Stable reorientation for the Forties dataset

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ABSTRACT

Traces from the nodal receivers used in the Forties platform undershoot require rotation to provide consistent multicomponent orientation. A quick quality control check on input data that were within just a few degrees of the desired North-East-Vertical orientation found vector magnitude changes approaching 25%. Inspecting the code in the existing module that applied nodal reorientation, the presence of an **IF** test against an arbitrary choice of epsilon, explicit unit vector renormalization, and angle transformations that their author questioned spurred us to derive and apply a stable and robust alternative that avoided those issues.

INTRODUCTION

The seafloor nodes used in the Forties platform undershoot deliver four output traces: one hydrophone and three geophone records. In addition to an X heading, three tilt angles, one each for X, Y and Z, are provided in the SEG Y trace headers. These angles provide sufficient information for reorienting the geophone outputs to Vertical, North and East. The challenge is to ensure that the transformation(s) derived from those angles preserves amplitude fidelity and component orthogonality. We emphasize that, while rotation matrices appear throughout the development, we are not "rotating" data but understanding out how to reexpress it in various useful choices of coordinate systems.

THEORY

The vertical components of the X, Y and Z unit direction vectors are given by the sines of their respective tilt angles $\theta_x, \theta_y, \theta_z$. With these nodes, positive is upward and X-Y-Z, like the original Galperin G_1-G_2-G_3 directions[1], are a right handed coordinate system. We need to determine how the tilted geophone axes relate to the global E-N-V coordinate system.

To help understand and translate it into a conventional matrix notation, we dredge up our basic linear algebra. Let $(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ be the orthonormal vector basis in which a vector $\mathbf{v}$ of interest (a, b, c) is expressed, i.e. $\mathbf{v} = a\mathbf{p}_1 + b\mathbf{p}_2 + c\mathbf{p}_3$, and let $(\mathbf{q}_1, \mathbf{q}_2,$

[1] See, for example, Grazier (2009) for a detailed discussion of the Galperin sensor configuration.

$\mathbf{q}_3$) be the orthonormal vector basis in which we want to reexpress $\mathbf{v}$. The coefficients $(\hat{a},\, \hat{b},\, \hat{c})$ are obtained via the dot product of the new basis with $\mathbf{v}$:

$$\begin{aligned}
\hat{a} &= a\,\mathbf{q}_1\cdot\mathbf{p}_1 + b\,\mathbf{q}_1\cdot\mathbf{p}_2 + c\,\mathbf{q}_1\cdot\mathbf{p}_3 \\
\hat{b} &= a\,\mathbf{q}_2\cdot\mathbf{p}_1 + b\,\mathbf{q}_2\cdot\mathbf{p}_2 + c\,\mathbf{q}_2\cdot\mathbf{p}_3 \\
\hat{c} &= a\,\mathbf{q}_3\cdot\mathbf{p}_1 + b\,\mathbf{q}_3\cdot\mathbf{p}_2 + c\,\mathbf{q}_3\cdot\mathbf{p}_3
\end{aligned}$$

which, in matrix form, says

$$\begin{pmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{pmatrix} = \begin{pmatrix} \mathbf{q}_1\cdot\mathbf{p}_1 & \mathbf{q}_1\cdot\mathbf{p}_2 & \mathbf{q}_1\cdot\mathbf{p}_3 \\ \mathbf{q}_2\cdot\mathbf{p}_1 & \mathbf{q}_2\cdot\mathbf{p}_2 & \mathbf{q}_2\cdot\mathbf{p}_3 \\ \mathbf{q}_3\cdot\mathbf{p}_1 & \mathbf{q}_3\cdot\mathbf{p}_2 & \mathbf{q}_3\cdot\mathbf{p}_3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} . \tag{1}$$

To go the other way, the matrix inverse is its transpose, and therefore,

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \mathbf{q}_1\cdot\mathbf{p}_1 & \mathbf{q}_2\cdot\mathbf{p}_1 & \mathbf{q}_3\cdot\mathbf{p}_1 \\ \mathbf{q}_1\cdot\mathbf{p}_2 & \mathbf{q}_2\cdot\mathbf{p}_2 & \mathbf{q}_3\cdot\mathbf{p}_2 \\ \mathbf{q}_1\cdot\mathbf{p}_3 & \mathbf{q}_2\cdot\mathbf{p}_3 & \mathbf{q}_3\cdot\mathbf{p}_3 \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{pmatrix} . \tag{2}$$

Given a rotation $\mathbf{R}$ that transforms the $\mathbf{p}_i$ to $\mathbf{q}_i$, we initially express the vectors $\mathbf{p}_i$ and $\mathbf{q}_i$ in terms of the $\mathbf{p}_i$ basis itself, and place them as matrix columns in $\mathbf{P} = \mathbf{I}$ and $\mathbf{Q}$ respectively. With this representation, we have simply $\mathbf{Q} = \mathbf{R}$, and the transformation matrix in equation (1) becomes $\mathbf{R}^{\mathrm{T}}$ and, correspondingly, the transformation matrix in equation (2) is simply $\mathbf{R}$, which tells us that when we construct a rotation, we should apply its inverse to get the proper coordinate transformation.

The key step for nodal reorientation is to determine the orientation of a horizontal axis of rotation that is consistent with both the X and Y tilts. Working backward from the local E-N-V right handed orientation, let $(\cos\alpha, \sin\alpha, 0)$ be the to-be-determined unit vector perpendicular to this initial axis of rotation. Changing coordinates for this vector to become $(1, 0, 0)$ is accomplished by rotation

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{pmatrix} . \tag{3}$$

Applying this rotation to the original $(1, 0, 0)$ and $(0, 1, 0)$ unit vectors converts them to $(\cos\alpha, -\sin\alpha, 0)$ and $(\sin\alpha, \cos\alpha, 0)$ respectively.

Rotating downward by the angle $\phi_z = \pi/2 - \theta_z$ is accomplished by multiplication with the matrix

$$\begin{pmatrix} \cos\phi_z & 0 & \sin\phi_z \\ 0 & 1 & 0 \\ -\sin\phi_z & 0 & \cos\phi_z \end{pmatrix} = \begin{pmatrix} \sin\theta_z & 0 & \cos\theta_z \\ 0 & 1 & 0 \\ -\cos\theta_z & 0 & \sin\theta_z \end{pmatrix}$$

and produces the matrix

$$\begin{pmatrix} \cos\alpha\sin\theta_z & \sin\alpha\sin\theta_z & \cos\theta_z \\ -\sin\alpha & \cos\alpha & 0 \\ -\cos\alpha\cos\theta_z & -\sin\alpha\cos\theta_z & \sin\theta_z \end{pmatrix} .$$

From the last row of this matrix we see that the E and N vertical components of the are $-\cos\alpha\cos\theta_z$ and $-\sin\alpha\cos\theta_z$. As the nominal vertical axis is tilted, the actual vertical measurements in the E and N directions are $v\sin\theta_x$ and $v\sin\theta_y$ respectively, where v is the measurement on the nominal vertical axis. Therefore α is given by *atan2* $(-\sin\theta_y, -\sin\theta_x)$, where *atan2* is the C or Fortran library routine. (If both θ_x and θ_y are zero, any value returned is fine and should the library raise a domain error [EDOM], we can set $\alpha = 0$.)

We now rotate back to global coordinates using the transpose of the matrix in equation (3), which results in the transformation matrix

$$\begin{pmatrix} \sin^2\alpha + \cos^2\alpha\sin\theta_z & \sin\alpha\cos\alpha\sin\theta_z - \sin\alpha\cos\alpha & \cos\alpha\cos\theta_z \\ \sin\alpha\cos\alpha\sin\theta_z - \sin\alpha\cos\alpha & \sin^2\alpha\sin\theta_z + \cos^2\alpha & \sin\alpha\cos\theta_z \\ -\cos\alpha\cos\theta_z & -\sin\alpha\cos\theta_z & \sin\theta_z \end{pmatrix} . \tag{4}$$

Because we are concerned that rotating by α and then by $-\alpha$ may result in precision cancellation, we applied trigonometric identities

$$\begin{aligned} \sin^2\frac{\gamma}{2} &= \frac{1-\cos\gamma}{2} \ , \\ \cos^2\frac{\gamma}{2} &= \frac{1+\cos\gamma}{2} \ , \\ \sin 2\gamma &= 2\sin\gamma\cos\gamma \ , \\ &\text{and} \\ \cos 2\gamma &= 2\cos^2\gamma - 1 = 1 - 2\sin^2\gamma \end{aligned}$$

to recast this matrix into the equivalent form

$$\begin{pmatrix} \cos^2\frac{\phi_z}{2} - \cos 2\alpha\sin^2\frac{\phi_z}{2} & -\sin 2\alpha\sin^2\frac{\phi_z}{2} & \cos\alpha\sin\phi_z \\ -\sin 2\alpha\sin^2\frac{\phi_z}{2} & \cos^2\frac{\phi_z}{2} + \cos 2\alpha\sin^2\frac{\phi_z}{2} & \sin\alpha\sin\phi_z \\ -\cos\alpha\sin\phi_z & -\sin\alpha\sin\phi_z & \cos\phi_z \end{pmatrix} , \tag{5}$$

where $\phi_z = \pi/2 - \theta_z$ as before.

After this transformation, the original E direction unit vector is generally no longer at a 90° azimuth. Call the counterclockwise rotation of that direction from due east β. From the first column of matrix (5), we immediately find that

$$\beta = atan2(-\sin 2\alpha\sin^2\frac{\phi_z}{2}, \cos^2\frac{\phi_z}{2} - \cos 2\alpha\sin^2\frac{\phi_z}{2}) ,$$

which says that we have calculated the E-N-V components of what the original X-Y-Z unit vector configuration would be if its heading was $\pi/2 - \beta$ instead of the actual heading h of the X component. The two differ by an angle of $(\pi/2 - h) - \beta$ radians measured counterclockwise from E.

This final correction is handled just the same as in (3) with α replaced by $\pi/2 - (h + \beta)$ and leads to the transformation matrix

$$\begin{pmatrix} \sin(h+\beta) & \cos(h+\beta) & 0 \\ -\cos(h+\beta) & \sin(h+\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{6}$$

By transposing the sequence of transformations, we have the following stable conversion of the X-Y-Z sensor orientation to E-N-V global coordinates:

$$\begin{pmatrix} \mathrm{E} \\ \mathrm{N} \\ \mathrm{V} \end{pmatrix} = \begin{pmatrix} \cos^2\frac{\phi_z}{2} - \cos 2\alpha \sin^2\frac{\phi_z}{2} & -\sin 2\alpha \sin^2\frac{\phi_z}{2} & -\cos\alpha \sin\phi_z \\ -\sin 2\alpha \sin^2\frac{\phi_z}{2} & \cos^2\frac{\phi_z}{2} + \cos 2\alpha \sin^2\frac{\phi_z}{2} & -\sin\alpha\sin\phi_z \\ \cos\alpha\sin\phi_z & \sin\alpha\sin\phi_z & \cos\phi_z \end{pmatrix} \times$$

$$\begin{pmatrix} \sin(h+\beta) & -\cos(h+\beta) & 0 \\ \cos(h+\beta) & \sin(h+\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{pmatrix} .$$

To apply the calculated rotation matrix to our data, we must remember that each data sample constitutes the time derivative of a 3-vector displacement from an origin. In terms of the displacements, the progressive columns of the rotation matrix are the original X, Y and Z unit vectors expressed in E-N-V coordinates. Writing the displacement as the linear combination $pX + qY + rZ$ of coordinate displacements says we should multiply (p, q, r) by the rotation matrix. Because the time derivative commutes with multiplication by a time-independent matrix, multiplying velocities (or accelerations) by the rotation matrix is also the correct operation.

Should we want, as is typical, the first component of our output to be northward motion and the second eastward motion, we need only flip the order of the E and N components to complete the transformation.

EXAMPLE

To confirm the theory, we applied it to nodes in the Forties platform undershoot survey. The first check was to see if, indeed, the transformations did preserve vector magnitudes, so we grabbed an arbitrary G_1-G_2-G_3 sample with tilts of $\theta_x = 0.27°$, $\theta_y = 1.79°$, $\theta_z = 89.99°$, a heading of 353° and converted it to X-Y-Z and N-E-V. The results in Table 1 confirm the norm preservation. Applying the transformation to a

Galperin symmetric		Local axis rotated		Global axis rotated	
G_1	-13805	X	11672	N	10674
G_2	-4078	Y	7478	E	-8846
G_3	6498	Z	-6159	V	-6156
Norm	15168	Norm	15168	Norm	15168

Table 1: Vector norm QC check for node geophone reorientation.

node that had 10 to 15 degrees of vertical tilt and a heading 12 degrees away from the North, yielded the correction from Figure 1 to Figure 2 where we have displayed the relative RMS amplitudes for each shot location around that node.

ACKNOWLEDGMENTS

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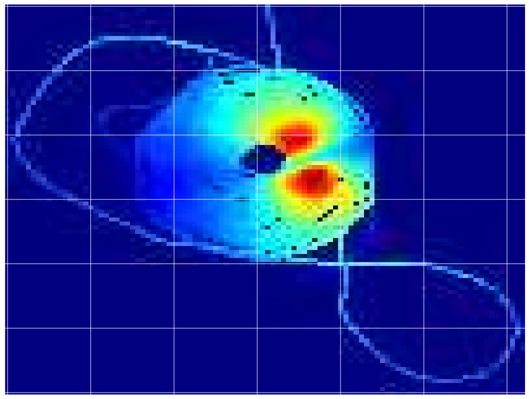

Figure 1: RMS of X component for node with heading 348°. [**ER**]

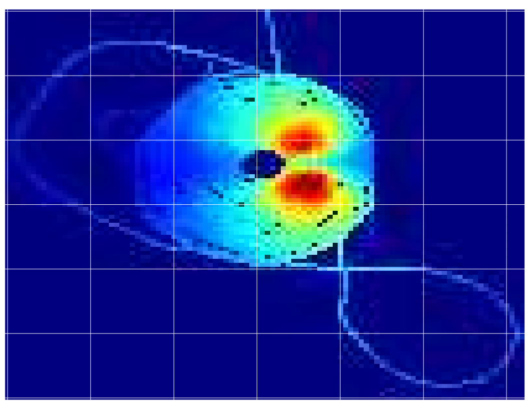

Figure 2: RMS of N component after reorientation. [**ER**]

REFERENCES

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