\sqrt{t}\;\; Data \qquad . \tag{2}$$

The job is to estimate the c 's.

ESTIMATION PROCEDURE

To simplify the task of estimating seafloor-consistent multiple suppression operators for the Barents Sea field data I am using, I make the following assumptions:

1. The seafloor is fairly flat with insignificant short wavelength texture. This lets me parametrize the seafloor reflection operator sparsely along the line and also lets me precompute the wave propagation operators.
2. The water is sufficiently deep that the shot waveform and the seafloor multiples are separated in time. This permits me to debubble (or deghost) at my convenience, either before or after multiple suppression, without having to incorporate it into the wave equation processing.
3. Seafloor reflection operators are not significantly angle dependent within the recording aperture. In deep water and with judicious muting, this is a reasonable assumption. This lets me replace seafloor reflection operators with convolutional seafloor reflection filters.
4. The sea surface is the trivial –1 free surface reflector. With this simplification, ghosting becomes a constant filter applied to all the traces and will not interfere with the multiple estimation.

Under these assumptions I linearize (2) around a reference model c_o to transforms it to

$$\begin{aligned} -(1 + \uparrow_g c_o \downarrow_g Mute\,)\,(1 + \uparrow_s c_o \downarrow_s)\,\sqrt{t}\;\; Data \;\; \approx \\ (1 + \uparrow_g c_o \downarrow_g Mute\,)\,(\uparrow_s \Delta c \downarrow_s)\,\sqrt{t}\;\; Data \;\; + \\ (\uparrow_g \Delta c \downarrow_g Mute\,)\,(1 + \uparrow_s c_o \downarrow_s)\,\sqrt{t}\;\; Data \end{aligned} \tag{3}$$

which must be solved for Δc . I will use conjugate gradient algorithm LSQR (Paige and Saunders, 1982) for the solution.

Equation (3) has the form

$$y \approx (A + B)\Delta c \tag{4}$$

with

$$\begin{aligned} A\ \Delta c &= (1 + \uparrow_g c_o \downarrow_g Mute\ ) \uparrow_s [\ \downarrow_s \sqrt{t}\ \ Data\ ] * \Delta c \quad , \\ B\ \Delta c &= \uparrow_g [\ \downarrow_g Mute\ (1 + \uparrow_s c_o \downarrow_s) \sqrt{t}\ \ Data\ ] * \Delta c \quad , \end{aligned} \tag{5}$$

and $*$ being trace-by-trace convolution. For the conjugate gradient solution we require the transpose operations as well. These are

$$\begin{aligned} A^T\ x &= [\ \downarrow_s \sqrt{t}\ \ Data\ ] \circledast \downarrow_s^- (1 + Mute\ \uparrow_g^- c_o^* \downarrow_g^-)\ x \qquad \text{and} \\ B^T\ x &= [\ \downarrow_g Mute\ (1 + \uparrow_s c_o \downarrow_s) \sqrt{t}\ \ Data\ ] \circledast \downarrow_g^- x \quad , \end{aligned} \tag{6}$$

where $\uparrow^-$ is backward (in time) propagation from the seafloor to the surface and $\circledast$ is trace-by-trace correlation.

This estimation procedure improves on past applications in one of two ways:

1. Fitting error is measured at the surface. This gives me an advantage over Riley, Estevez, Morley, or Wiggins because they measure fitting error at a seafloor, where data is neither recorded nor processed.
2. Reflection operators are placed at the seafloor. This gives us an advantage over Bernth and Sonneland or Berryhill and Kim who posit a fixed seafloor location, and design adaptive filters at the surface to try to compensate for errors in location, strength, and duration of their seafloor reflection model.

APPLICATION: BARENTS SEA

Figure 2 is a near-offset section from the Barents Sea. This line features a hard, flat seafloor at 0.4 s, a gently dipping primary at about 1.5 s, and prominent multiple trains following both. The pegleg stacks in strongly.

From the center of this line, I take a window of 56 cdp gathers, each 48-fold and 4.1 seconds (1 024 samples) long. Figure 3 shows a stack and some representative these gathers. The pure seabottom multiple beginning at 0.8 s is attenuated by the stacking, but the pegleg multiple at 2 s remains quite strong. As Figures 4 through 6 show, conventional processing does not successfully remove these multiples. Figure 4 is the best result of several runs of F-K multiple attenuation. Figure 5 is the result of gapped deconvolution before normal moveout. Figure 6 is the result of gapped deconvolution after normal moveout. The gap is 380 ms, just above the seafloor arrival; the filter extends 128 ms below that. Even with the intermediate moveout velocity positioned at

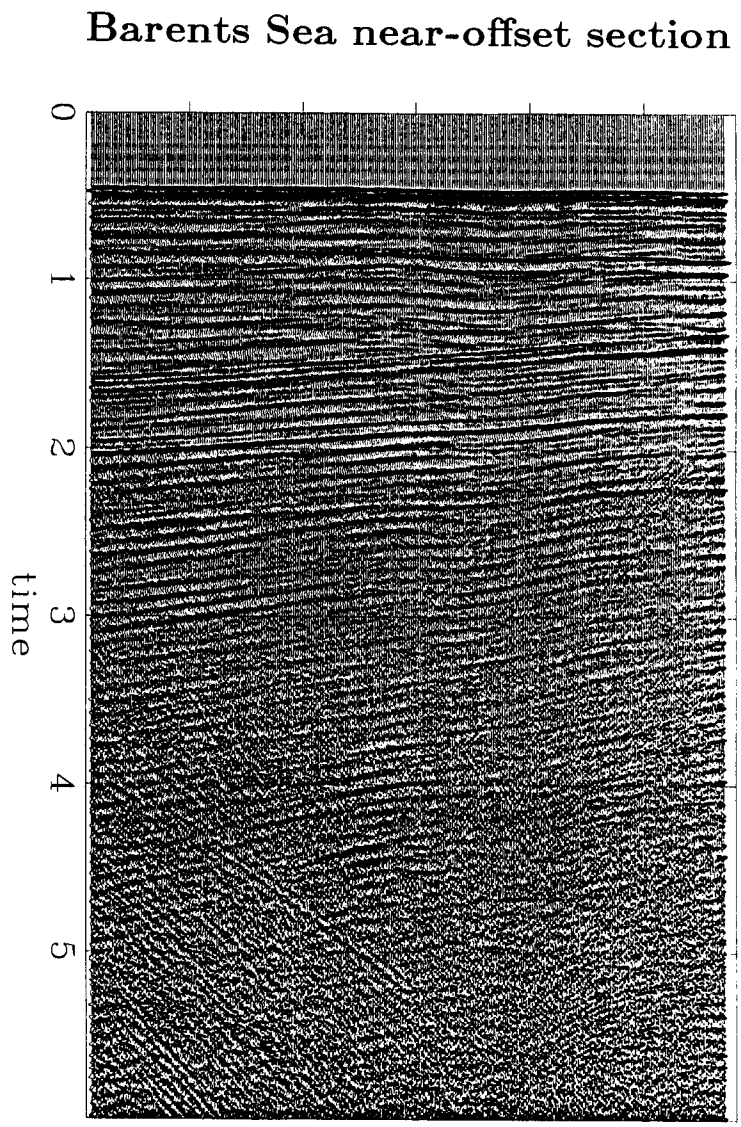


FIG. 2. Near offset section from the Barents Sea. Trace spacing is 25 m, time sampling interval is 4 ms. The dipping primary at about 1.5 s is followed by a strong pegleg near 2 s. The offset is 294 m.

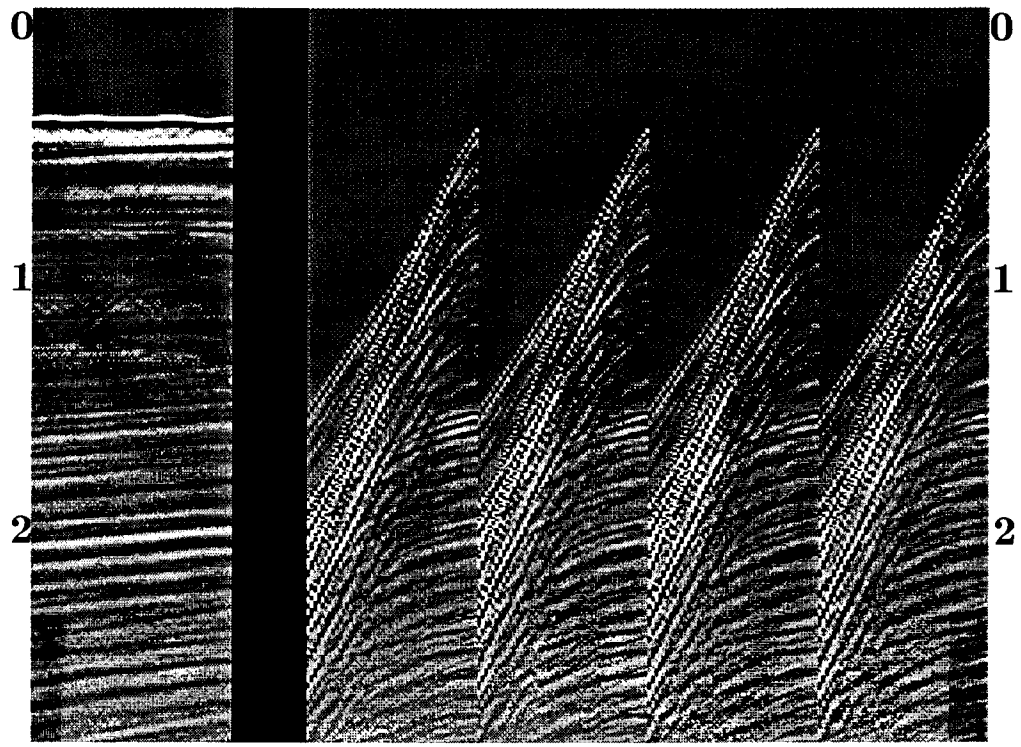


FIG. 3. 56 CDP stack and selected CDP gathers from the center of the Barent Sea profile of Figure 2. CDP interval is 12.5 m, time sampling interval 4 ms. Gathers are 48 fold with a 25 m trace interval.

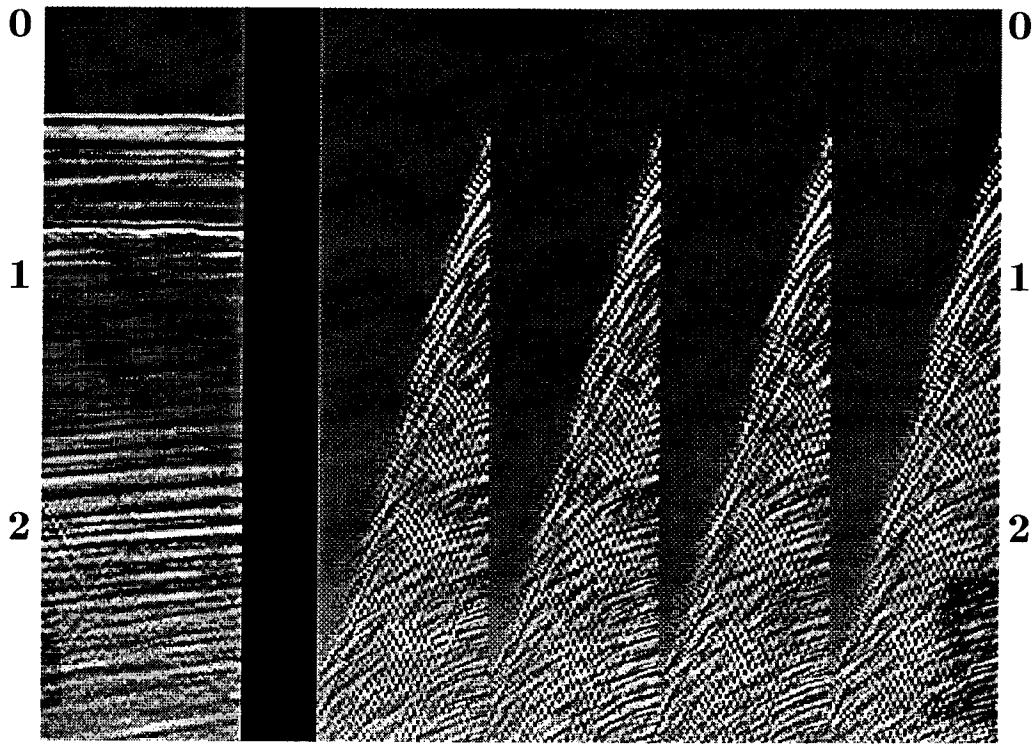


FIG. 4. Result of conventional F-K moveout filtering to attenuate multiples. The primary at 1.5 s is undesirably attenuated because its moveout velocity is too close to that of the pegleg at 2 s.

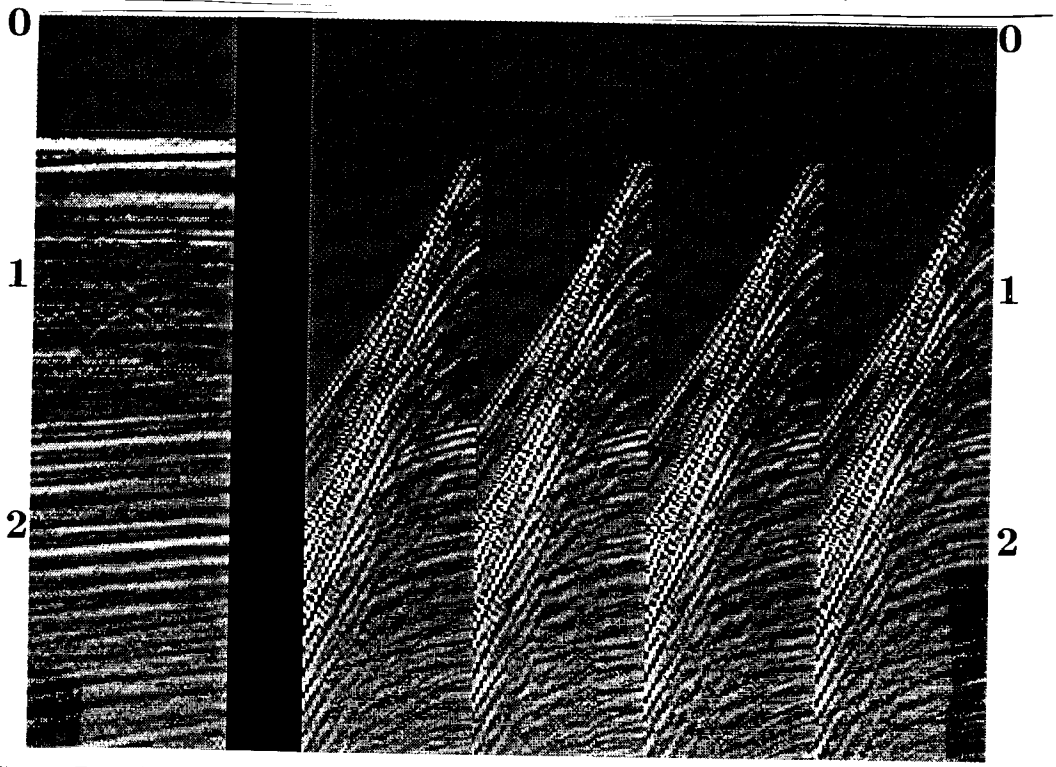


FIG. 5. Result of gapped deconvolution before normal moveout correction. The gap is 380 ms, the filter length extends an additional 128 ms. The pegleg at 2 s remains strong on the stack.

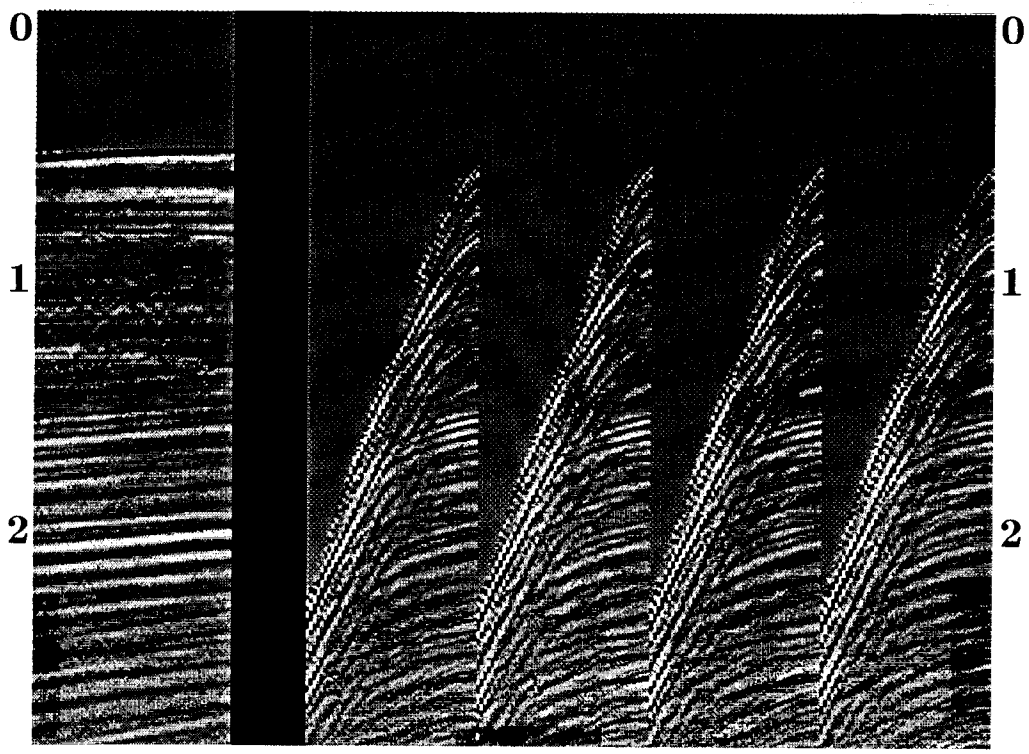


FIG. 6. Result of gapped deconvolution after normal moveout correction. Filter parameters are the same as for Figure 5. Normal moveout is removed from the CDP gathers before display. Again the pegleg at 2 s remains strong on the stack.

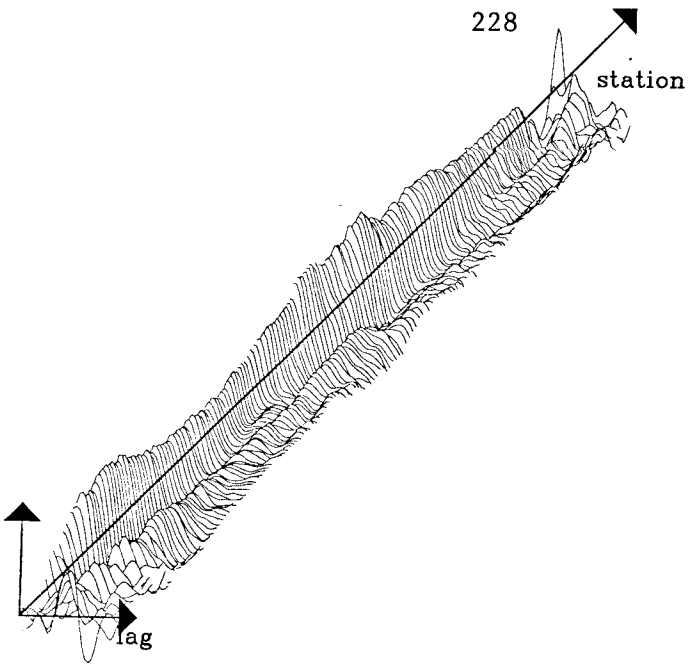


FIG. 7. Seafloor reflection operators designed by least squares. The result of five conjugate-gradient iterations, these filters are convolved with the extrapolated gathers at the seafloor to suppress water borne multiples.

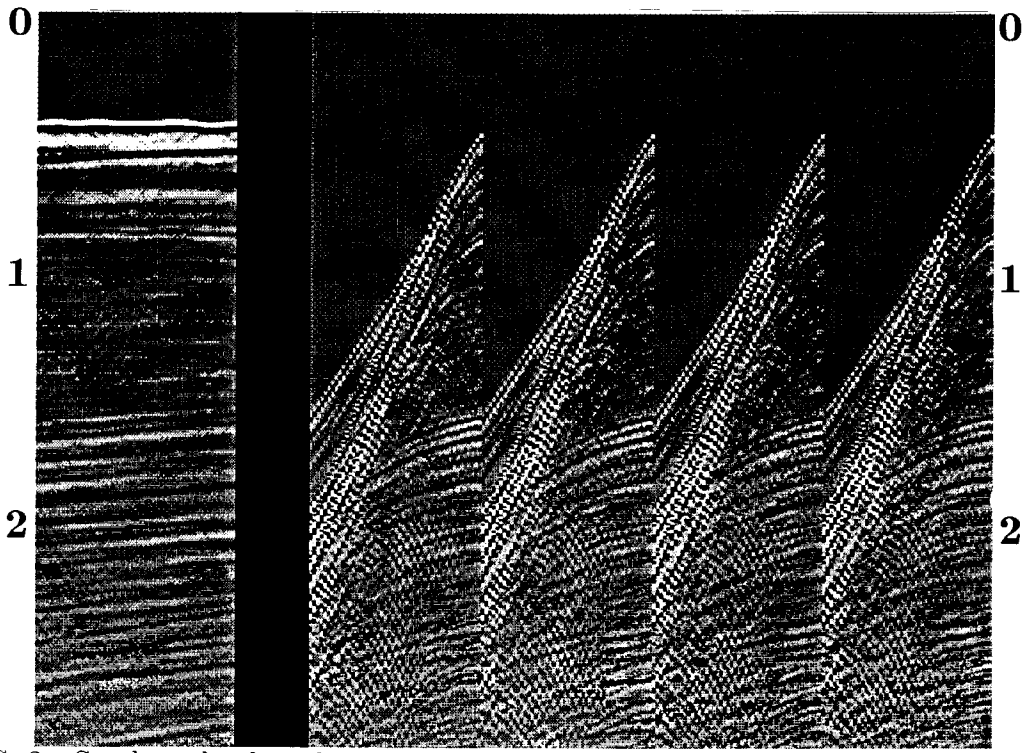


FIG. 8. Stack and selected CDP gathers after seafloor-consistent multiple suppression. The pegleg at 2 s is significantly attenuated on both the gathers and the stack. The pure seafloor multiple is attenuated by the stacking but remains strong on the CDP gathers.

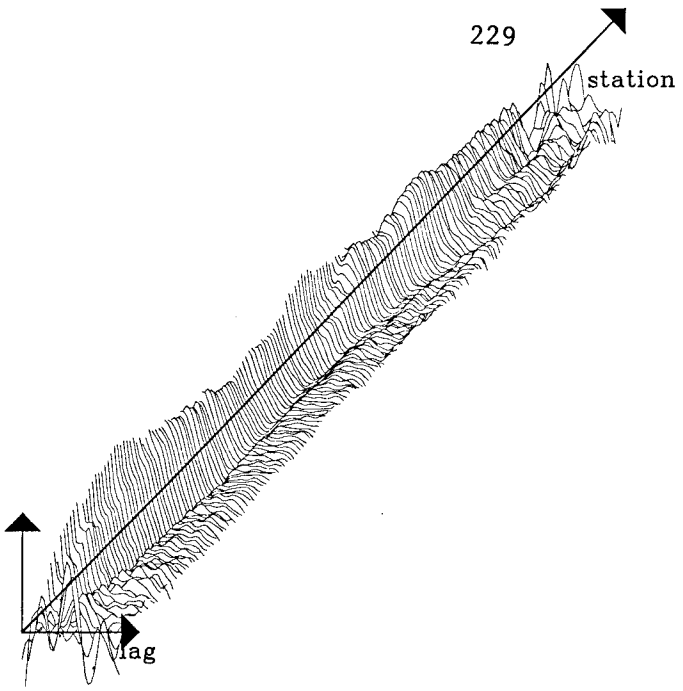


FIG. 9. Seafloor reflection operators designed using the filters of Figure 7 as a starting point.

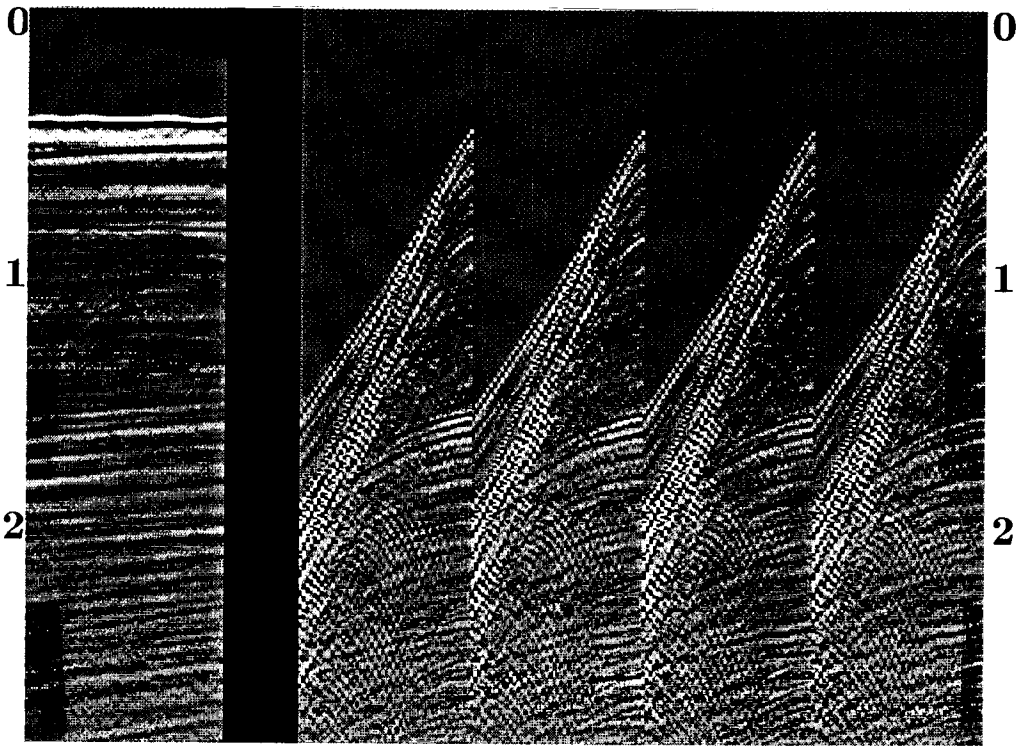


FIG. 10. Stack and selected CDP gathers after seafloor-consistent multiple suppression with the filters of Figure 9. Comparing with Figure 8, we see virtually no change.

water velocity, FK processing attenuates the primary at 1.5 s. The gapped deconvolutions did at least some good in attenuating the pegleg multiple.

At this point I turn to wave-equation multiple suppression. Projecting the shot and receiver locations for the data window downward to the seafloor, I specify 156 seafloor stations at which to estimate 128 ms reflection filters. The resulting least-squares problem has 5 184 unknown reflection filter coefficients to be estimated from 2 752 512 equations. The wave operator is precomputed using dip-limited phase shifts (Levin, 1983) to extrapolate between the surface and a datum time of 380 ms, just above the seafloor.

Figure 7 shows the seafloor reflection filters resulting from 5 conjugate-gradient iterations to solve equation (3). The starting point for the design is $c_0 \equiv 0$. Figure 8 shows the corresponding stack and some CDP gathers after this processing. We see a marked improvement in pegleg multiple suppression.

Of course, one may argue, starting with $c_0 \equiv 0$ is an abysmally poor first guess. $c_0 \equiv 0$ means we think the seafloor is transparent. If this were so, we wouldn't be worrying about seafloor multiples. It does have a practical advantage. Half the terms in the matrix-vector products in equations (5) and (6) go away thereby saving about the same fraction of computer time. It also takes no data preprocessing to specify.

How much better can we do starting with a reasonable first guess? To examine this, I use the filters from Figure 7, i.e. the output of the first run, as the starting guess for a new filter design. The results, shown in Figures 9 and 10, are nearly the same as starting from $c_0 \equiv 0$, but cost nearly twice as much to compute. Overall, it would be more cost effective to let the $c_0 \equiv 0$ design proceed for twice the number of iterations instead.

THE MULTIPLES THAT GOT AWAY

Wave-equation multiple suppression has done a superior, but not a perfect, job of removing water reverberations. On the stack, we still see some residual pegleg at 2 s and only a little attenuation of the pure water bottom multiple at 0.8 s. If we look at the individual CDP gathers before and after multiple attenuation, we find: a) the pure seabottom multiple is suppressed at all but the near offsets; b) the deeper pegleg is attenuated at all offsets; c) the aliased, steeply dipping refractions, wide-angle reflections, and multiply-reflected refractions are producing hyperbolic artifacts on the gathers that do not appear on the stack; and d) the multiples of the seafloor diffractions are not attenuated on the stack.

Explanations are not difficult; cures may be much harder. The shallow seabottom multiple remains on the inner offsets because of the finite recording aperture of the cable – specifically the nonzero inner offset. The direct seafloor reflection needed to predict the first multiple at the inner offsets arrives at the surface at half the inner offset and is therefore not in the recorded data. Berryhill and Kim recommend some amount of

interpolation into the missing inner offset range to alleviate the problem. This is much less of a problem for deeper events when the raypaths are closer to vertical. This is why we observe the greater pegleg attenuation in item b) – the deeper peglegs at 2 s (and 2.4 s) are more gently sloping at the inner offsets than the water bottom multiple at 0.8 s. Berryhill and Kims also recommend using reciprocity to extend the gathers. This is most useful for processing arrivals from dipping beds and diffractions.

The hyperbolic artifacts from the wide-angle reflections and refractions are not apparent on the stack for two reasons. First, and foremost, they do not line up along primary stacking hyperbolas. They're more nearly perpendicular than parallel. The stack will therefore discriminate against them. Second, the hyperbolas are spreading the energy on the wide-angle arrivals over a much greater area, giving them correspondingly lower amplitudes. This makes the original linear events (at least those remaining after processing mute) stack in weaker as well. The hyperbolic smearing of the aliased wide-angle events also helps reduce their influence on the reflection filter estimates. For the results I've shown this is fortunate as I did not mute these events prior to processing. I should have; these trapped modes do not dissipate energy into the subsurface and so remain very strong throughout the record after $\sqrt{t}$ scaling.

I cannot say with certainty why diffractions remain attached to the seafloor multiple after processing. Certainly some of this an aperture problem as diffracted energy arrives at the surface at all possible angles. For this Berryhill and Kim's reciprocity trick should help. Also the spherical divergence of diffractions is proportional to $1/t^2$ whereas 2D wave extrapolation expects them to decay as $1/t$. Thus the $\sqrt{t}$ correction in equation (2) is inappropriate for the diffractions. An additional multiple reflection will therefore predict an incorrect amplitude. Also, being very weak on the gathers, they will have little influence on the early least-squares filter design iterations. Wiggins, however, shows an example where there was a significant attenuation of these diffraction multiples. Since he uses an L^1 minimization instead of L^2 minimization, this suggests that the filter design method plays a role. However it could also arise from dip-filtering in his Kirchhoff wave extrapolation operator as his processed gathers are obtained by extrapolation back to the surface after minimizing the fitting error at the seafloor.

To better predict both amplitude and phase of the multiples on the data, I have tried using 3D wave extrapolation instead of $\sqrt{t}$ scaling and 2D extrapolation. I used a 2.5D code, courtesy of Norm Bleistein and Paul Docherty at the Center for Wave Phenomena, Colorado School of Mines, to model an impulse response for a point diffractor on the seafloor. I substituted this (and its transpose) for the operators used to extrapolate to and from the seafloor in equations (3), (5), and (6). The results were poor and got worse as the number of design iterations increased. The problem is a combination of wraparound artifacts and aliasing. The modeling program is simply too good. The high frequency and dip content of the synthetics make them unsuitable as wave extrapolation operators until frequency and dip filtering comparable to those I used in my

2D phase-shift operators is applied. I have yet to rerun this test.

One other possible direction is to incorporate some angle dependence in my model of seafloor reflection. This can be done in several ways. Conceptually, the simplest way is to work in the slant $p-\tau$ domain. There we can directly specify a model of reflection coefficient as a function of angle. A similar model can also be incorporated into the phase-shift operators I currently use. One must remember these, being plane wave reflection coefficients will only be correct for a flat seafloor and cannot account for local inhomogeneities at the seafloor. Alternatively we can design a 2D convolutional reflection filter at each seafloor station in order to accommodate angular dependence. Since the number of equations is several hundred times the number of parameters in the 1D filter design, the 2D convolutional filters can be allowed to be quite wide.

CONCLUSIONS

Seafloor-consistent multiple suppression has worked well. This confirms that wave-equation modeling can do a superior job of predicting he timing and amplitude of water-path multiples. My choice of modeling and filter estimation procedure differs from previous applications in some or all of the following aspects: truly seafloor-consistent filters; incorporation of the pure seabottom multiple; fitting error minimized at the surface; and simultaneous design using all the recorded data.

REFERENCES

Backus, M.M., 1959, Water reverberations – their nature and elimination: Geophysics 24, 233-261.

Bernth, H., and Sonneland, L., 1983, Wave field extrapolation techniques for prestack attenuation of water reverberations: presented at the 53rd Ann. Internat. Mtg., Soc. Explor. Geophys., Las Vegas.

Berryhill, J.R., and Kim, Y.C., 1986, Deep-water peglegs and multiples: Emulation and suppression: Geophysics 51, 2177-2184.

Clærbout, J.F., 1986, Deconvolution in velocity space: SEP-48, 329-344.

Estevez, R.J., (1977), Wide-angle diffracted multiple reflections: Stanford Univ. Ph.D. thesis (SEP-12).

Hampson, D., 1986, Inverse velocity stacking for multiple elimination: presented at the 56th Ann. Internat. Mtg., Soc. Explor. Geophys., Houston.

Levin, S.A., 1983, Avoiding artifacts in phase-shift migration: SEP-37, 27-35.

Loewenthal, D., Lu., L., Roberson, R., and Sherwood, J., 1974, The wave equation applied to migration and water bottom multiples: presented at the 44th Ann. Internat. Mtg., Soc. Explor. Geophys., Dallas.

Morley, L.C., 1982, Predictive techniques for marine multiple suppression: Stanford Univ. Ph.D. thesis (SEP-29).

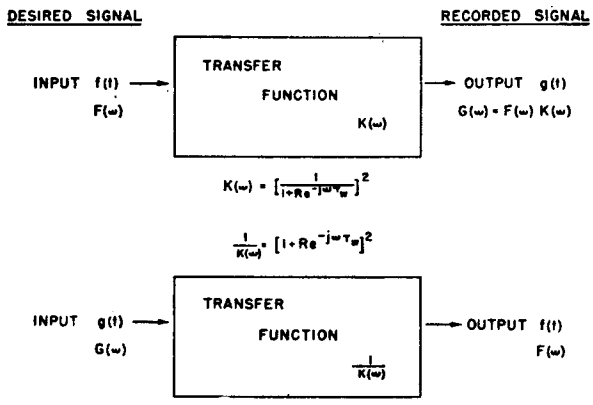
Morley, L., and Clæbout, J.F., 1983, Predictive deconvolution in shot-receiver space: Geophysics 48, 515-531.

Ozdemir, H., 1981, Optimum hyperbolic moveout filters with applications to seismic data: Geoph. Prosp. 29, 702-714.

or

$$H(\omega) = (1 + Re^{-j\omega\tau_w})^2. \tag{19}$$

The difference between this filtering concept and the usual seismic filtering concept requires emphasis. In general, we record a desired seismic signal, $f(t, x)$, to which is added noise, $N(t, x)$. The characteristics of f and N are examined in terms of frequency and wave-length. We then use a filter with a pass band limited to the region where $F(\omega)/N(\omega)$ is large, or we design a multiple-seismometer array with a pass band where $F(k)/N(k)$ is large. We filter out those frequencies or wave lengths where the signal-to-noise ratio is low and accept the resulting degradation in the signal. An equivalent approach to singing records would be to regard the water reverberations as noise, and use a narrow band-pass filter



WATER REVERBERATIONS—
AN INVERSE FILTERING PROBLEM

DESIRED SIGNAL	SEISMIC NOISE	RECORDED SIGNAL	
f(t)	N(t)	f(t) + N(t)	time
F(ω)	N(ω)	F(ω) + N(ω)	frequency
F(k)	N(k)	F(k) + N(k)	wave length

SEISMIC NOISE – A SELECTIVE FILTERING PROBLEM

FIG. 15. Block diagram contrasting the water reverberation and the seismic noise problems.

"*Water Reverberations–Their Nature and Elimination,*" M.M. Backus, *Geophysics*, XXIV, 2(1959), 233-261.

Paige, C.C., and Saunders, M.A., 1982, LSQR: an algorithm for sparse linear equations: SIAM J. Num. Anal., 12, 617-629.

Riley, D.C., and Clærbout, J.F., 1976, 2-D multiple reflections: Geophysics 41, 592-620.

Schneider, W.A., Prince, E.R. Jr., and Giles, B.F., 1965, A new data-processing technique for multiple attenuation exploiting differential normal moveout: Geophysics 30, 348-362.

Wiggins, J.W., 1985, A demonstration of long-period multiple attenuation by wave extrapolation: presented at the 55th Ann. Internat. Mtg., Soc. Explor. Geophys., Washington, D.C. Also Western Geophysical brochure "Wave-equation-based multiple suppression" (1986).