Stanford Exploration Project, Report 129, May 6, 2007, pages 1–148

Stanford Exploration Project, Report 129, May 6, 2007, pages 1–148

Aligned vertical fractures, HTI reservoir symmetry, and Thomsen seismic anisotropy parameters

James G. Berryman

ABSTRACT

The Sayers and Kachanov (1991) crack-influence parameters are shown to be directly related to Thomsen (1986) weak-anisotropy seismic parameters for fractured reservoirs when the crack/fracture density is small enough. These results are then applied to the problem of seismic wave propagation in reservoirs having HTI symmetry due to the presence of aligned vertical fractures. The approach suggests a method of inverting for fracture density from wave speed data.

INTRODUCTION

Aligned vertical fractures provide one commonly recognized source of azimuthal (surface angle dependent) seismic anisotropy in oil and gas reservoirs (Lynn *et al.*, 1995). While layering in the earth also results in seismic anisotropy (Backus, 1962), horizontal layering of isotropic rock produces vertical transversely isotropic (VTI) media, and could not produce horizontal transversely isotropic (HTI) symmetry without some very significant uplift phenomena being present simultaneously. Of course, anisotropic layers such as shale beds bring seismic anisotropy with them, but again this anisotropy will more typically be VTI, rather than HTI.

On the other hand, VTI earth media seem much easier to understand and analyze than HTI media. Nevertheless, when the source of the anisotropy is aligned vertical fractures, we can make very good use of the simpler case of horizontal fracture analysis by making a rather minor change of our point of view that easily gives all the needed results.

Together with the simplifications already noted, we can also understand very directly the sources of the anisotropy due to fractures by considering a method introduced by Sayers and Kachanov (1991). We find that elastic constants, and therefore the Thomsen (1986) parameters, can be very conveniently expressed in terms of the Sayers and Kachanov (1991) formalism. Furthermore, in the low crack density limit [which is also consistent with the weak anisotropy approach of Thomsen (1986)], we obtain direct links between the Thomsen parameters and the fracture properties. These links suggest a method of inverting for fracture density from wave speed data.

THOMSEN'S SEISMIC WEAK ANISOTROPY METHOD

Thomsen's weak anisotropy method (Thomsen, 1986), being an approximation designed specifically for use in velocity analysis for exploration geophysics, is clearly not exact. Approximations incorporated into the formulas become most apparent for greater angles θ from the vertical, especially for compressional and vertically polarized shear velocities $v_p(\theta)$ and $v_{sv}(\theta)$, respectively. Angle θ is measured from the $\hat{z}$-vector pointing into the earth.

For reference purposes, we include here the exact velocity formulas for P, SV, and SH seismic waves at all angles in a VTI elastic medium. These results are available in many places (Rüger, 2002; Musgrave, 2003), but were taken specifically from Berryman (1979) with some minor changes of notation. The results are:

$$v_p^2(\theta) = \frac{1}{2\rho}\left\{\left[(c_{11}+c_{44})\sin^2\theta + (c_{33}+c_{44})\cos^2\theta\right] + R(\theta)\right\} \tag{1}$$

and

$$v_{sv}^2(\theta) = \frac{1}{2\rho}\left\{\left[(c_{11}+c_{44})\sin^2\theta + (c_{33}+c_{44})\cos^2\theta\right] - R(\theta)\right\}, \tag{2}$$

where

$$R(\theta) = \sqrt{\left[(c_{11}-c_{44})\sin^2\theta - (c_{33}-c_{44})\cos^2\theta\right]^2 + 4(c_{13}+c_{44})^2\sin^2\theta\cos^2\theta} \tag{3}$$

and, finally,

$$v_{sh}^2(\theta) = \frac{1}{\rho}\left[c_{44} + (c_{66}-c_{44})\sin^2\theta\right]. \tag{4}$$

Expressions for phase velocities in Thomsen's weak anisotropy limit can be found in many places, including Thomsen (1986, 2002) and Rüger (2002). The pertinent expressions for phase velocities in VTI media as a function of angle θ, measured as before from the vertical direction, are

$$v_p(\theta) \simeq v_p(0)\left(1 + \delta\sin^2\theta\cos^2\theta + \epsilon\sin^4\theta\right), \tag{5}$$

$$v_{sv}(\theta) \simeq v_s(0)\left(1 + [v_p^2(0)/v_s^2(0)](\epsilon-\delta)\sin^2\theta\cos^2\theta\right), \tag{6}$$

and

$$v_{sh}(\theta) \simeq v_s(0)\left(1 + \gamma\sin^2\theta\right). \tag{7}$$

In our present context, $v_s(0) = \sqrt{c_{44}/\rho_0}$, and $v_p(0) = \sqrt{c_{33}/\rho_0}$, where c_{33}, c_{44}, and ρ_0 are two stiffnesses of the cracked medium and the mass density of the isotropic host elastic medium. We assume that the cracks have insufficient volume to affect the mass density ρ_0 significantly.

In each case, Thomsen's approximation has included a step that removes the square on the left-hand side of the equation, by expanding a square root of the right hand side. This step introduces a factor of $\frac{1}{2}$ multiplying the $\sin^2\theta$ terms on the right hand side, and — for example — immediately explains how equation (7) is obtained from (4). The other two equations for $v_p(\theta)$ and $v_{sv}(\theta)$, *i.e.*, (5) & (6), involve additional approximations as well that we will not attempt to explain here.

The three resulting Thomsen (1986) seismic parameters for weak anisotropy with VTI symmetry are $\gamma = (c_{66} - c_{44})/2c_{44}$, $\epsilon = (c_{11} - c_{33})/2c_{33}$, and

$$\delta = \frac{(c_{13}+c_{44})^2 - (c_{33}-c_{44})^2}{2c_{33}(c_{33}-c_{44})} = \left(\frac{c_{13}+c_{33}}{2c_{33}}\right)\left(\frac{c_{13}+2c_{44}-c_{33}}{c_{33}-c_{44}}\right). \tag{8}$$

All three of these parameters can play important roles in the velocities given by (5)-(7) when the crack densities are high enough. If crack densities are very low, then the SV shear wave will actually have no dependence on angle of wave propagation. Note that the so-called anellipticity parameter $A = \epsilon - \delta$, vanishes when $\epsilon \equiv \delta$, which we will soon see does happen for low crack densities.

FRACTURED RESERVOIRS AND CRACK-INFLUENCE PARAMETERS

To illustrate the Sayers and Kachanov (1991) crack-influence parameter method, consider the situation in which all the cracks in the system have the same vertical (or z-)axis of symmetry. (We use 1,2,3 and x,y,z notation interchangeably for the axes.) Then, the cracked/fractured system is not isotropic, and we have the first-order compliance correction matrix for horizontal fractures, which is:

$$\Delta S_{ij}^{(1)} = \rho_c \begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1+\eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}, \tag{9}$$

where $i, j = 1,2,3$. The two lowest order crack-influence parameters from the Sayers and Kachanov (1991) approach are η_1 and η_2. The scalar crack density parameter is defined, for penny-shaped cracks having number density $n = N/V$ and radius in the plane of the crack equal to a, to be $\rho_c = na^3$. The aspect ratio of the cracks is b/a.

Now it is also not difficult to see that, if the cracks were oriented instead so that all their normals were pointed horizontally along the x-axis, then we would have one permutation of this matrix and, if instead they were all pointed horizontally along the y-axis, then we would have a third permutation of the matrix. To obtain an isotropic compliance correction matrix, we can simply average these three permutations: just add the three ΔS's together and then divide by three. [Note that this method of averaging, although correct for contributions linear in ρ_c, does not necessarily work for higher order corrections (Berryman, 2007).] This construction shows in part both the power and the simplicity of the Sayers and Kachanov (1991) approach. The connection to the isotropic case is of great practical importance, because it permits us to estimate the parameters η_1 and η_2 by studying isotropic cracked/fractured systems, using well-understood effective medium theories (Zimmerman, 1991; Berryman and Grechka, 2006).

HORIZONTAL FRACTURES AND VTI SYMMETRY

Next consider horizontal fractures, as illustrated by the correction matrix (9). The axis of fracture symmetry is uniformly vertical, and so such a reservoir would exhibit VTI symmetry. The resulting expressions for the Thomsen parameters in terms of the Sayers and Kachanov (1991) parameters η_1 and η_2 are given by

$$\gamma_h = \frac{c_{66} - c_{44}}{2c_{44}} = \rho_c \eta_2 G_0, \tag{10}$$

and

$$\epsilon_h = \frac{c_{11} - c_{33}}{2c_{33}} = \rho_c[(1+\nu_0)\eta_1 + \eta_2]\frac{E_0}{(1-\nu_0^2)} \simeq \frac{2\rho_c\eta_2 G_0}{1-\nu_0}. \tag{11}$$

The background shear modulus is G_0, and the corresponding Poisson ratio is ν_0. Young's modulus is $E_0 = 2(1+\nu_0)G_0$. We also find that $\delta = \epsilon$ to the lowest order in the crack density parameter. We have chosen to neglect the term in η_1 in the final expression of (11), as this is on the order of a 1% correction to the term retained. Values of η_1 and η_2 can be determined from simulations and/or effective medium theories (Zimmerman, 1991; Berryman and Grechka, 2006). They depend on the elastic constants of the background medium, and on the shape of the cracks (assumed to be penny-shaped in these examples).

HTI RESERVOIR SYMMETRY FROM ALIGNED VERTICAL FRACTURES

Now the trick to get from horizontal fractures and VTI to aligned vertical fractures and HTI symmetry is relatively simple. We will not need to make any effort to relabel the c_{ij}'s. Rather we just change the meaning of the labels. As long as we stay mentally oriented in the reference frame of the fractures themselves, we can continue to view the z-direction as the symmetry axis and the xy-plane, as the plane of the fractures. The only change we need to make arises from the fact that the surface, where we shoot our seismic survey, is now at 90^o from the fracture plane, whereas for horizontal fractures the surface was at 0^o from the fracture plane. This observation implies that, wherever the angle θ (measured in radians) appeared in our previous formulas, now we must replace it by $\frac{\pi}{2} - \theta$ radians. Thus, $\sin^2\theta \to \cos^2\theta$ and vice versa in the formulas. This algorithm is exactly right only for those planes that are vertical and also perpendicular to the fracture plane, *i.e.*, at azimuthal angles $\phi = \pm\frac{\pi}{2}$. For all angles, we actually need to replace $\sin^2\theta$ by $\cos^2\theta\sin^2\phi$. Then, when $\phi = 0$ or π, there is no angular dependence since we are in the plane of the fracture.

For the θ dependence, taking $\sin^2\theta \to 1 - \sin^2\theta$, is actually a handier way to proceed, because then we can reduce all the formulas to the same equivalent form as the one Thomsen had originally chosen — if we choose to do so. It is also helpful to backup one step in

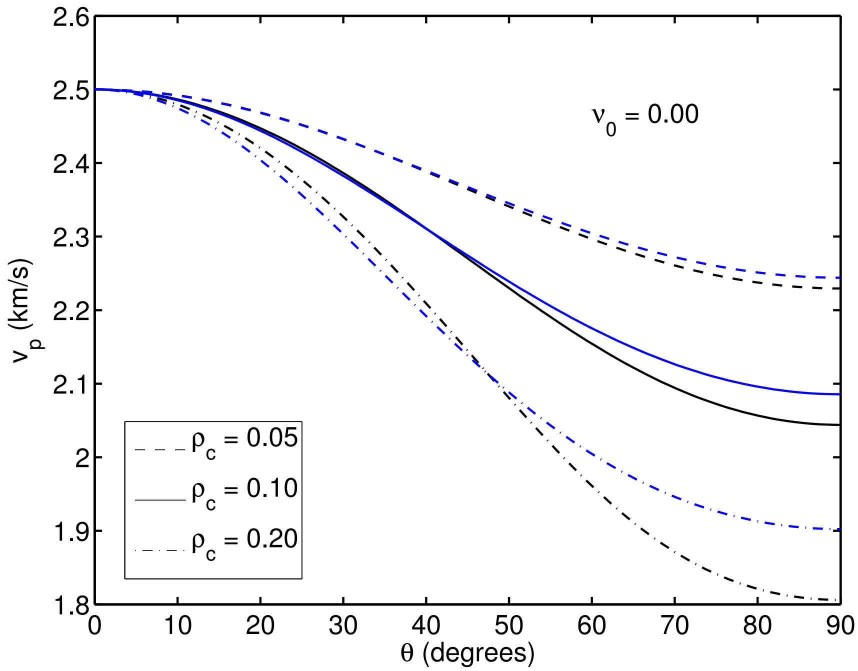

Figure 1: For aligned vertical cracks: examples of anisotropic compressional wave speed (v_p) for Poisson's ratio of the host medium $\nu_0 = 0.00$. Velocity curves in black are exact for the fracture model discussed in the text. The Thomsen weak anisotropy velocity curves for the same fracture model were then overlain in blue. jim1-FIG1 [NR]

the Thomsen derivation and restore squares, thereby "unexpanding" the square root. Certain approximations are then undone, and the final formulas we obtain will be more accurate.

If ϵ, δ, and γ are the Thomsen parameters for the VTI symmetry (horizontal fracture), then, for example,

$$v_{sh}^2(\frac{\pi}{2}-\theta) = v_s^2(0)\left[1+2\gamma\sin^2(\frac{\pi}{2}-\theta)\right] = v_s^2(0)(1+2\gamma)\left[1-\frac{2\gamma}{1+2\gamma}\sin^2\theta\right]. \qquad (12)$$

From this result, we deduce that $\gamma \to -\gamma/(1+2\gamma)$. This is a rigorous statement for the form of the equation considered. Then, the weak anisotropy limit will be $\gamma \to -\gamma$, but this final step is not necessary or recommended for some of the higher crack densities considered here.

Similar calculations for v_p^2 and v_{sv}^2 give

$$v_p^2(\frac{\pi}{2}-\theta) = v_p^2(0)\left[1+2\delta\sin^2\theta\cos^2\theta+2\epsilon\sin^2(\frac{\pi}{2}-\theta)\right] \qquad (13)$$

and

$$v_{sv}^2(\frac{\pi}{2}-\theta) = v_s^2(0)\left[1+2[v_p^2(0)/v_s^2(0)](\epsilon-\delta)\sin^2\theta\cos^2\theta\right], \qquad (14)$$

which lead to the results $\epsilon \to \frac{-\epsilon}{1+2\epsilon} \simeq -\epsilon$, and $\delta \to \frac{\delta-2\epsilon}{1+2\epsilon} \simeq \delta-2\epsilon$. As a consistency check, note that $\epsilon-\delta \to (\epsilon-\delta)/(1+2\epsilon) \simeq (\epsilon-\delta)$. Similarly, the pertinent wave speeds are: $v_p(0) \to$

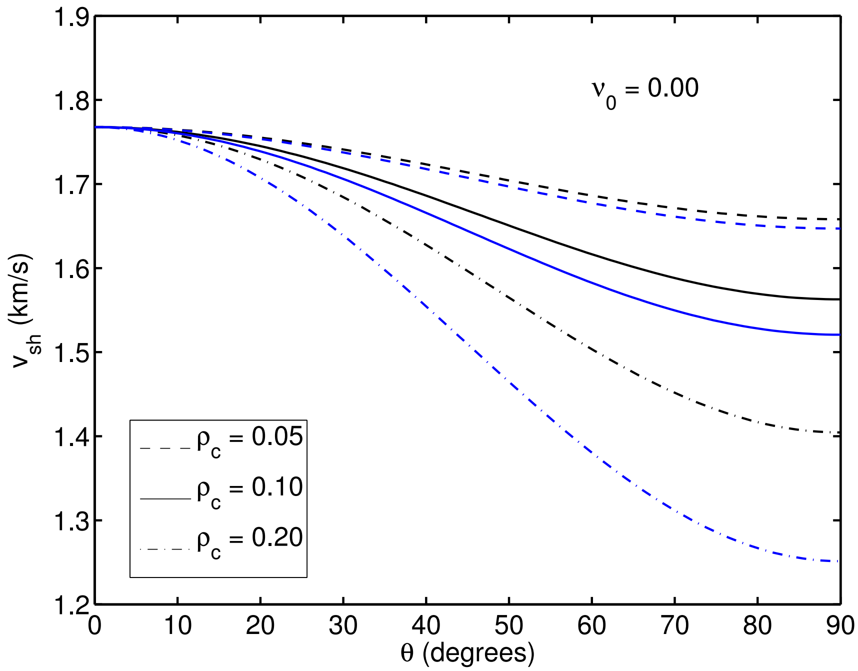

Figure 2: Same as Figure 1 for SH shear wave speed (v_{sh}). jim1-FIG2 [NR]

$\sqrt{c_{33}(1+2\epsilon)/\rho} = \sqrt{c_{11}/\rho}$ and $v_s(0) \to \sqrt{c_{44}(1+2\gamma)/\rho} = \sqrt{c_{66}/\rho}$ in (12), but the remaining velocity $[v_{sv}(0)]$ does not change since $v_{sv}(\theta)$ (within Thomsen's weak anisotropy approximation) is completely symmetric in θ and therefore has to remain so, also with the same end points, after the switch from θ to $\frac{\pi}{2} - \theta$. These results were all known previously and can be found in Rüger (2002), p. 75.

Examples of these results for small ($\rho_c = 0.05$) and higher ($\rho_c = 0.1, 0.2$) crack densities [see Berryman and Grechka (2006) for details of the methods used to obtain all the Sayers and Kachanov crack-influence parameters from simulation data and Berryman (2007) for a full discussion of the reservoir application] are presented in Figures 1-6.

CONCLUSIONS

We find that the Sayers and Kachanov (1991) crack-influence parameters are ideally suited to analyzing the role of fracture mechanics in producing anisotropic elastic constants for aligned fractures in a reservoir exhibiting HTI symmetry. Discussion of the results obtained for the higher crack density examples presented in Figures 1-6 will be provided in a later publication. But the main ideas are already contained in Berryman and Grechka (2006) and Berryman (2007). One important conclusion from the modeling presented here is that the Thomsen weak anisotropy method is valid for crack densities up to about $\rho_c \simeq 0.05$, but should be replaced by better approximations, or exact calculations, if the crack density is $\rho_c \simeq 0.1$ or higher.

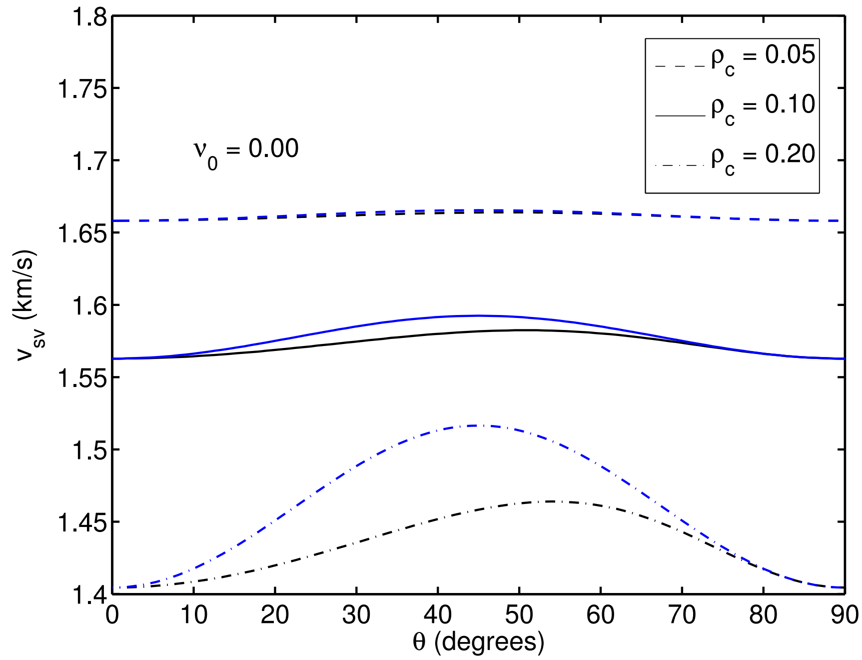

Figure 3: Same as Figure 1 for SV shear wave speed (v_{sv}). jim1-FIG3 [NR]

ACKNOWLEDGMENTS

The author thanks V. Grechka, M. Kachanov, S. R. Pride, and M. Schoenberg for helpful collaborations and conversations.

REFERENCES

Backus, G. E., 1962, Long-wave elastic anisotropy produced by horizontal layering: *J. Geophys. Res.*, **67**, 4427–4440.

Berryman, J. G., 1979, Long-wave elastic anisotropy in transversely isotropic media: *Geophysics*, **44**, 896–917.

Berryman, J. G., 2007, Seismic waves in rocks with fluids and fractures: Submitted for publication.

Berryman, J. G., and Grechka, V., 2006, Random polycrystals of grains containing cracks: Model of quasistatic elastic behavior for fractured systems: *J. Appl. Phys.*, **100**, 113527.

Lynn, H. B., Simon, K. M., Layman, M., and Schneider, R., 1995, Use of anisotropy in *P*-wave and *S*-wave data for fracture characterization in a naturally fractured gas reservoir: *The Leading Edge*, **14**, 887–893.

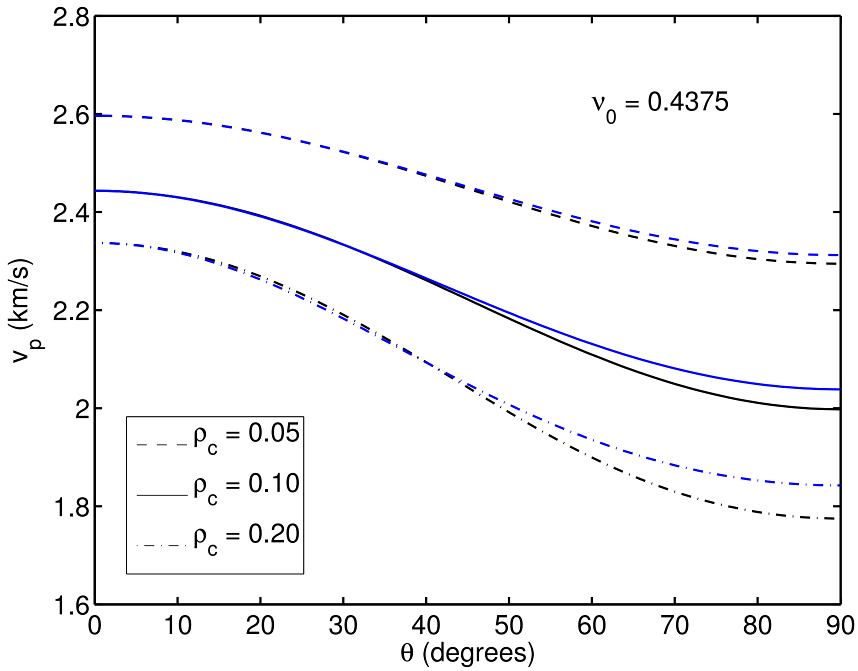

Figure 4: Same as Figure 1, but the value of $\nu_0 = 0.4375$. jim1-FIG4 [NR]

Musgrave, M. J. P., 2003, *Crystal Acoustics*: Acoustical Society of America, Leetsdale, Pennsylvania, Chapter 8.

Rüger, A., 2002, *Reflection Coefficients and Azimuthal AVO Analysis in Anisotropic Media*: Geophysical Monographs Series, Number 10, SEG, Tulsa, OK.

Sayers, C. M., and Kachanov, M., 1991, A simple technique for finding effective elastic constants of cracked solids for arbitrary crack orientation statistics: *Int. J. Solids Struct.*, **27**, 671–680.

Thomsen, L., 1986, Weak elastic anisotropy: *Geophysics*, **51**, 1954–1966.

Thomsen, L., 2002: *Understanding Seismic Anisotropy in Exploration and Exploitation*, 2002 Distinguished Instructor Short Course, Number 5, SEG, Tulsa, OK.

Zimmerman, R. W., 1991, *Compressibility of Sandstones*: Elsevier, Amsterdam.

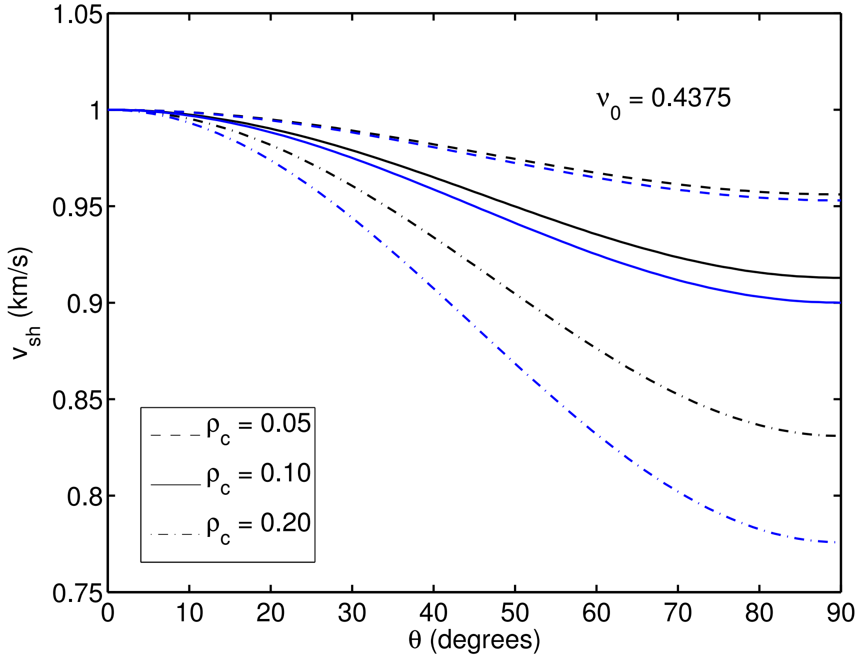

Figure 5: Same as Figure 2, but the value of $\nu_0 = 0.4375$. jim1-FIG5 [NR]

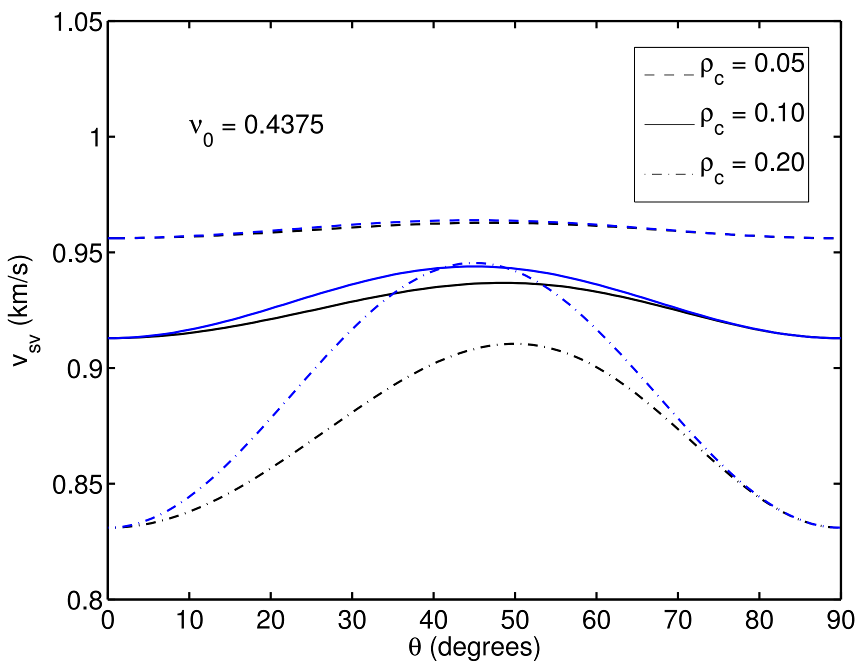

Figure 6: Same as Figure 3 for a different background medium having Poisson's ratio $\nu_0 = 0.4375$. jim1-FIG6 [NR]