Blocky velocity inversion by hybrid norm

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ABSTRACT

Inverting a regularized Dix equation using the $L2$ norm produces smooth interval velocity models. To get a blocky interval velocity estimate that is more geologically reasonable, a regularized Dix inversion needs to be done using an $L1$-like norm. In this paper, we compare different 2D regularizations using both the $L2$ norm and the hybrid $L1/L2$ norm to find the regularization that produces blocky velocity models while maintaining high accuracy and resolution. The results show that the hybrid $L1/L2$ norm successfully achieves blockiness when using the first derivative in multiple directions as a regularization.

INTRODUCTION

Most inversion problems try to correct for the fact that the adjoint is different than the inverse. However, in the case of the Dix formula (Dix, 1952), an exact inverse exists. The problem with the Dix formula is that it requires noise-free RMS velocity as an input. Real data always contains some level of noise. In addition, stacking velocity is measured from the data and used as the RMS velocity, even though these values can differ significantly. Therefore, the Dix formula is usually cast as an inverse problem. The conventional regularized Dix inversion (Koren and Ravve, 2006; Harlan, 1999; Clapp, 2001) produces smooth results because the inversion is optimized using the $L2$ norm. However, there are many situations, such as salt boundaries and faults, in which sharp boundaries (i.e. blocky models) are more geologically realistic.

Previous work has shown that the $L1$ norm can produce sparse and blocky results. However, $L1$ norm solvers can encounter stability and convergence issues, since the objective function defined by that norm is at the verge of convexity. Claerbout (2009) has proposed a hybrid $L1/L2$ norm in which a smooth transition between $L1$ and $L2$ can be set at any desired percentile; the hybrid norm has better convergence and stability properties than a pure $L1$ norm. Also, Maysami and Moussa (2009) describe a conjugate-direction solver in which the hybrid $L1/L2$ norm is optimized using Taylor's expansion.

Previously, Li and Maysami (2009) showed that blockiness can be achieved in 1D using a first derivative operator to regularize the problem. In this paper, we expand blockiness to 2D by testing different regularizations on two field datasets.

DIX INVERSION AS AN $L1$-OPTIMIZATION PROBLEM

The Dix equation can be made linear by relating the square of interval velocity v to the square of RMS velocity V,

$$v_\tau^2 \;=\; \tau V_\tau^2 - (\tau-1)V_{\tau-1}^2, \tag{1}$$

where τ is the two-way traveltime. By defining $u_\tau = v_\tau^2$ and $d_\tau = \tau V_\tau^2$, we can set up the Dix inversion problem in an $L1$ sense as follows:

$$||\mathbf{W}_d(\mathbf{Cu}-\mathbf{d})||_{\text{hybrid}} \approx 0, \tag{2}$$

where $\mathbf{W}_d$ is a weight function proportional to the pick strength in the velocity scan divided by τ, $\mathbf{C}$ is the causal integration operator, and $\mathbf{u}$ and $\mathbf{d}$ are vectors containing all the values of u_τ and d_τ, respectively. The division by τ reduces the strength of the later events to balance the data fitting strength along the time axis.

The hybrid norm above defines the cost function as follows:

$$\mathbf{C}(\mathbf{r}) = \sqrt{\mathbf{r}^2+\mathbf{R}^2} - \mathbf{R}, \tag{3}$$

where $\mathbf{r}$ is the residual and $\mathbf{R}$ is a threshold which defines a smooth transition between the $L1$ and $L2$ norms (Claerbout, 2009).

Fitting goal (2) is not enough to fully constrain the inversion, because it has a large null space (Li and Maysami, 2009). Moreover, picking errors can lead to incorrect RMS velocities and unreasonable interval velocities. Therefore, a second fitting goal (i.e. a regularization term) is required to constrain this inversion. The regularization term can be written as follows:

$$||\epsilon\mathbf{Au}||_{\text{hybrid}} \approx 0, \tag{4}$$

where $\mathbf{A}$ is typically a roughening operator, and ϵ is a scalar to balance the two fitting goals.

Notice that the norm in fitting goal (2) has a different effect than the norm in fitting goal (4). Using the hybrid norm in data fitting makes the inversion less sensitive to outliers. On the other hand, using the hybrid norm in model styling affects the general shape of the estimated model, which is the goal of this paper.

Li and Maysami (2009) successfully produced blockiness in 1D when using the first derivative as a regularization operator. In the following sections, we will try different regularization operators to achieve the same goals in 2D.

REGULARIZATION BY THE LAPLACIAN OPERATOR

First, we will use the Gulf of Mexico data provided by WesternGeco. This is a four-second dataset, at a sampling interval of 4 ms. The offset axis has 24 traces starting at 264 m with an increment of 134 m. There are 125 CMP gathers with a spacing of 67 m. Figure 1(b) shows the results of autopicking the dataset, and Figure 1(a) shows the results of direct Dix conversion. The direct conversion is done trace by trace after vertical smoothing which prevents the minimum interval velocity to go below water velocity. Figure 2 shows the strength of the picks in the velocity scans, which will be used as a weight as described above.

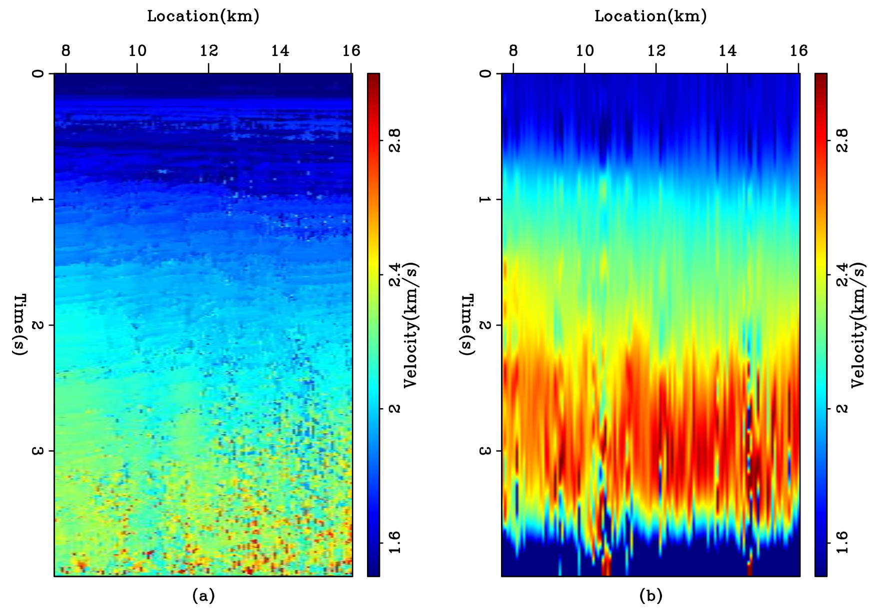

Figure 1: The WG dataset. (a) The input RMS velocity, which is automatically picked from the CMP gathers. (b) The interval velocity by direct dix conversion. [**ER**]

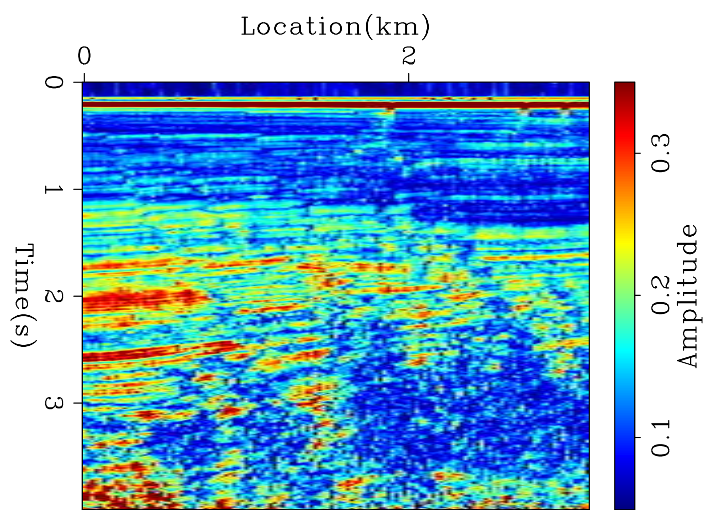

Figure 2: The strength of the picks in the velocity scan of the WG dataset, which is used as the weight before dividing by time. [**ER**]

We start by choosing the symmetric Laplacian operator. Figure 3 shows the results of using the $L2$ norm and Figure 4 shows the results of using the hybrid norm. Although the two Figures look similar, the hybrid norm shows less smoothing and more detail than the $L2$ norm. The hybrid norm results are still not blocky, because a linear trend in velocity will also result in a zero second derivative. In the next section we attempt to more closely approach the first derivative by using the helix derivative.

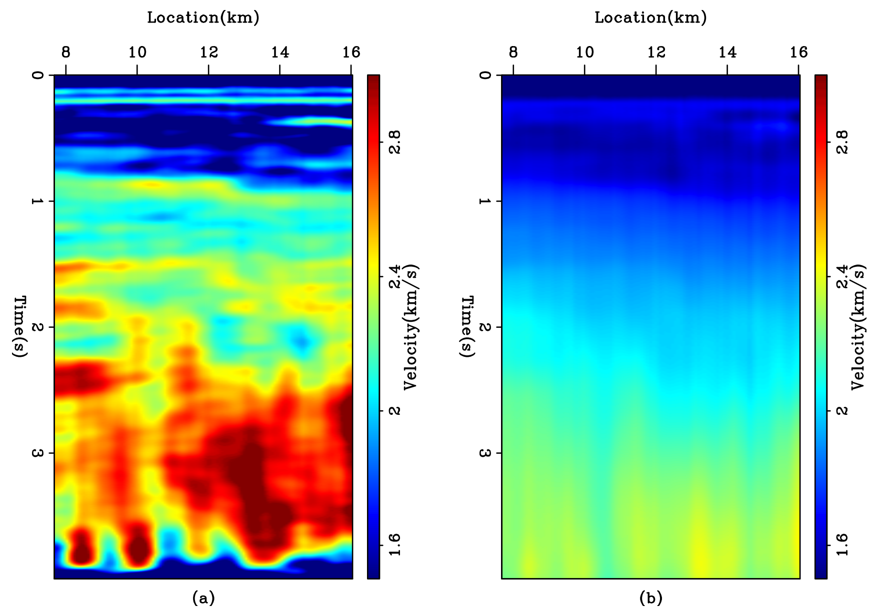

Figure 3: The WG dataset. (a) The interval velocity estimated by using the Laplacian operator as a regularization in the $L2$ norm. (b) The reconstructed RMS velocity. [**ER**]

REGULARIZATION BY THE HELIX DERIVATIVE OPERATOR

Now we consider the helix derivative (Claerbout, 1997) as a regularization operator. Figure 5 shows the results of using the $L2$ norm, and Figure 6 shows the results of using the hybrid norm. In this case, we see a dramatic difference between the two results. In the hybrid norm case, we can see the beginnings of blockiness, but only in one direction (toward the right). The reason for this asymmetry is that using an L1-like norm is similar to applying the regularization only once. On the other hand, we do not see this effect in the $L2$ norm results, because the regularization in that norm is similar to applying the forward and the adjoint of an operator, which is a symmetric procedure.

REGULARIZATION BY THE FIRST DERIVATIVE OPERATOR IN TWO DIRECTIONS

The previous regularizations show that only a first derivative can create blockiness. However, using the first derivative means that we must pick a direction each time

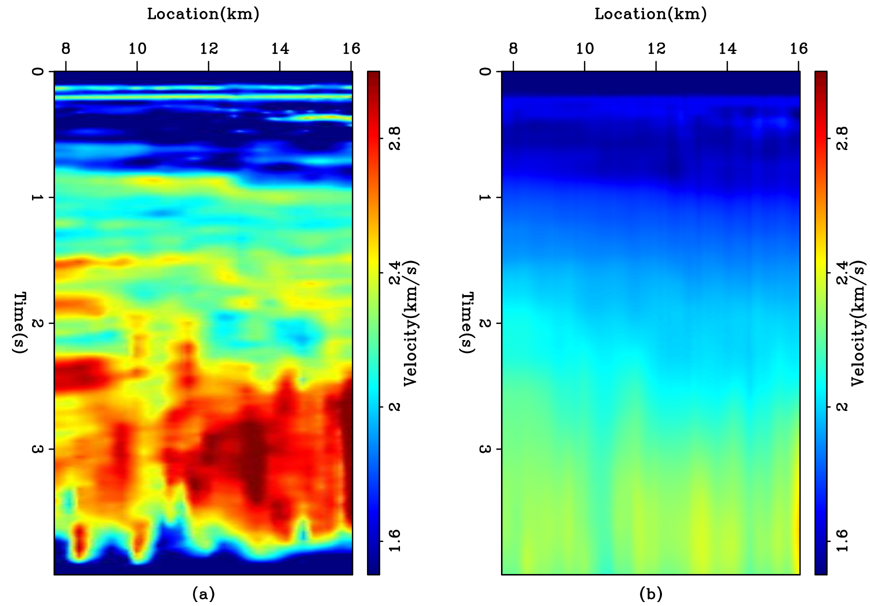

Figure 4: The WG dataset. (a) The interval velocity estimated by using the Laplacian operator as a regularization in the hybrid norm. (b) The reconstructed RMS velocity. [**ER**]

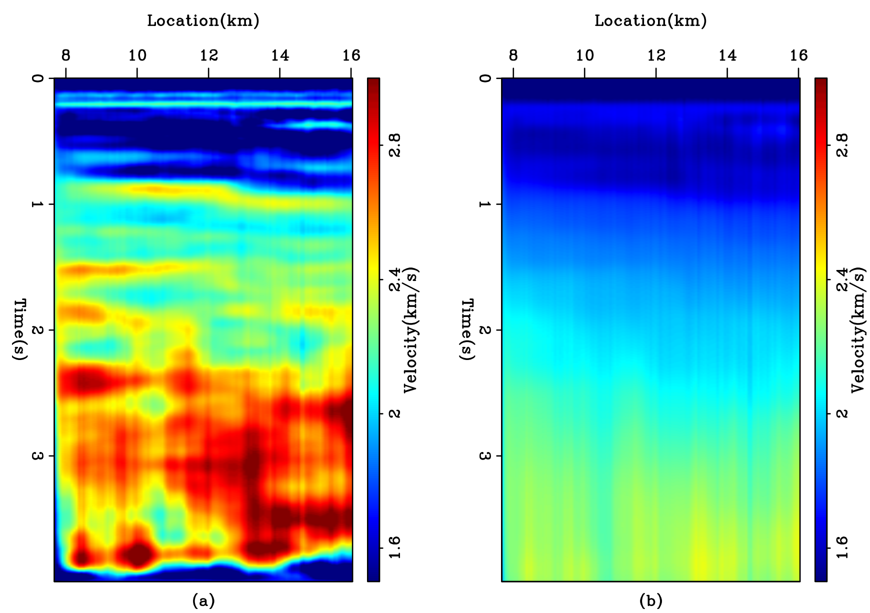

Figure 5: The WG dataset. (a) The interval velocity estimated by using the helix derivative operator as a regularization in the $L2$ norm. (b) The reconstructed RMS velocity. [**ER**]

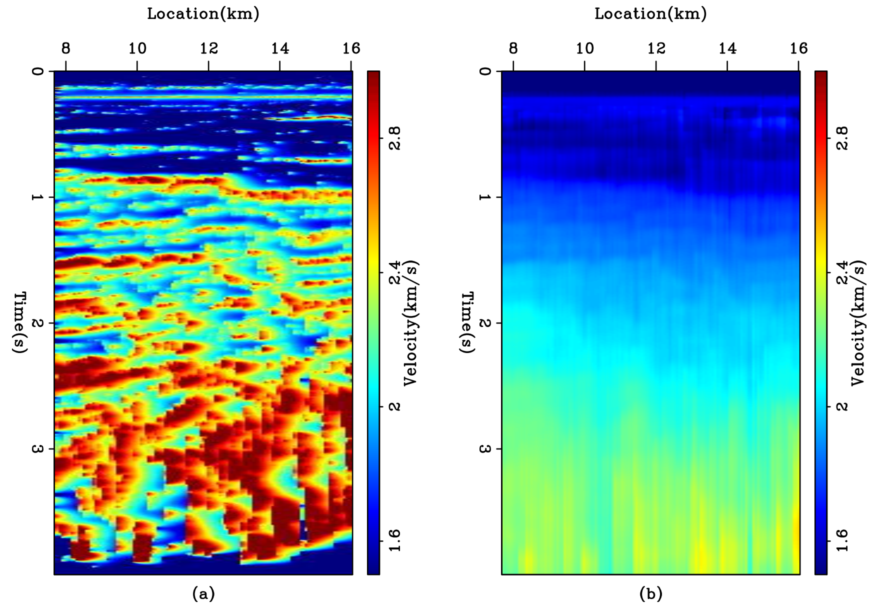

Figure 6: The WG dataset. (a) The interval velocity estimated by using the helix derivative operator as a regularization in the hybrid norm. (b) The reconstructed RMS velocity. [**ER**]

we apply the derivative. As a first test, we pick two directions: the vertical and horizontal as follows:

$$||\epsilon \mathbf{D}_z \mathbf{u}||_{\text{hybrid}} \approx 0, \tag{5}$$

$$||\epsilon \mathbf{D}_x \mathbf{u}||_{\text{hybrid}} \approx 0, \tag{6}$$

where $\mathbf{D}_z$ and $\mathbf{D}_x$ are the first derivative operators along the z- and x-axis, respectively . The derivative of each direction is applied in a separate regularization equation (i.e. we have two regularization equations in this case) in order to maintain symmetry. Combining these two filters in one regulariztion will cause an asymmetry in blockiness, similar to the previous result from the helix derivative regularization.

Figure 7 shows the results of using the $L2$ norm with two first derivative applications, and Figure 8 shows the results of using the hybrid norm. Blockiness is clearly present in the hybrid norm results. However, there seems to be a preference for the sharp boundaries to be either horizontal or vertical, which is due to the directions of the derivatives we chose.

REGULARIZATION BY THE FIRST DERIVATIVE OPERATOR IN FOUR DIRECTIONS

To reduce the bias in blockiness directions, we increased the directions of the first derivative to four: the previous two directions, plus two directions at 45 degrees to the vertical and horizontal axes. Figure 9 shows the results of using the $L2$ norm,

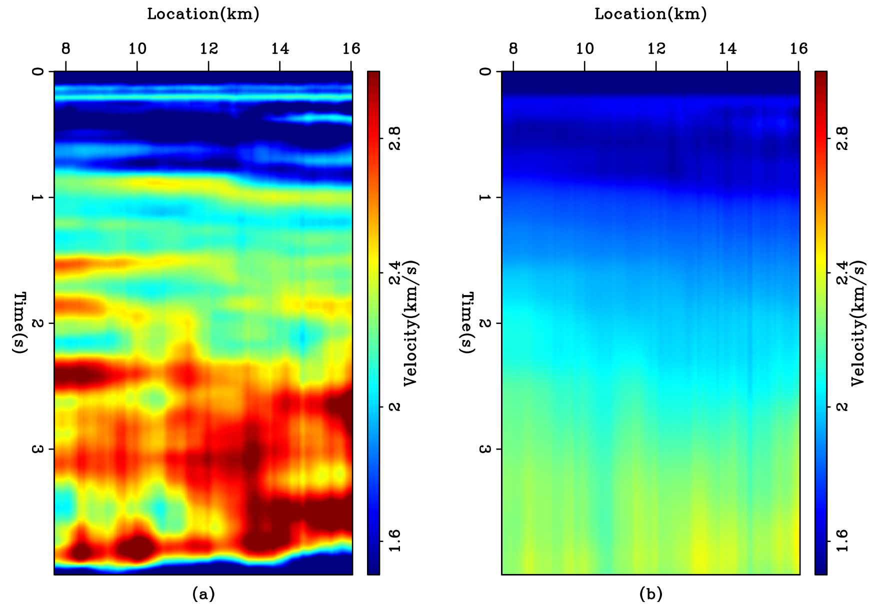

Figure 7: The WG dataset. (a) The interval velocity estimated by using the first derivative operator in two directions as a regularization in the $L2$ norm. (b) The reconstructed RMS velocity. [**ER**]

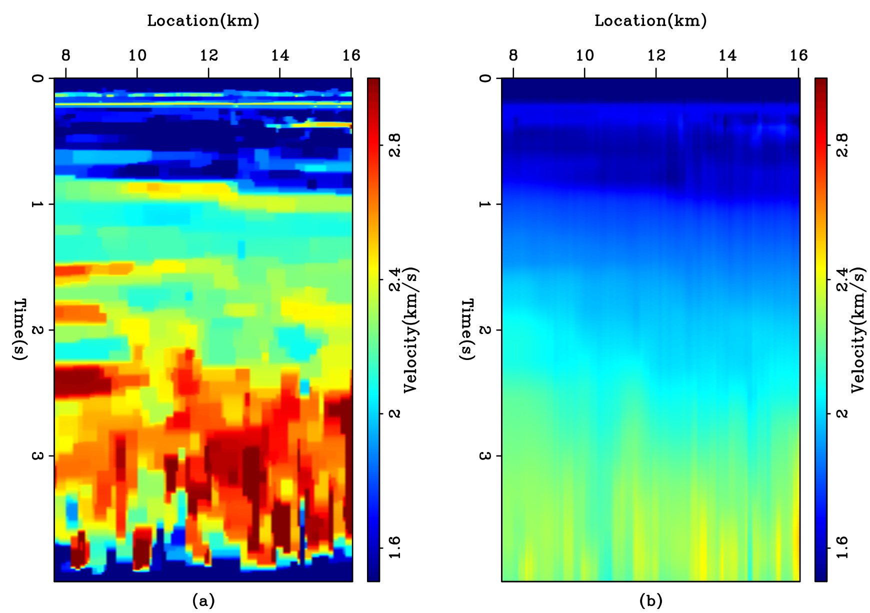

Figure 8: The WG dataset. (a) The interval velocity estimated by using the first derivative operator in two directions as a regularization in the hybrid norm. (b) The reconstructed RMS velocity. [**ER**]

and Figure 10 shows the results of using the hybrid norm. By comparing Figure 10 to Figure 8, we can clearly see a large improvement in the model. The details and blockiness are still preserved. However, the inversion now has more "freedom" to pick the direction of blockiness out of four directions instead of two directions.

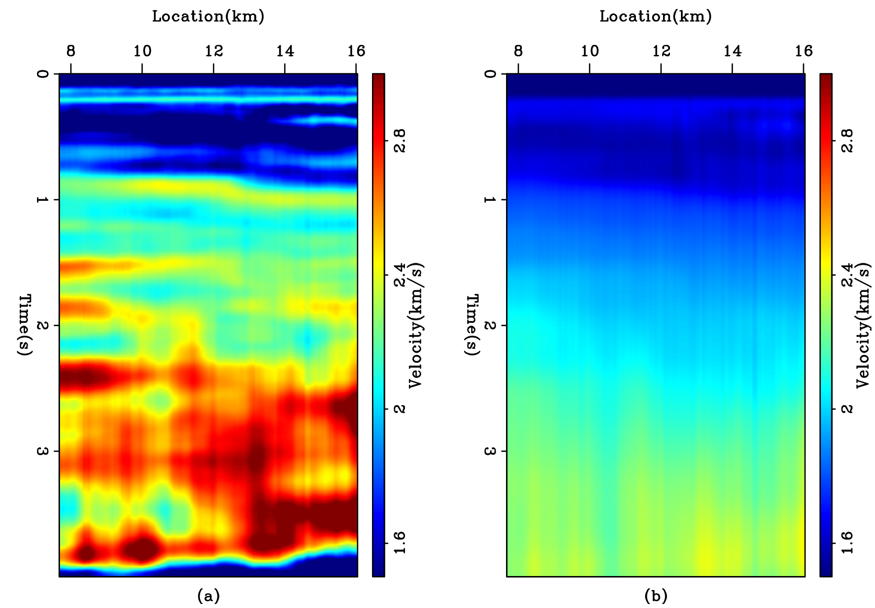

Figure 9: The WG dataset. (a) The interval velocity resulted by using the first derivative operator in four directions as a regularization in the $L2$ norm. (b) The reconstructed RMS velocity. [**ER**]

ELF DATASET

In this section, we will estimate an interval velocity model of the North Sea data provided by ELF. There is a known salt body in the middle of this data, which makes it a proper test case for our blockiness goals. Also, this dataset has better spatial sampling than the previous dataset, and can thus better illustrate the differences between the different inversion results. This is also four-second dataset, at a sampling of 5.9 ms. The offset axis has 143 traces starting at 0 m with an increment of 25 m. There are 537 CMP gathers with a spacing of 25 m. Figure 11(b) shows the results of autopicking the dataset, which was constrained by the background RMS velocity, and Figure 11(a) shows the results of direct Dix conversion, as defined above. Figure 12 shows the strength of the picks in the velocity scans.

For this dataset, we will only repeat the last two regularizations, which are the first derivative in two and four directions, since they showed the best results with the most symmetric blockiness. Figure 13 shows the results of regularizing in two directions in the $L2$ norm and Figure 14 shows the results of the same regularization in the hybrid norm. Figures 15 and 16 show the results of using four-direction regularization.

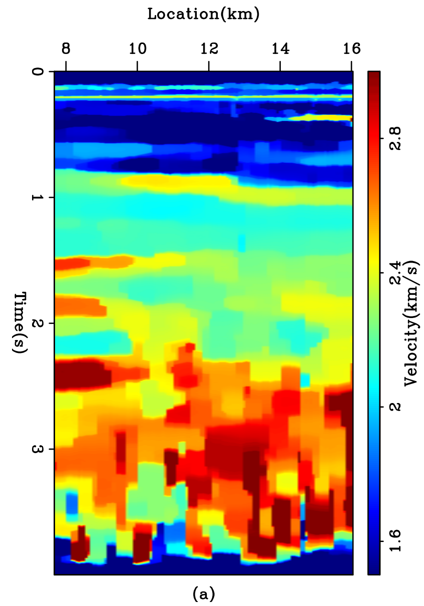

Figure 10: The WG dataset. (a) The interval velocity resulted by using the first derivative operator in four directions as a regularization in the hybrid norm. (b) The reconstructed RMS velocity. [**ER**]

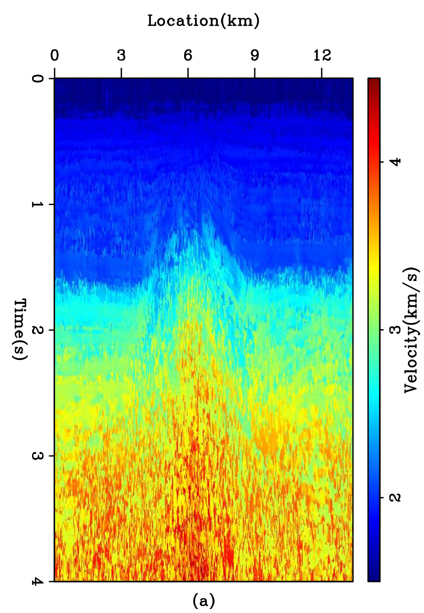

Figure 11: The ELF dataset. (a) The input RMS velocity which is automatically picked from the CMP gathers. (b) The interval velocity by direct Dix conversion. [**ER**]

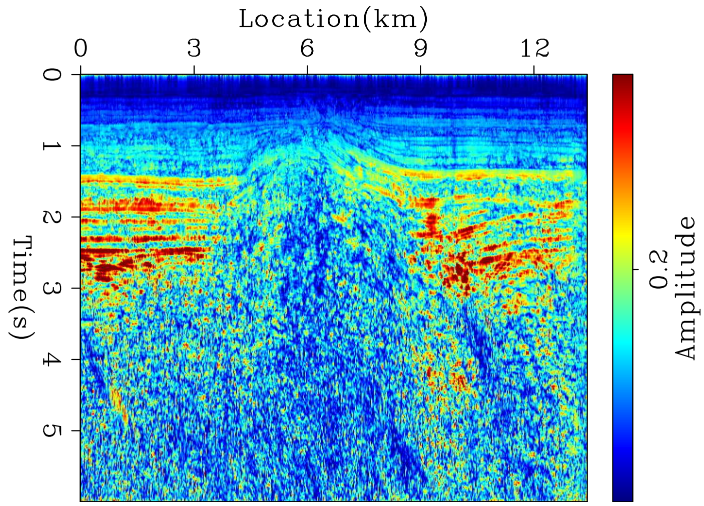

Figure 12: The strength of the picks in the velocity scan of ELF dataset, which is used as the weight before dividing by time. [**ER**]

Since the dataset is larger with smaller sampling, the improvement of using more directions is evident. Forcing the inversion to pick between two directions has an apparent effect of reducing the resolution. One obvious example is the chalk layer, which looks very horizontal in Figure 14 but more detailed in Figure 16. In all cases, $L2$ always gives smooth results, which smear the model and lower the resolution of the inversion.

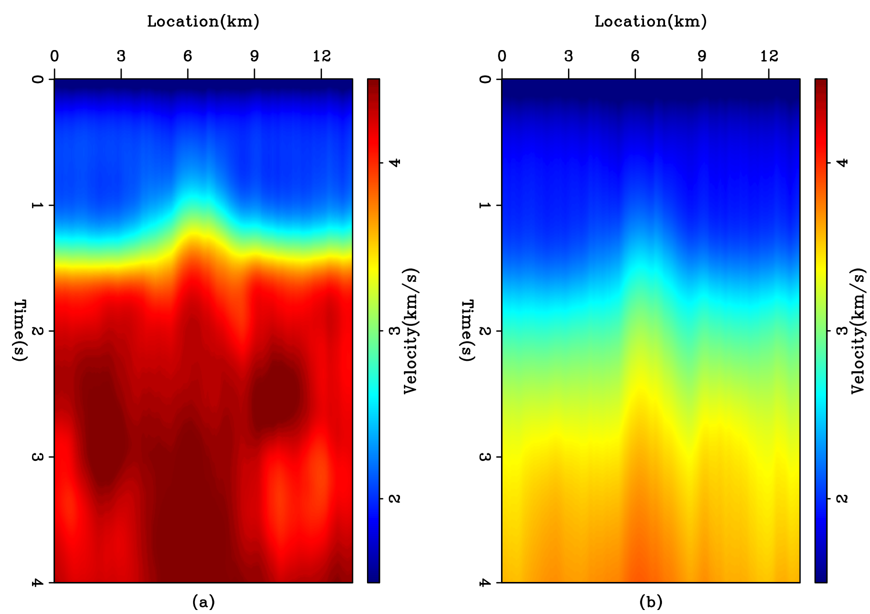

Figure 13: The ELF dataset. (a) The interval velocity estimated by using the first derivative operator in two directions as a regularization in the $L2$ norm. (b) The reconstructed RMS velocity. [**ER**]

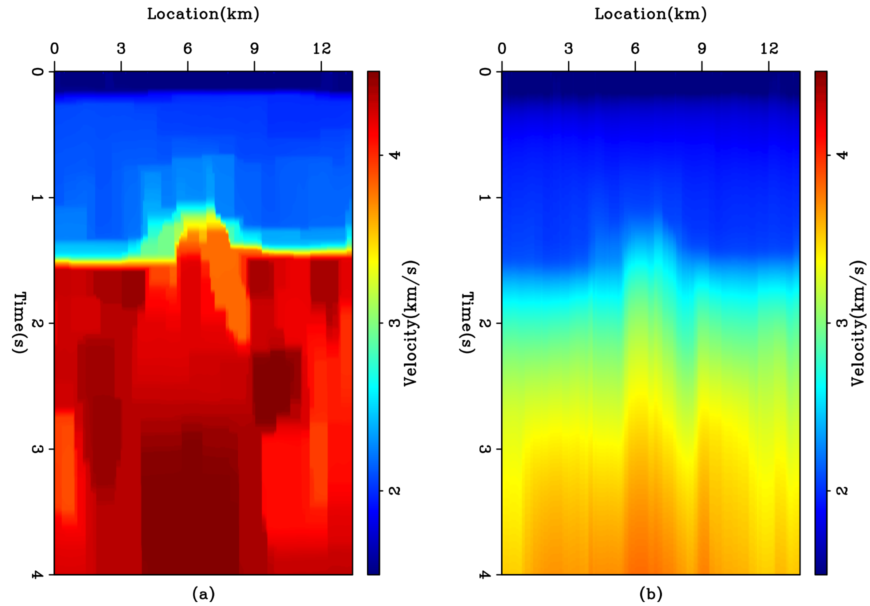

Figure 14: The ELF dataset. (a) The interval velocity estimated by using the first derivative operator in two directions as a regularization in the hybrid norm. (b) The reconstructed RMS velocity. [**ER**]

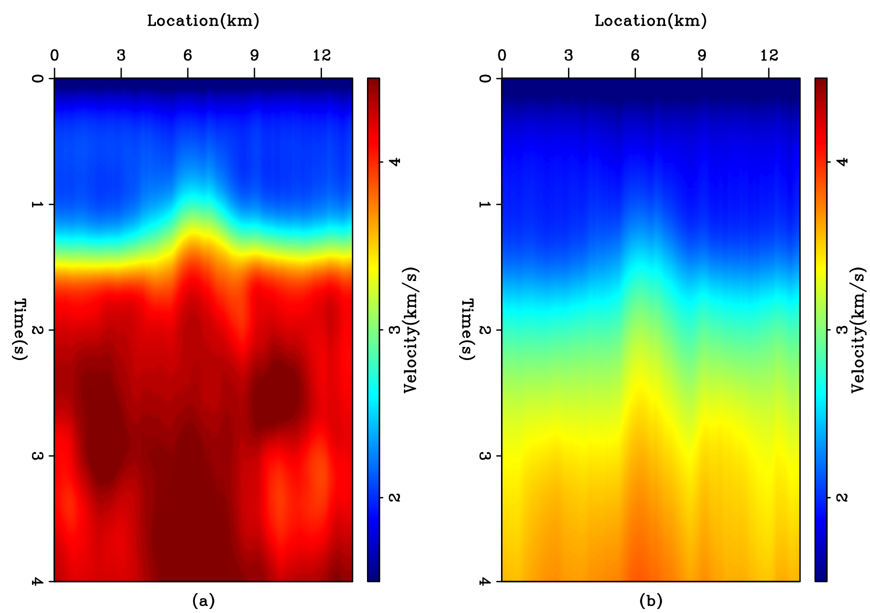

Figure 15: The ELF dataset. (a) The interval velocity estimated by using the first derivative operator in four directions as a regularization in the $L2$ norm. (b) The reconstructed RMS velocity. [**ER**]

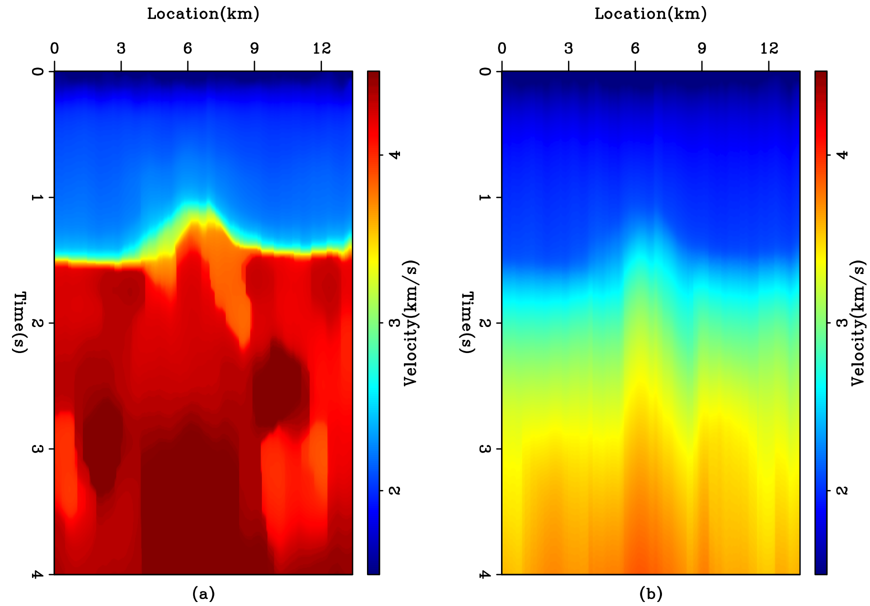

Figure 16: The ELF dataset. (a) The interval velocity estimated by using the first derivative operator in four directions as a regularization in the hybrid norm. (b) The reconstructed RMS velocity. [**ER**]

CONCLUSIONS AND DISCUSSIONS

We successfully achieved blocky velocity models by using the hybrid norm. We also showed that the choice of the regularization operator has a great impact on how blocky the results are. Nonetheless, the hybrid norm always showed more detail and resolution than the $L2$ norm, even when blockiness was not achieved. An example of this can be seen by comparing Figures 4 and 9. Although the first Figure uses a Laplacian operator for regularization and the later Figure uses the first derivative in four directions (which we showed has the best results), the hybrid norm was still superior to the $L2$ norm in preserving more details and showing higher resolution.

Another point to keep in mind is that the hybrid norm has a flexibile threshold. In all previous cases, we set that threshold to 0.20, meaning that 80 percent of residuals (both data residuals and model residuals) are going to be in the $L1$ region and the rest in the $L2$ region. However, this is a parameter that can be adjusted based on the desired degree of blockiness.

FUTURE WORK

As we have seen, increasing the number of directional derivatives will increase the resolution and flexibility of the inversion results. However, increasing the number of drections will also slow the inversion, because each direction has a model residual the size of the model. Instead of using many directions, it is possible to use steering filters to pre-define the local direction of maximum variance and then use that information to align the directions of the regularization to be parallel and perpendicular to it. This way, we might only need two directions in the regularization.

ACKNOWLEDGMENTS

We thank Jon Claerbout, Robert Clapp, Antoine Guitton, Louis Vaillant, Elita Li and Yang Zhang for their helpful suggestions and discussions throughout this project. We also thank WesternGeco and TotalFinaElf for providing the data.

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