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Simultaneous adaptive matching of primaries and multiples with non-stationary filters

Gabriel Alvarez and Antoine Guitton

ABSTRACT

We develop a method to match estimates of primaries and multiples to data containing both. It works with prestack data in either data space or image space and addresses the well-known issue of cross-talk between the estimates of the primaries and the multiples. The method iteratively computes non-stationary filters with micro-patches and its cost is a negligible fraction of the cost of computing the estimates of the primaries and multiples with SRME. We show, with several synthetic and real data examples, that the matched estimates of both primaries and multiples are essentially free of cross-talk. We also apply the method to the separation of ground-roll and body waves and show that most residual ground-roll contaminating the estimate of the body waves can be eliminated.

INTRODUCTION

Most methods to attenuate multiples perform, in one way or another, two complementary but clearly distinguishable steps: first, estimate a model for the multiples and second, adaptively match and subtract the estimate of the multiples from the data to get the estimate of the primaries. Surface Related Multiple Elimination (Berhout and Verschuur, 1997; Verschuur and Berkhout, 1997; Weglein et al., 1997; Dragoset and Jericevic, 1998; Dragoset, 1999) uses the auto-convolution of the data to estimate the multiples whereas moveout-based methods use filtering in either frequency-wavenumber or Radon domain (Hampson, 1986; Sava and Guitton, 2003; Alvarez et al., 2004) to estimate the multiple model. Whatever the method, the estimate of the multiples is likely to be contaminated with residual primary energy and have errors in amplitude and phase. After adaptive subtraction, the estimated primaries are likely to suffer from undesired residual multiple energy, or weakened primaries, or both (Guitton and Verschuur, 2004).

In this paper we assume that the multiple model has already been estimated by whatever method. We concentrate on the adaptive subtraction step to match and subtract the multiples from the data to get the estimated-matched primaries.

In the next section we present our adaptive-matching algorithm. It estimates non-stationary filters (Rickett et al., 2001) that simultaneously match both the estimates of the primaries and the multiples to the data. These filters act on micro-patches (Claerbout and Fomel, 2002) and can handle inaccuracies in the estimated multiples in terms of both amplitudes and kinematics. The filters are estimated iteratively by re-estimating the multiple and primary models until the residual (the difference between the sum of the matched primaries and multiples and the data) is zero. Only a few iterations (three to five) seem to be necessary.

In the following section we apply the new method to two synthetic datasets contaminated with multiples. In the first test we match kinematically perfect estimates of primaries and multiples contaminated with 40% of cross-talk and show that the method produces a cross-talk-free result. Then we apply the method to a very inaccurate estimate of both the primaries and the multiples obtained via migration-demigration as described in a previous report (Alvarez, 2006). Even with such a poor estimate of both primaries and multiples, and the strong cross-talk on both, the matched results are very good, with little cross-talk. To illustrate the method with stacked data, we apply it to a migrated section of the Sigsbee model. Here the multiples were estimated with an image space version of SRME (**?**). The results show that the method attenuated most of the multiples and produced a largely multiple-free estimate of the primaries.

In the last section we apply the method to an angle-domain common-image gather taken from a real 2D line in the Gulf of Mexico. The estimate of the multiples was obtained by Radon filtering in the image space (Alvarez et al., 2004). In the final estimate of the primaries, most of the residual energy from the diffracted multiples was eliminated. Finally, we apply the method to a different problem, namely the separation of ground-roll and body-waves. We use a real land shot gather contaminated with strong ground-roll and show that most of the residual ground-roll can be attenuated in the final estimate of the body waves. This is a more challenging problem because the non-stationarity characteristics of the ground-roll and the body waves are different requiring different filter lengths and patch sizes to match them to the data.

DESCRIPTION OF THE METHOD

Conceptually, the first step of our method is to form the convolutional matrices of both the estimated multiples $\mathbf{M}$ and the estimated primaries $\mathbf{P}$. In practice, these matrices are not explicitly formed but computed with linear operators (Claerbout and Fomel, 2002). Next, we compute non-stationary filters in micro-patches (that is, filters that act locally on overlapping two-dimensional partitions of the data) to match the estimated multiples and the estimated primaries, to the data containing both. We compute the filters by solving the following least-squares inverse problem:

$$\begin{bmatrix}\mathbf{M} & \mu\mathbf{P}\end{bmatrix}\begin{bmatrix}\mathbf{f_m}\\ \mathbf{f_p}\end{bmatrix} \approx \mathbf{d} \quad (1)$$

$$\epsilon\mathbf{A}\begin{bmatrix}\mathbf{f_m}\\ \mathbf{f_p}\end{bmatrix} \approx \mathbf{0} \quad (2)$$

where $\mathbf{f_m}$ and $\mathbf{f_p}$ are the matching filters for the multiples and primaries respectively, μ is a parameter to balance the relative importance of the two components of the fitting goal (Guitton, 2005), **d** is the data (primaries and multiples), **A** is a regularization operator, (in our implementation a Laplacian operator), and ϵ is the usual way to control how strong a regularization we want.

Once convergence is achieved, each filter is applied to its corresponding convolutional matrix, and new estimates for M and P are computed:

$$M \leftarrow M\mathbf{f_m} \tag{3}$$
$$P \leftarrow \mu P\mathbf{f_p}. \tag{4}$$

These updated versions of M and P are then plugged into equations 1 and 2 and the process repeated until the cross-talk has been eliminated or significantly attenuated.

EXAMPLES WITH SYNTHETIC DATA

As a first example, we will consider the synthetic dataset shown in panel (a) of Figure 1. There are two primaries (black) and four multiples (white). The traveltimes of both primaries and multiples were computed analytically from a three-layer model: water layer, a sedimentary layer and a half space. The estimates of the multiples (b) and primaries (c) were computed by adding 40% of the primaries to the multiples and 40% of the multiples to the primaries, respectively. The goal is to simulate a situation in which the kinematics of the estimates of primaries and multiples are both correct but there is strong cross-talk (leakage) which may happen, for example, after the multiples are estimated via filtering in the Radon domain. Figure 2 shows the estimated multiples after one, two and three non-linear iterations of the algorithm. The corresponding results for the estimated primaries are shown in Figure 3. In both figures we see that the cross-talk is substantially reduced after the first non-linear iteration and is completely eliminated after the third. Notice the hole in the top multiple and the bottom primary in the final estimates. This is actually present in the data (panel (a) in Figure 1) and is an artifact because both primaries and multiples were modeled with the same amplitude and opposite polarity. Consider now the more realistic situation of kinematic and offset-dependent amplitude errors and noise as shown in Figure 4. The multiple and primary estimates were obtained via migration-demigration as described in a previous report (**?**). Clearly these are imperfect estimates with cross-talk on primaries and multiples and other noises. Panel (a) of Figures 5 and 6 show the results after one iteration of the non-linear inversion whereas panels (b) and (c) of the same figures show the results after three and five non-linear iterations respectively. There is still some localized cross-talk from the multiples into the primaries, but given how imperfect the initial estimates were, the result is encouraging.

The next example uses the well-known Sigsbee model to illustrate the method in the image space on an angle stack. The estimate of the multiples was computed with an image space version of SRME (**?**). Figure 7 shows the modeled data after shot profile migration (a) and the estimated multiples (b). Both panels are plotted at the exact same clip value. Notice that the estimate of the multiples is accurate only in kinematics, not in amplitudes or frequency content.

In contrast with the previous examples, in this case we do not have an independent initial estimate of the primaries. We could subtract the estimates of the multiples from the data, but the corresponding estimate of the primaries is too distorted and using it actually hurts the chances of matching the multiples to the data (since this example is in the image space, "data" means the migrated image with primaries and multiples). Other option is to use the data itself as the initial estimate of the primaries. We found, however, that a better alternative is to do a first iteration setting $\mu = 0$, meaning only the multiples need to be matched. Once matched, the multiples are subtracted from the data to get the estimate of the primaries for the next iteration Figures 8 and 9 show a close-up view of the matched primaries and multiples, respectively, after one, two and three non-linear iterations. After the first iteration, the most obvious multiples contaminating the estimate of the primaries have been attenuated (compare panels (a) of Figures 7 and 8). The second iteration helps attenuate the multiples further, although is hard to appreciate in these small figures. See, for example the multiple inside the salt and in the bottom right corner of panel (b). The third iteration does not help appreciably. The remaining multiples have too much dip and would require a long filter that could also match the primaries. On the estimate of the multiples, again the first iteration extracts the most significant multiples and the second iteration locally correct the amplitudes. The third iteration actually hurts the estimate of the multiples because the effect of the regularization term becomes significant as the match of both the primaries and the multiples to the data improves. The net result is an estimate of the primaries that is close to the primaries in the original image. The estimate of the multiples, however, is weaker than it should. If for some reason we wanted the multiples, we could subtract the estimated primaries from the original image.

EXAMPLES WITH REAL DATA

We will now illustrate the method with an Angle-Domain Common-Image Gather (ADCIG) from a seismic line acquired in the Gulf of Mexico. Figure 10 shows the ADCIG together with the estimates of the multiples and primaries. The estimate of the multiples was obtained with an apex-shifted Radon transform in the image space (Alvarez et al., 2004) and the estimate of the primaries was obtained simply by subtracting it from the data. Notice the residual primary energy just below 3000 m in the estimate of the primaries. Note also the residual energy from the multiples in the estimate of the primaries. Figure 11 shows the ADCIG after one, five and ten non-linear iterations. The first iteration attenuates the strongest residual multiples (compare panel (a) of Figure 11 with panel (c) of Figure 10). Subsequent iterations further reduce the residual multiples. Also, although hard to see in the hard copy, the primary energy that contaminated the estimate of multiples below 3000 m has been mapped back to the primaries. Figure 12 shows the corresponding results for the multiples. Notice again that the residual primary energy has been severely attenuated.

As a final example, consider the problem of separating ground-roll from body waves in land data. This a more challenging application of our implementation of the algorithm because the signal has curvature that changes rapidly with both offset and time so to match it we need small filters in relatively small patches. The ground-roll, on the other hand, has little global curvature (although it may have strong local curvature due to aliasing) and matching it is more successful with large filters in large patches. Figure 13 shows the original shot as well as the initial estimates of the body waves and the ground-roll.

The ground-roll estimate was computed simply by high-cut filtering the data to 24 Hz using a Butterworth filter with six poles. We allowed significant energy from the body waves to leak into the estimate of the ground-roll to illustrate the problem described in the previous paragraph. Similarly, the estimate of the body waves was computed by low-cut filtering the data to 18 Hz also with a Butterworth filter with 6 poles. Since we don't want to reduce the low frequency components of the signal too much, we allowed strong ground-roll to leak into the estimate of the body waves. The purpose is to eliminate this ground-roll without hurting the signal and ideally, mapping back some of the body-waves from the estimate of the ground-roll.

Figure 14 shows the estimate of the body-waves after one, five and 20 non-linear iterations of the proposed algorithm. Even after just the first iteration, most of the ground-roll has been eliminated and after five iterations it is almost gone. For this example we used just two patches in time and one in offset. Figure 15 shows similar results for the ground-roll. Since the patches were so large, the energy of the leaked body-waves were only slightly attenuated (see the reflector at about 1.7 secs). This energy was mapped back to the estimate of the body-waves.

DISCUSSION AND CONCLUSIONS

The issue of cross-talk is a very important one when separating signal from noise and in particular primaries from multiples. The standard approach is to match the estimated multiples directly to the data and obtain the primaries by subtraction of the matched multiples. This approach often leads to weakened primaries and/or contamination with residual multiples. By exploiting the estimates of both, multiples and primaries, we prevent the matching algorithm from attempting to match primaries into the multiples which is almost unavoidable otherwise. Furthermore, we obtain simultaneous estimates of both the primaries and the multiples that are guaranteed to be consistent with the original data.

It should be emphasized that the algorithm, as presented, is independent of the method employed to obtain the initial estimates of the multiples and the primaries. It should also be stressed that the algorithm does not rely on explicit knowledge of the moveouts of the primaries or the multiples. It only relies on the fact that the data is the sum of the multiples and the primaries.

The method presented in this paper can be used not only to match primaries and multiples but in general to match estimates of noise and signal to data containing both. We showed an example with the separation of ground-roll and body-waves with land data, but other applications may also be possible.

REFERENCES

Alvarez, G., B. Biondi, and A. Guitton, 2004, Attenuation of diffracted multiples in angle-domain common-image gathers:, *in* 74th Ann. Internat. Mtg. Soc. of Expl. Geophys., 1301–1304.

Alvarez, G., 2006, Attenuation of 2d specularly-reflected multiples in image space: SEP–**124**.

Berhout, A. and D. Verschuur, 1997, Estimation of multiple scattering by iterative inversion, part i: theoretical considerations: Geophysics, **62**, no. 5, 1586–1595.

Claerbout, J. and S. Fomel, 2002, Image Estimation by Example: Geophysical soundings image construction: Class notes, http://sepwww.stanford.edu/sep/prof/index.html.

Dragoset, W. and Z. Jericevic, 1998, Some remarks on multiple attenuation: Geophysics, **63**, no. 2, 772–789.

Dragoset, W., 1999, A practiacal approach to surface multiple attenuation: The Leading Edge, **3**, no. 2, 772–789.

Guitton, A. and D. Verschuur, 2004, Adaptive subtraction of multiples using the l1-norm: Adaptive subtraction of multiples using the l1-norm:, Eur. Assn. Geosci. Eng., Geophysical Prospecting, 27–38.

Guitton, A., 2005, Multidimensional seismic noise attenuation: Ph.D. thesis, Stanford University.

Hampson, D., 1986, Inverse velocity stacking for multiple elimination:, *in* 56th Ann. Internat. Mtg. Soc. of Expl. Geophys., Session:S6.7.

Rickett, J., A. Guitton, and D. Gratwick, 2001, Adaptive multiple subtraction with non-stationary helical shaping filters:, *in* 63rd Meeting Eur. Assn. Geosci. Eng., Session: P167.

Sava, P. and A. Guitton, 2003, Multiple attenuation in the image space:, *in* 73rd Annual International Meeting Soc. of Expl. Geophys., 1933–1936.

Verschuur, D. and A. Berkhout, 1997, Estimation of multiple scattering by iterative inversion, part ii: practical aspects and examples: Geophysics, **62**, no. 5, 1596–1611.

Weglein, A., F. Gasparotto, P. Carvalho, and R. Stolt, 1997, An inverse scattering series method for attenuating multiples in seismic reflection data: Geophysics, **62**, 1975–1989.

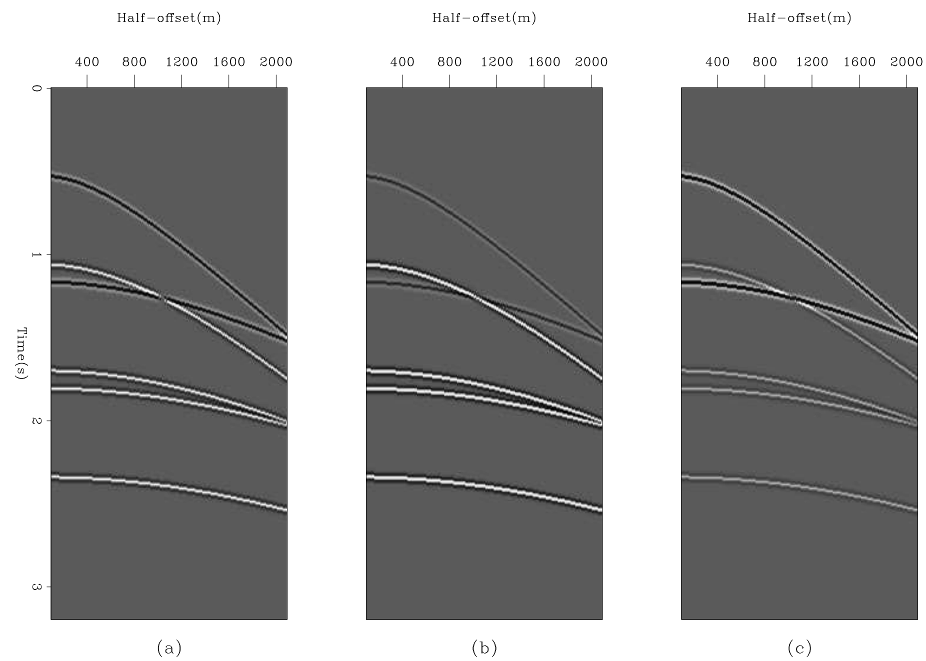


Figure 1: Synthetic CMP gather (a) showing two primaries (black) and four multiples from a three horizontal layer model. The initial estimates of multiples (b) and primaries (c) are contaminated with 40% cross-talk. gabriel1-syn1_estimates1 [ER]

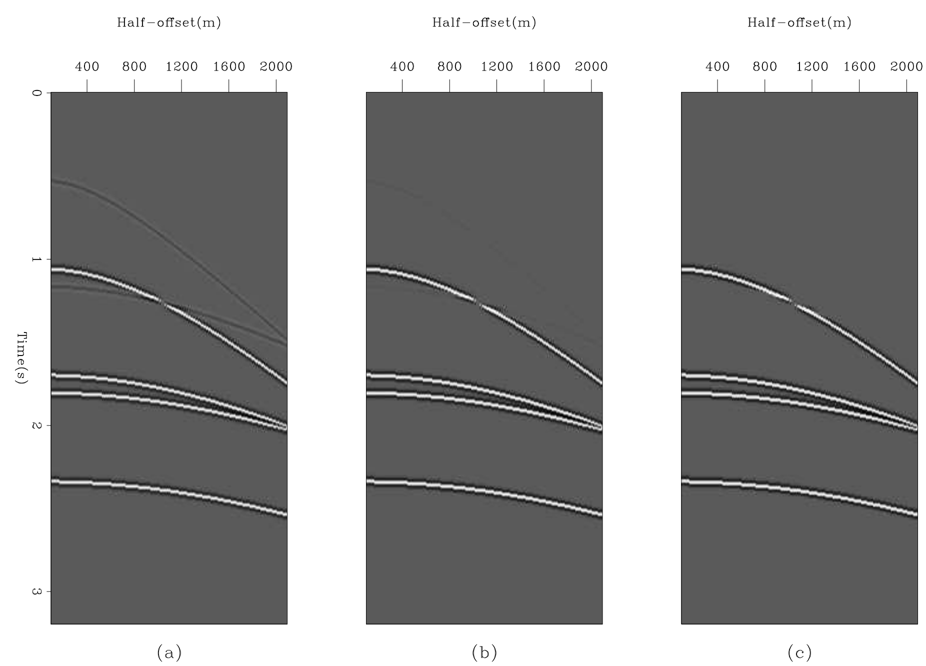


Figure 2: Matched estimates of multiples after one (a), three (b) and five (c) non-linear iterations of the algorithm. gabriel1-syn1_matched_muls [ER]

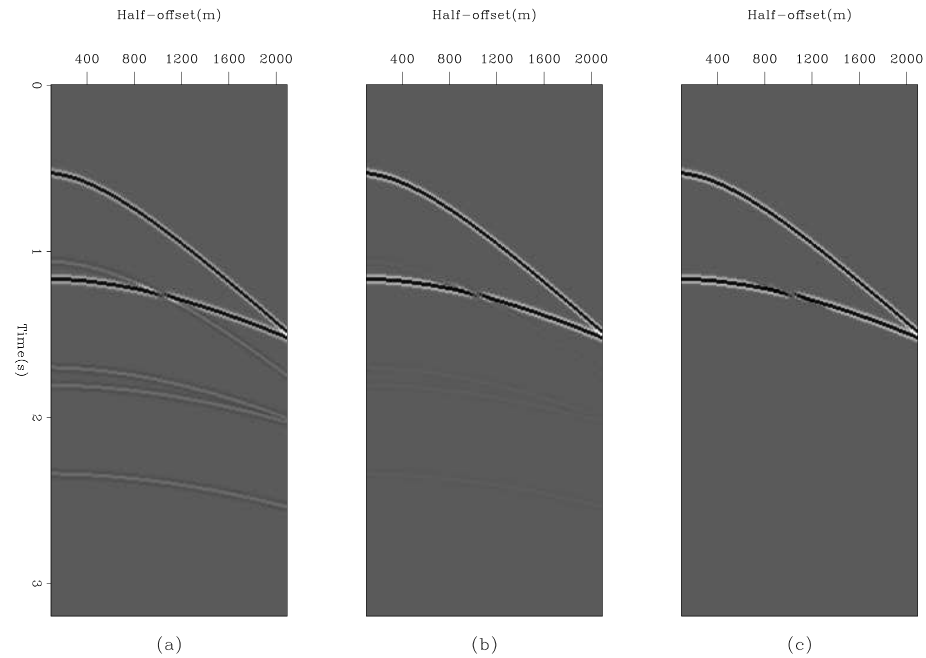


Figure 3: Matched estimates of primaries after one (a), two (b) and three (c) non-linear iterations of the algorithm. gabriel1-syn1_matched_prims [ER]

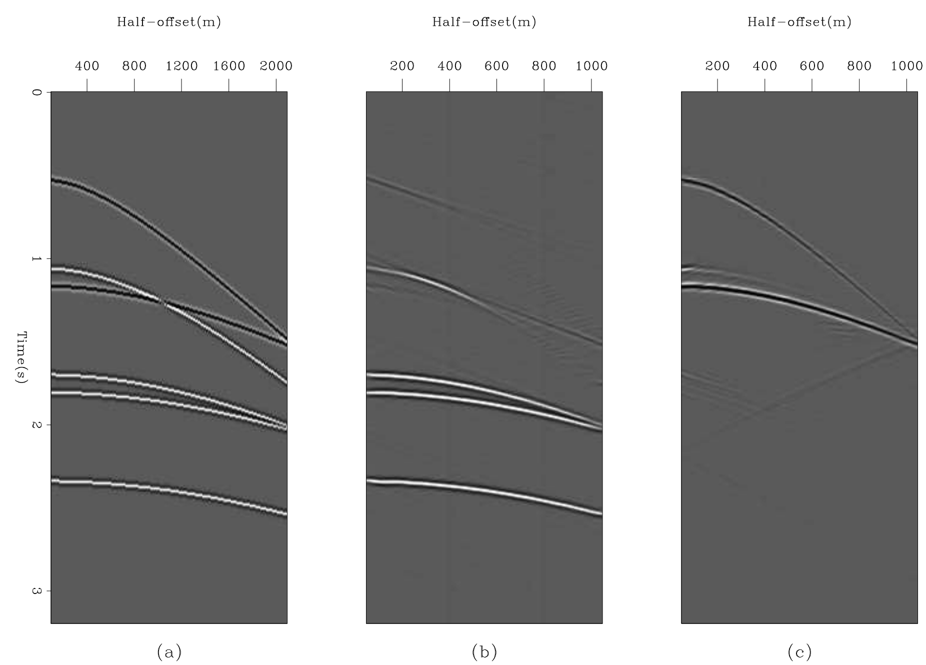


Figure 4: Original CMP gather (a), initial estimate of multiples (b) and primaries (c). gabriel1-syn2_estimates1 [ER]

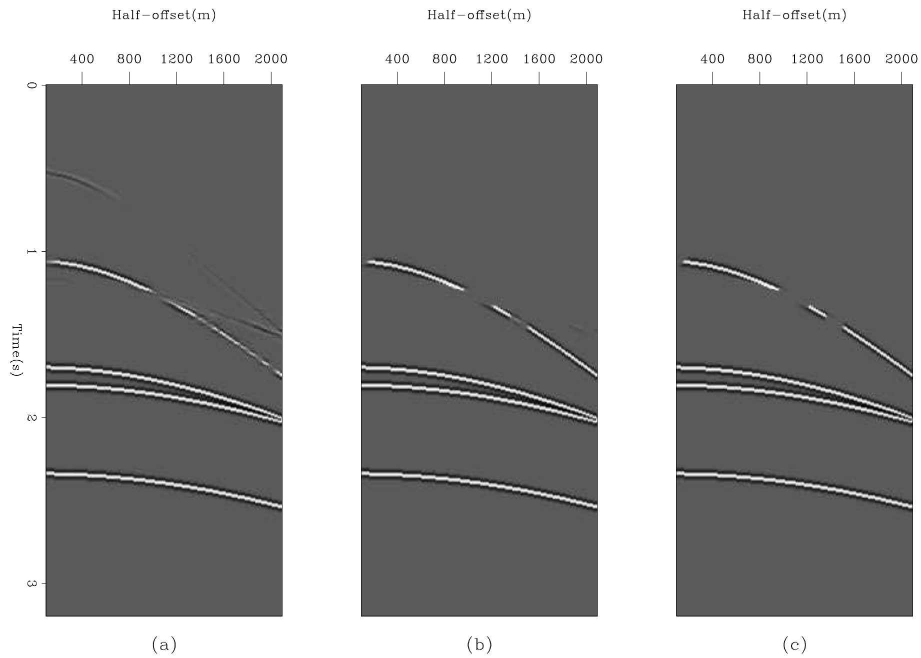


Figure 5: Matched estimates of multiples after one (a), three (b) and five (c) non-linear iterations of the algorithm. gabriel1-syn2_matched_muls [ER]

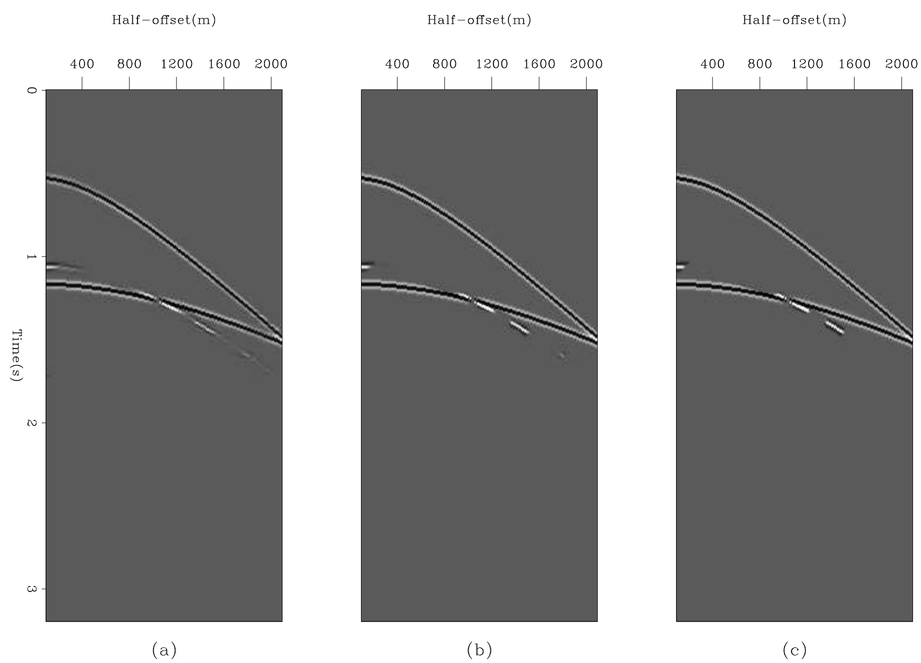
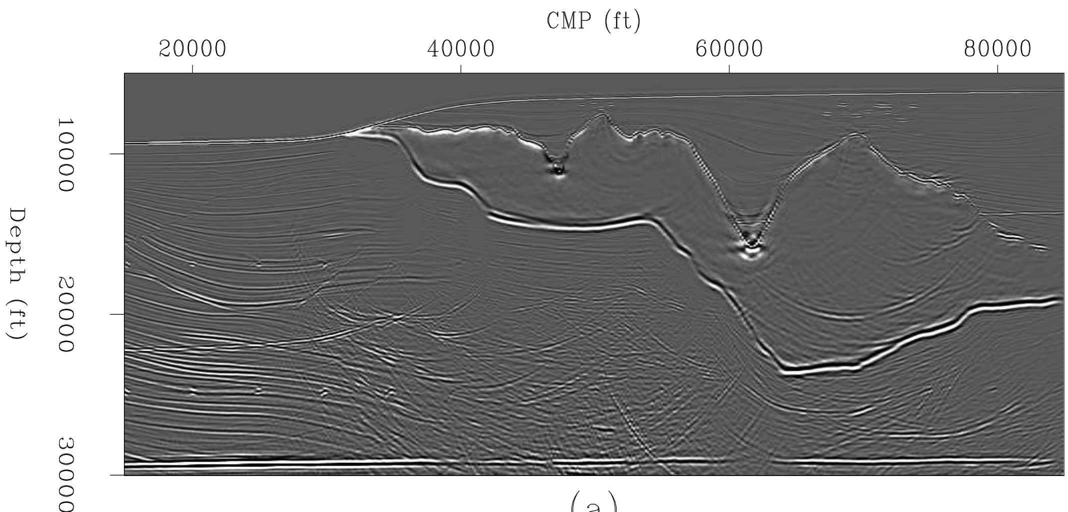
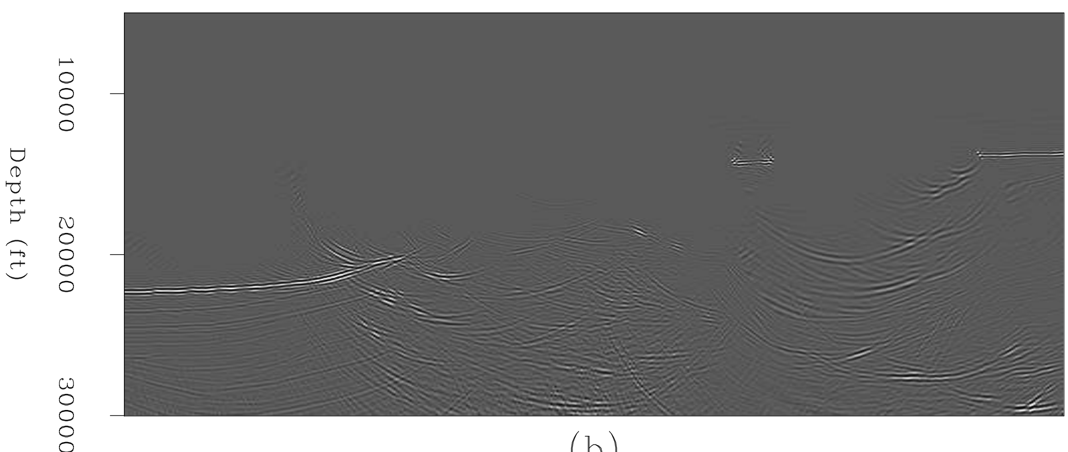


Figure 6: Matched estimates of primaries after one (a), three (b) and five (c) non-linear iterations of the algorithm. gabriel1-syn2_matched_prims [ER]



(a)



(b)

Figure 7: Migrated angle stack of the Sigsbee model (a), initial estimates of multiples (b). gabriel1-sgsb_estimates1 [ER]

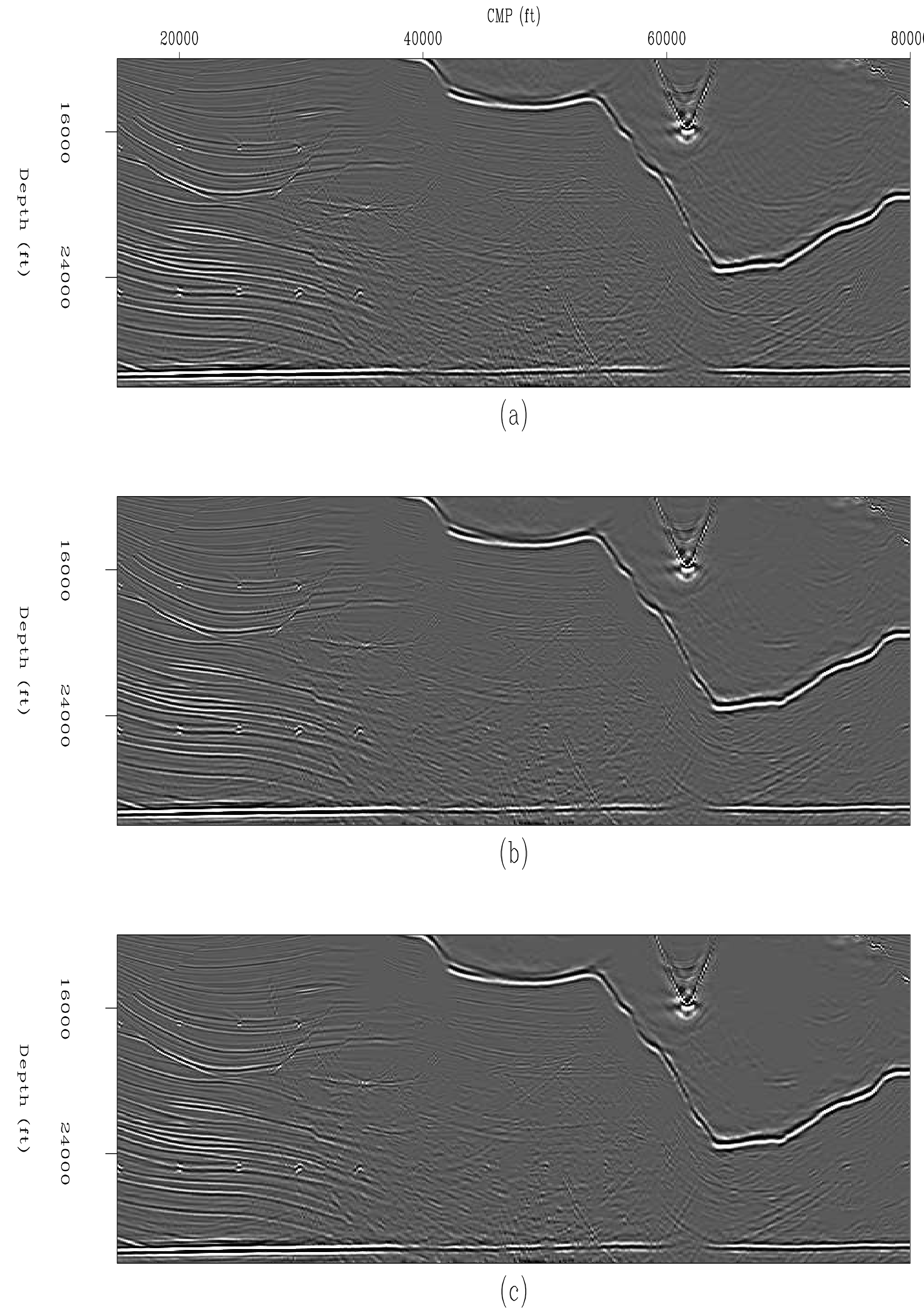


Figure 8: Estimated primaries after one (a), two (b) and three (c) iterations of the non-linear inversion. gabriel1-sgsb_matched_prims [ER]

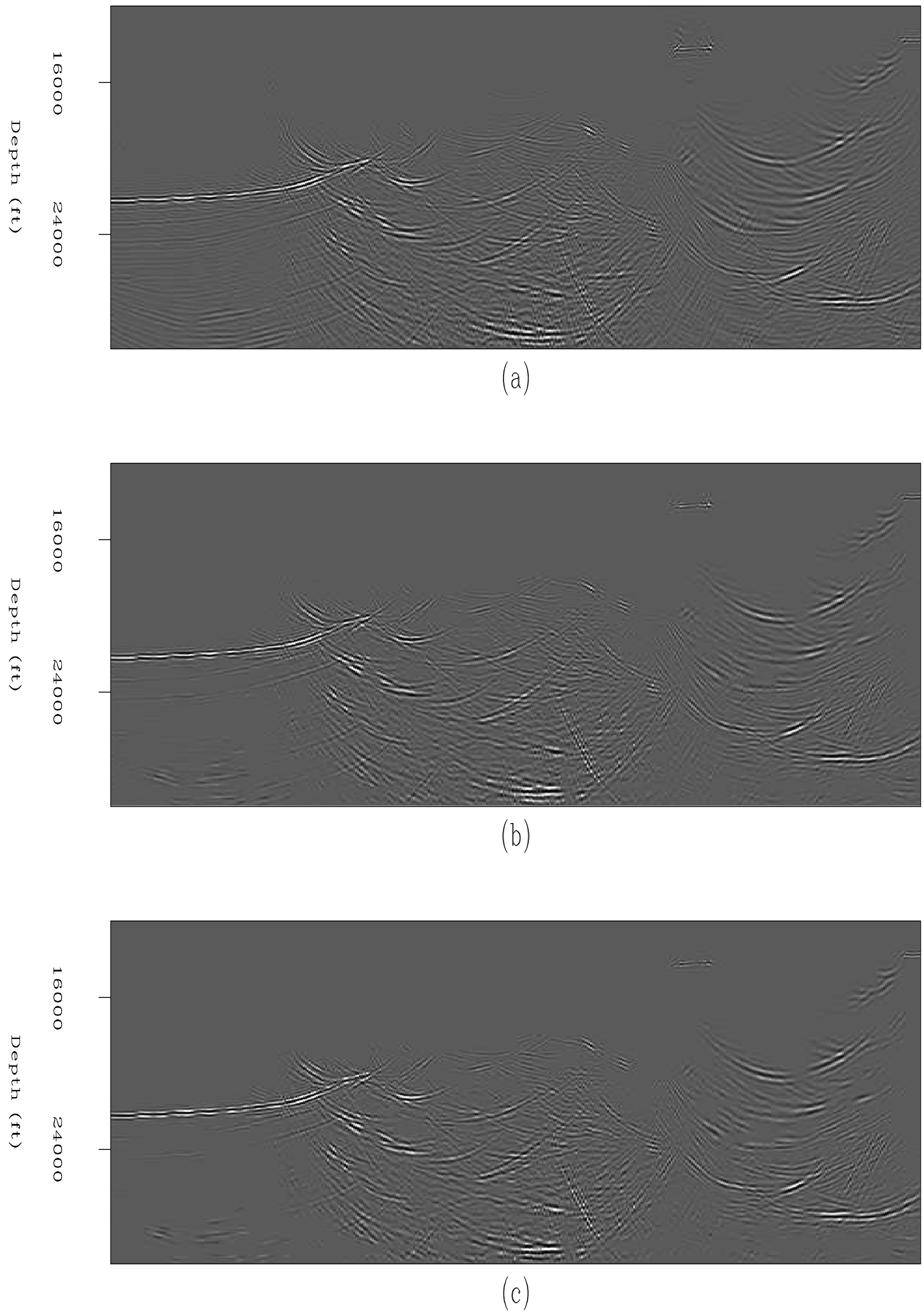


Figure 9: Estimated multiples after one (a), two (b) and three (c) iterations of the non-linear inversion. gabriel1-sgsb_matched_muls [ER]

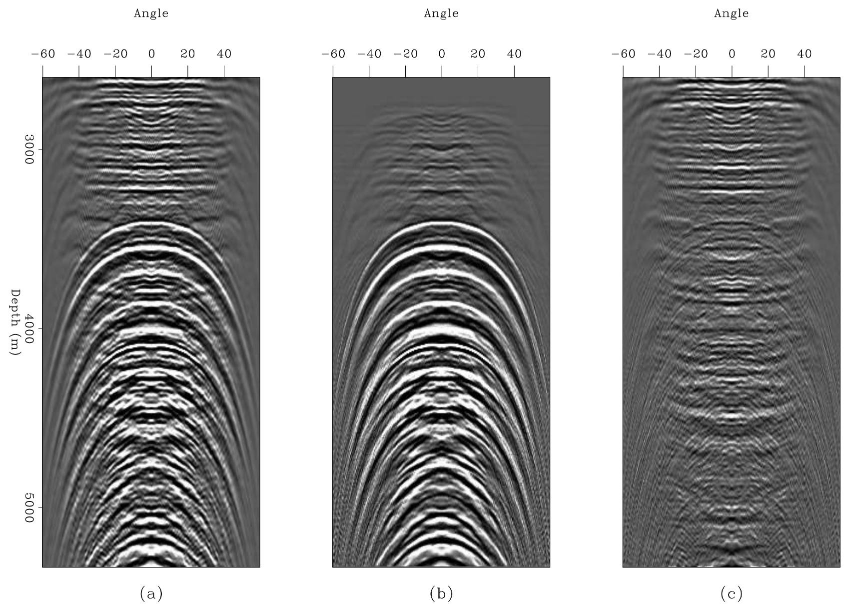


Figure 10: ADCIG (a), initial estimate of the multiples (b), and the primaries (c). Note the crosstalk on both panels. gabriel1-adcig1_estimates1 [ER]

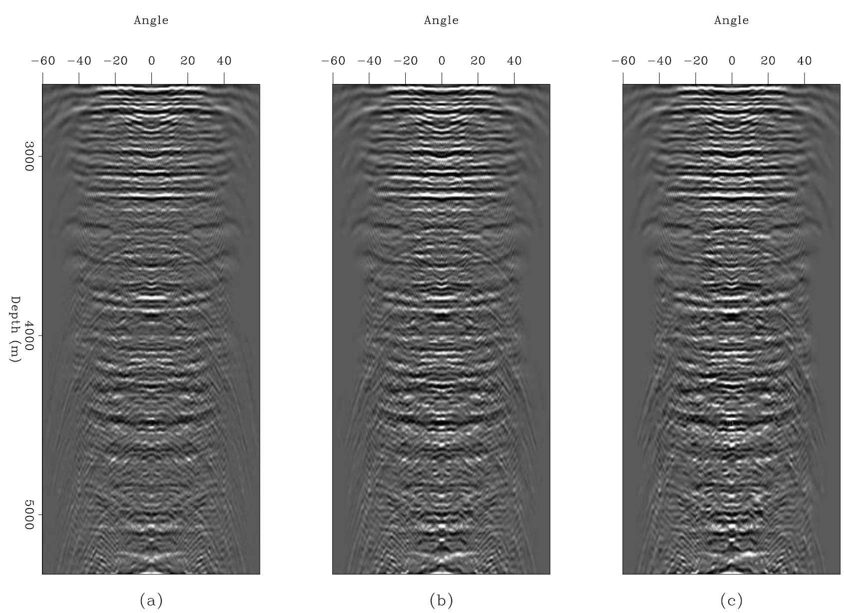


Figure 11: Estimated primaries after one (a), five (b) and ten (c) non-linear iterations. gabriel1-adcig1_matched_prims [ER]

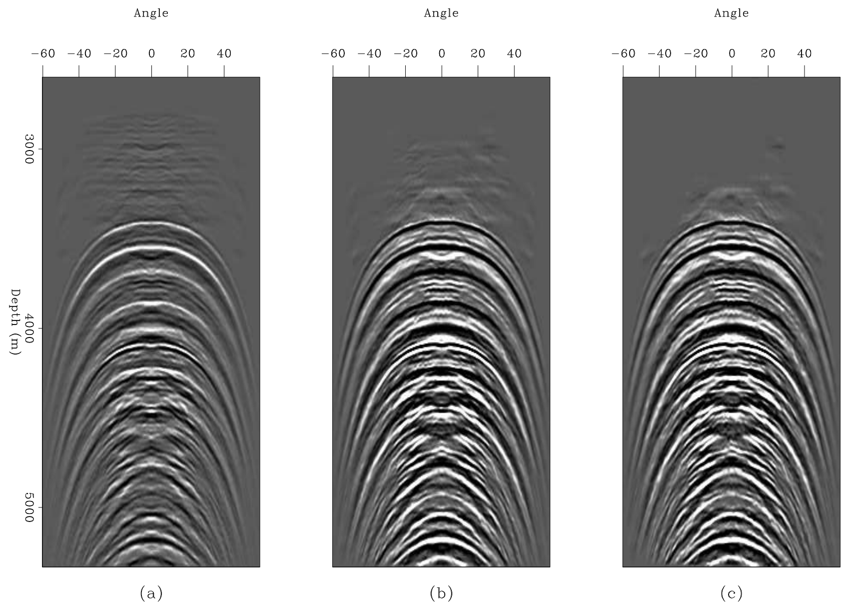


Figure 12: Estimated multiples after one (a), five (b) and ten (c) non-linear iterations. gabriel1-adcig1_matched_muls [ER]

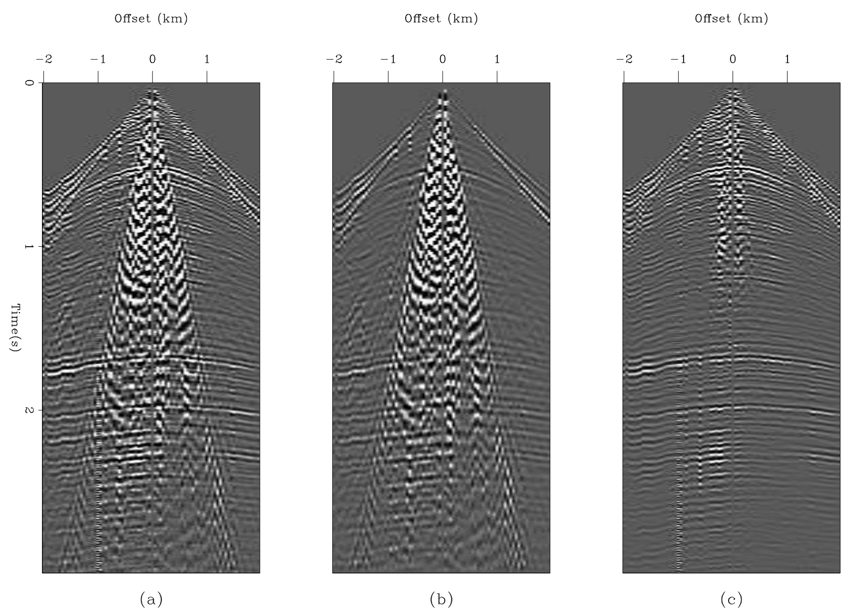


Figure 13: Land shot gather with strong ground-roll (a), initial estimate of ground-roll (b), and body waves (c). gabriel1-shot1_estimates1 [ER]

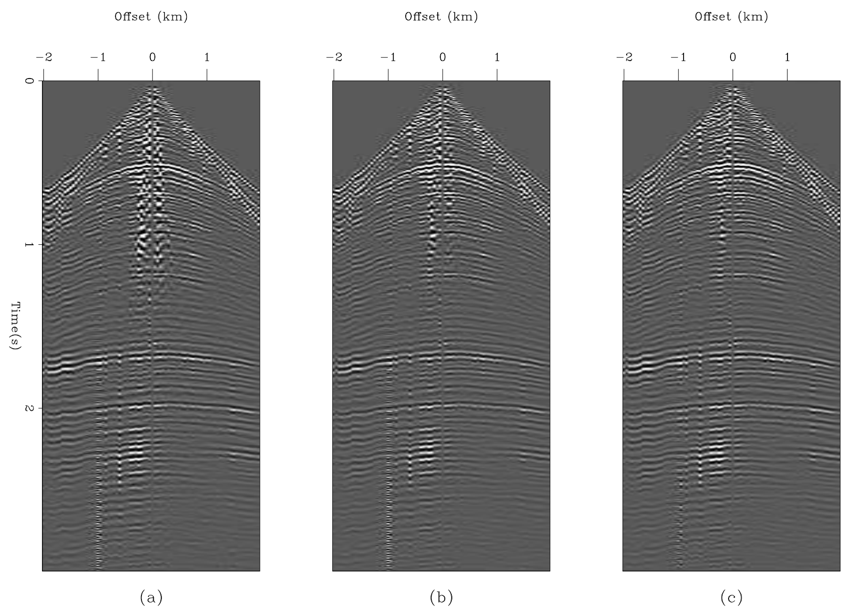


Figure 14: Estimate of body waves after one non-linear iteration (a), after 5 non-linear iterations (b) and after 20 non-linear iterations (c). Notice how after the fifth iteration the ground-roll is essentially gone. gabriel1-shot1_matched_bw [ER]

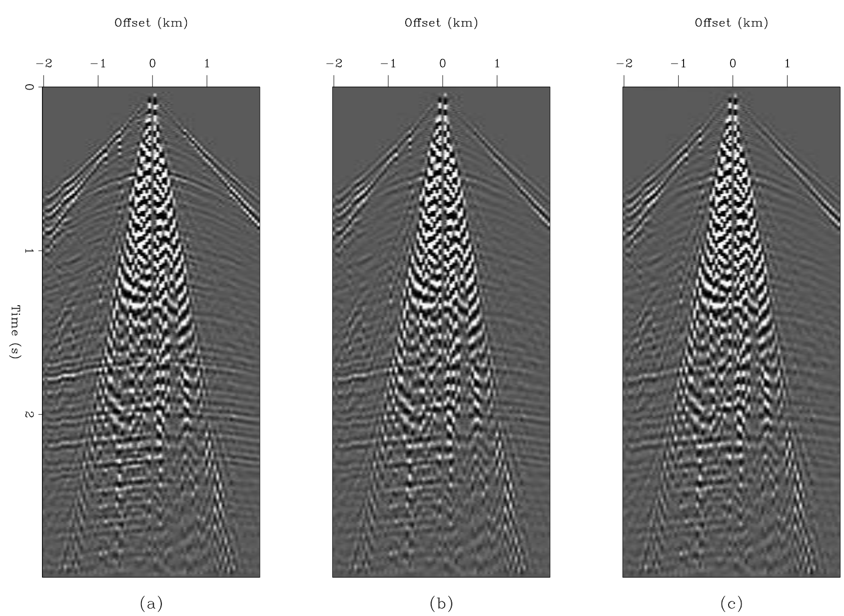


Figure 15: Estimate of ground-roll after one non-linear iteration (a), after 5 non-linear iterations (b) and after 20 non-linear iterations (c). Some of the body waves have been removed in panel (c) but much still remains. gabriel1-shot1_matched_gr [ER]