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Application of Least-Squares Joint Imaging of Multiples and Primaries on Shallow Water-Bottom Data Sets

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ABSTRACT

Data contaminated with strong shallow water-bottom multiples is rife with challenges. Application of Least-Squares Joint Imaging of Multiples and Primaries (LSJIMP) on such data sets yields mixed results. LSJIMP solves both the separation and integration simultaneously, as a global least-squares inverse problem. We point out some limitations of LSJIMP by testing it on synthetic data sets that emulate shallow water-bottom marine environments. Some slight modifications have been made, and we suggest some strategies that might make LSJIMP an effective algorithm.

INTRODUCTION

Multiples are often the most significant impediment to the successful construction and interpretation of an image of the primaries, especially in regions with anomalously strong reflectors (e.g., "hard" water bottom or salt bodies). But, since they penetrate deeply enough into the earth and illuminate different angular ranges and reflection points, a primary and its multiples are more than simply redundant.

The problem of handling multiples becomes more challenging when we encounter shallow water-bottom with a large impedence contrast. In such cases a large amount of multiple energy is generated, most of which is surface related. Offshore Australia is a prime example of such a marine environment. An important class of multiple-suppression techniques create from the data a "model" of the multiples, which may then be adaptively subtracted from the data. Verschuur and Prein (1999) use Surface-Related Multiple Elimination (SRME) for multiple-suppression on an Australian data set infected with shallow water bottom multiples. The multiple-generating boundaries were more or less horizontal or gently dipping. Long et al. (2005) use SRME to revisit a poor-data-quality area in the northern Carnarvon Basin, offshore Western Australia, where both short- and long-period multiple energy prohibit imaging of the underlying geology. In some sense, shallow-water multiple suppression is easier. The multiples look a lot like primaries, so deconvolution with a gapped filter (gap size = water-bottom time) will solve the problem nicely. However, some of the more advanced multiple suppression methods, like high-resolution Radon and SRME, have a trouble with shallow water-bottom. The Radon transform relies on moveout differences between multiples and primaries, which are not present in shallow data, except near the surface, and SRME often requires many non-linear iterations in shallow-water environments.

Brown (2004) introduces LSJIMP (Least-Squares Joint Imaging of Multiples and Primaries) for using information embedded in primaries and multiples simultaneously to enhance the signal-to-noise ratio and fill illumination gaps by averaging the images constructed from primaries and multiples. He has effectively demonstrated the use of his method on a variety of deep water-bottom data sets.

In this paper, we discuss implementation of LSJIMP on two shallow water-bottom synthetic data sets, for which we get mixed results. Some slight modifications have been made and we point out some improvements that might make this method perform better.

THEORY

LSJIMP models the recorded data as the superposition of primary reflections and p orders of pegleg multiples from $n_{\rm surf}$ multiple-generating surfaces. A schematic for LSJIMP is given in Figure 1. An $i^{\rm th}$ order pegleg splits into $i+1$ legs. If we denote the primaries as $\mathbf{d}_0$ and the $k^{\rm th}$ leg of the $i^{\rm th}$ order pegleg from the $m^{\rm th}$ multiple generator as $\mathbf{d}_{i,k,m}$, the modeled data takes the following form:

$$\mathbf{d}_{\rm mod} = \mathbf{d}_0 + \sum_{i=1}^{p}\sum_{k=0}^{i}\sum_{m=1}^{n_{\rm surf}} \mathbf{d}_{i,k,m}. \tag{1}$$

If we have designed an imaging operator that produces primary and multiple images with consistent signal (kinematics and angle-dependent amplitudes), then we assume that we can model the important events in the data. We can rewrite equation (1) as

$$\begin{aligned} \mathbf{d}_{\rm mod} &= \mathbf{L}_0\mathbf{m}_0 + \sum_{i=1}^{p}\sum_{k=0}^{i}\sum_{m=1}^{n_{\rm surf}} \mathbf{L}_{i,k,m}\mathbf{m}_{i,k,m} && (2)\\ &= \mathbf{L}\mathbf{m} && (3) \end{aligned}$$

where $\mathbf{L}_0$ is the modeling operator of the primaries, and $\mathbf{m}_0$ is the image of primaries. Similarly, for the $k^{\rm th}$ leg of the $i^{\rm th}$ order pegleg from the $m^{\rm th}$ multiple generator, $\mathbf{L}_{i,k,m}$ and $\mathbf{m}_{i,k,m}$ are modeling operator and image respectively.

Brown (2004) derived appropriate imaging and amplitude correction operators to map the primary image, $\mathbf{m}_0$, into data-space primary events using the Normal Move-Out (NMO) operator, $\mathbf{N}_0$. Similarly, a given pegleg image, $\mathbf{m}_{i,k,m}$, is mapped into data space by sequentially applying the differential geometric spreading correction ($\mathbf{G}_{i,m}$), Snell resampling ($\mathbf{S}_{i,m}$), the Hetrogeneous-Earth Multiple NMO operator (HEMNO) ($\mathbf{N}_{i,k,m}$), and finally, a reflection coefficient ($\mathbf{R}_{i,k,m}$). HEMNO is a slight improvement over NMO that takes into account kinematics of mildly dipping reflectors. Using these operators, we can rewrite equation (2) as follows:

$$\mathbf{d}_{\rm mod} = \mathbf{N}_0\mathbf{m}_0 + \sum_{i=1}^{p}\sum_{k=0}^{i}\sum_{m=1}^{n_{\rm surf}} \mathbf{R}_{i,k,m}\mathbf{N}_{i,k,m}\mathbf{S}_{i,m}\mathbf{G}_{i,m}\mathbf{m}_{i,k,m}. \tag{4}$$

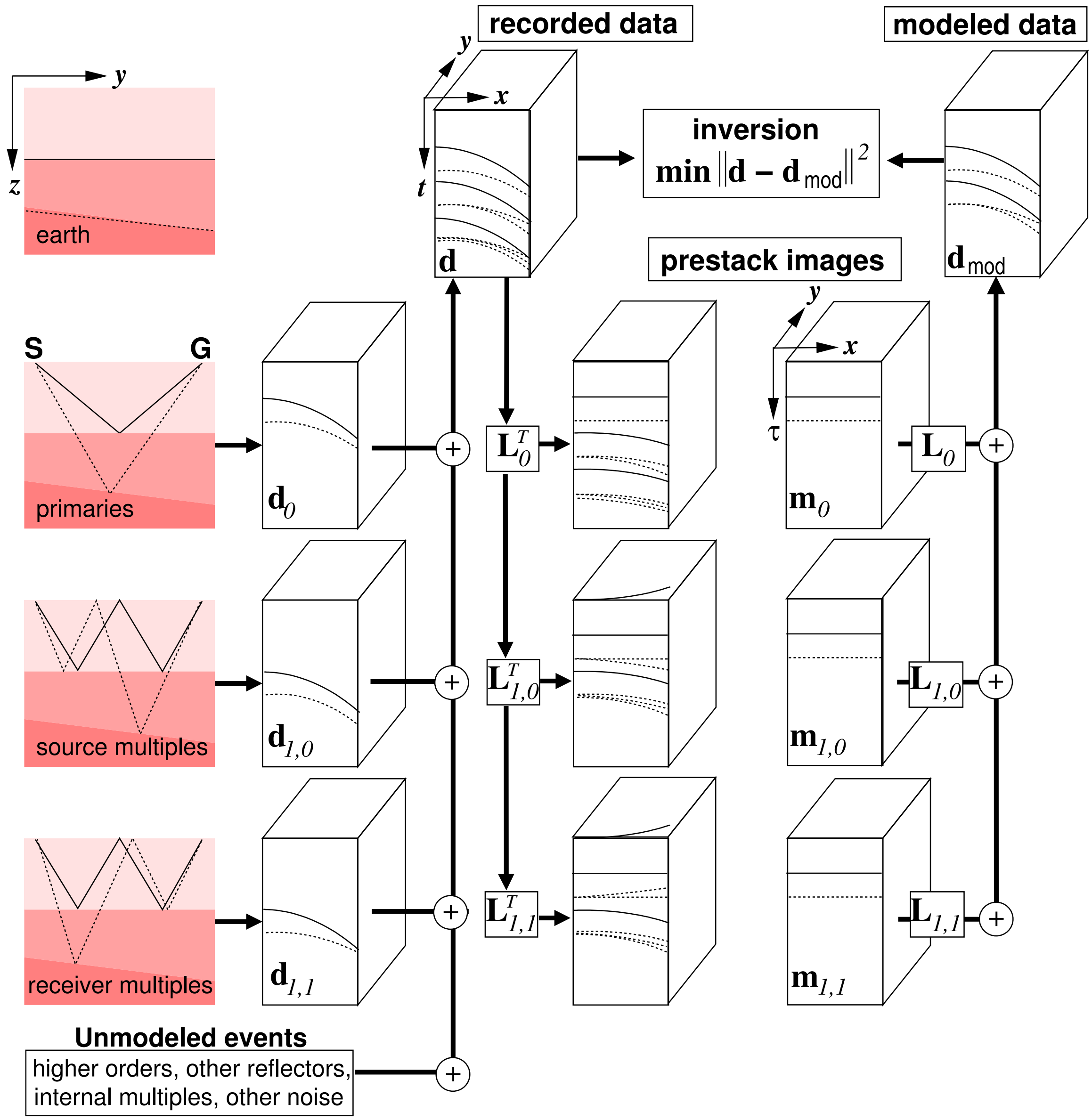

Figure 1: LSJIMP schematic. Assume that the recorded data consist of primaries and peg-leg multiples. Prestack imaging alone (applying adjoint of modeling operator $\mathbf{L}_{i,k}$) focuses signal events in zero-offset traveltime (or depth) and offset (or reflection angle), but leaves behind crosstalk events. If the $\mathbf{m}_{i,k}$ images contain only signal, then we can model all the events in the data that we desire. The LSJIMP inversion suppresses crosstalk and endeavors to fit the recorded data in a least-squares sense. The model regularization operators used to suppress crosstalk simultaneously enable LSJIMP to exploit the intrinsic redundancy between and within the images to increase signal fidelity. madhav1-schem-LSJIMP-seg [NR]

The LSJIMP seeks to optimize the primary and multiple images, $\mathbf{m}$, by minimizing the ℓ_2 norm of the data residual, defined as the difference between the recorded data, $\mathbf{d}$, and the modeled data, $\mathbf{d}_{\text{mod}}$ [equation (3)]:

$$\min_{\mathbf{m}} \; \|\mathbf{d} - \mathbf{L}\mathbf{m}\|^2. \tag{5}$$

Minimization (5) is under-determined for many choices of prestack imaging operator, which implies an infinite number of least-squares-optimal solutions. The problem of infinite optimal solutions manifests itself as crosstalk leakage. Of this infinity of possible $\mathbf{m}$'s, we seek the one which not only fits the recorded data, but which also has minimum crosstalk leakage and maximum consistency between signal events on different images. In general we compensate for a correlated or poorly scaled data residual by adding a residual weighting operator, $\mathbf{W_d}$:

$$\min_{\mathbf{m}} \; \|\mathbf{W_d}\left[\mathbf{d} - \mathbf{L}\mathbf{m}\right]\|^2, \tag{6}$$

where strictly speaking,

$$\left(\mathbf{W_d}^T \mathbf{W_d}\right)^{-1} = \text{cov}[\mathbf{d}]. \tag{7}$$

To effect the final step of LSJIMP and penalize crosstalk, we use three regularization operators. As discussed in detail by Brown (2004), the operators penalize roughness in reflection angle and between images, and also penalize the model after weighting with a prior model of the crosstalk on each $\mathbf{m}_{i,k,m}$. For estimation of the optimal set of $\mathbf{m}_{i,k,m}$, we minimize a quadratic objective function which consists of the sum of the weighted ℓ_2 norms of the data residual [equation(6)] and of the three model residuals:

$$\min_{\mathbf{m}} Q(\mathbf{m}) \;=\; \|\mathbf{W_d}\left[\mathbf{L}\mathbf{m} - \mathbf{d}\right]\|^2 \;+\; \epsilon_1^2\|\mathbf{r}_m^{[1]}\|^2 \;+\; \epsilon_2^2\|\mathbf{r}_m^{[2]}\|^2 \;+\; \epsilon_3^2\|\mathbf{r}_m^{[3]}\|^2, \tag{8}$$

where ϵ_1, ϵ_2, and ϵ_3 are scalars that balance the relative weight of the three model residuals (damping factors) with the data residual. These three residuals are calculated by differencing across images, differencing across offset, generating a crosstalk model, and calculating their penalty weights.

IMPLEMENTATION OF LSJIMP

In this section we discuss implementation of LSJIMP on two synthetic shallow water-bottom data sets SYN-1 and HASK.

SYN-1

A synthetic data set, which we call SYN-1, was generated to simulate very shallow and hard water-bottom marine environment. The model has four horizontal layers. The depth of the water bottom is about 150 m. The data-set, shown in Figure 2, is contaminated with first- and higher-order water-bottom peglegs. Brown (2004) discusses kinematic imaging of pegleg

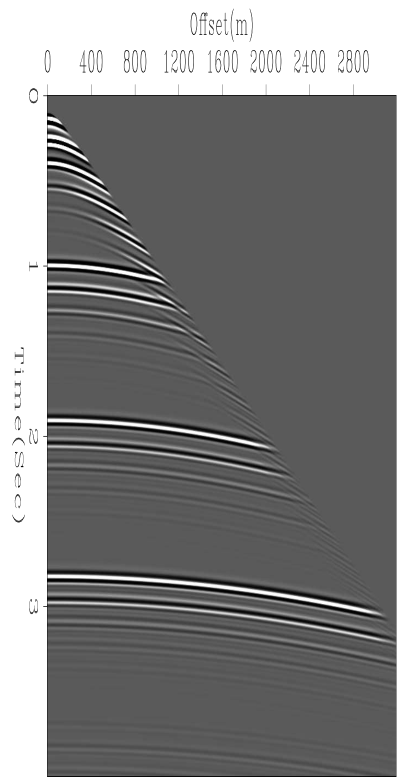

Figure 2: Synthetic data-set (SYN-1) infected with shallow water bottom multiples. madhav1-data [ER]

multiples and applies NMO correction for a j^{th} order pegleg using the following equation:

$$t = \sqrt{(\tau + j\tau^*)^2 + \frac{x^2}{V_{\text{eff}}^2}}, \tag{9}$$

where τ and τ^* are the zero-offset travel-time and travel-time depth of the multiple generating surface respectively and V_{eff} is the effective velocity, which is given by

$$V_{\text{eff}}^2 = \frac{\left(j\tau^* V_{\text{rms}}^2(\tau^*) + \tau V_{\text{rms}}^2(\tau)\right)}{\tau + j\tau^*}. \tag{10}$$

We use these kinematic equations in our NMO operator to construct images from primaries as well as from multiples. When we apply the NMO operator with V_{rms} as the velocity function, primaries get perfectly corrected for the moveout, but multiples have a residual moveout, and they appear as crosstalk. The same holds true for primaries and higher-order multiples when we correct with a V_{eff} appropriate for first-order multiples. Thus, in each image we should ideally be able to distinguish the components (primaries or different order of multiples) we correct for the moveout from others. These equations work fine for most cases, but for SYN-1 we actually evaluate a limiting case of these equations. In this case, the travel-time depth of the water bottom is very small, so for deeper layers we have

$$\tau^* \ll \tau. \tag{11}$$

Using this in equations (9) and (10), we get

$$t \Longrightarrow \sqrt{\tau^2 + \frac{x^2}{V_{\text{eff}}^2}}, \tag{12}$$

$$V_{\text{eff}} \Longrightarrow V_{rms}. \tag{13}$$

When we construct the image from primaries, we use V_{rms} to apply the NMO operator. In this case where $V_{rms} \approx V_{\text{eff}}$, we do not expect water-bottom peglegs to have a residual moveout. This is precisely the reason why in Figure 3 all the primaries are imaged to their perfect positions, but the multiples, which we were expecting to appear as crosstalk, look like perfectly flat reflectors.

Likewise, when we construct images from first- and higher-order multiples, we do not see any residual moveout for primaries and other multiples, except for shallow ones. In an ideal case, we would have a consistent signal across all the images, but the crosstalk would be inconsistent, a fact we could use to enhance signal-to-noise ratio and penalize crosstalk. But for this special case at hand, crosstalk is as consistent as the signal itself. LSJIMP uses three regularization operators as discussed in equation (8); two of these, differencing across offset (designed to penalize events with residual moveout) and differencing between images (designed to penalize inconsistent crosstalk) do not seem to work effectively for SYN-1. Moreover, the crosstalk-modeling operator fails to perfectly model the crosstalk and subsequently

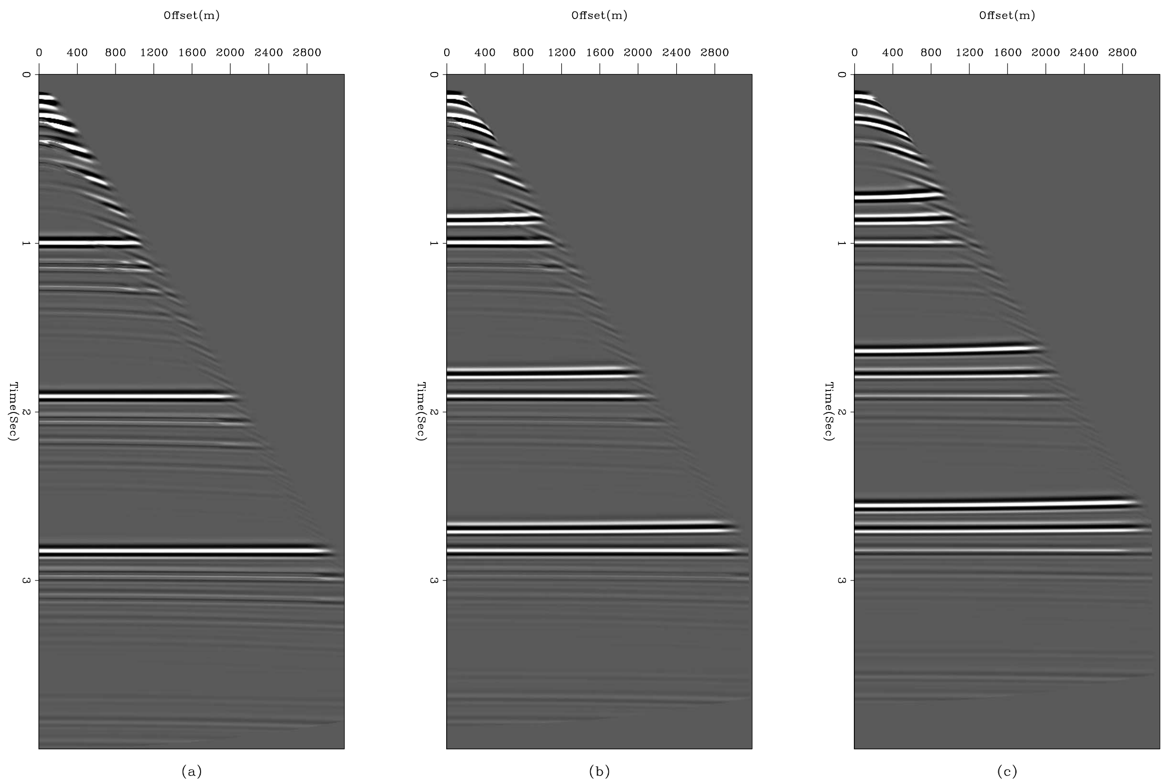

Figure 3: Images constructed from (a) primary, (b) first- and (c) second-order multiples. A strong primary crosstalk is present on images constructed from first- and second-order multiples, moreover, its difficult to distinguish between signal and crosstalk on the basis of residual moveout. $\boxed{\text{madhav1-syn1i}}$ [ER]

penalize the multiple energy. In Figure 3 we can observe, a very strong primary crosstalk in images constructed from first- and second-order multiples.

Thus, the application of LSJIMP to such a data set can indeed image the primaries but cannot eliminate water-bottom peglegs completely, as shown in Figure 4. Since most of the multiple energy present here is in water-bottom peglegs, we can use some simple tricks like predictive deconvolution to suppress this energy. We can also try to model the crosstalk by shifting the dataset by the travel-time depth of water bottom. The results for this exercise are given in Figure 5, though our results have improved a lot, we still have not been able to get rid of all the multiple energy in the shallow part(first-order water-bottom multiple). The primary reason is that most of the zone is muted, and we have few data points in that time range to perfectly model crosstalk.

The first regularizaton operator, $\mathbf{r}_m^{[1]}$ in equation 8, was a result of differencing between images of model panels generated by primaries and different orders of multiples. Ideally the difference between two of them should be small where signal is present and large where crosstalk dominates. Application of such a scheme ensures some degree of smoothness and consistency across images. LSJIMP computes the difference across two consecutive images, for instance between $\mathbf{m}_0$ and $\mathbf{m}_{101}$, $\mathbf{m}_{101}$ and $\mathbf{m}_{111}$ and likewise. Another possible alternative is to compute the difference between a multiple image and the primary image, as that would make all other images consistent with the primary, where we have maximum signal-to-noise ratio. We implemented this approach, but the results as given in Figure 6 are not pleasing. The images we obtained have lots of ringing. One of the main reasons for this was the presence of strong primary crosstalk across all the panels which appears as a spurious event on the primary image after regularization.

Hask Data Set

The "Hask" data set refers to Haskell-Thompson synthetic modeling. The dataset was modeled to resemble a North Sea data set donated by Mobil. We successfully apply LSJIMP to the Hask data set given in Figure 7 for imaging and suppressing the multiple energy. In Figure 8, we compare the raw data to the data generated by applying a forward modeling operator to our image, notice that much of the multiple energy is removed. Next, we compare our present result with results presented by Brown (2002) in Figure 9. It can be observed that now we are doing a better job of multiple suppression in shallow parts. The primary reason being inclusion of crosstalk modeling operator in LSJIMP. The method used by Brown (2002) is equivalent to current method , with ϵ for crosstalk in equation (8) set equal to zero. To improve our performance on the Hask data set, we then took advantage of the fact that hask is also a shallow water-bottom data set and used the improved crosstalk-modeling strategy discussed in the previous section. Unfortunately, the results as given in Figure 10 do not seem to improve a lot.

We also tried a non-linear scheme (Brown, 2004) for updating reflection coefficients between two runs of LSJIMP. The presence of correlated events in the data residual ($\mathbf{r}_d$) hints at the likelihood for further improvements in estimates of the reflection coefficients. The main

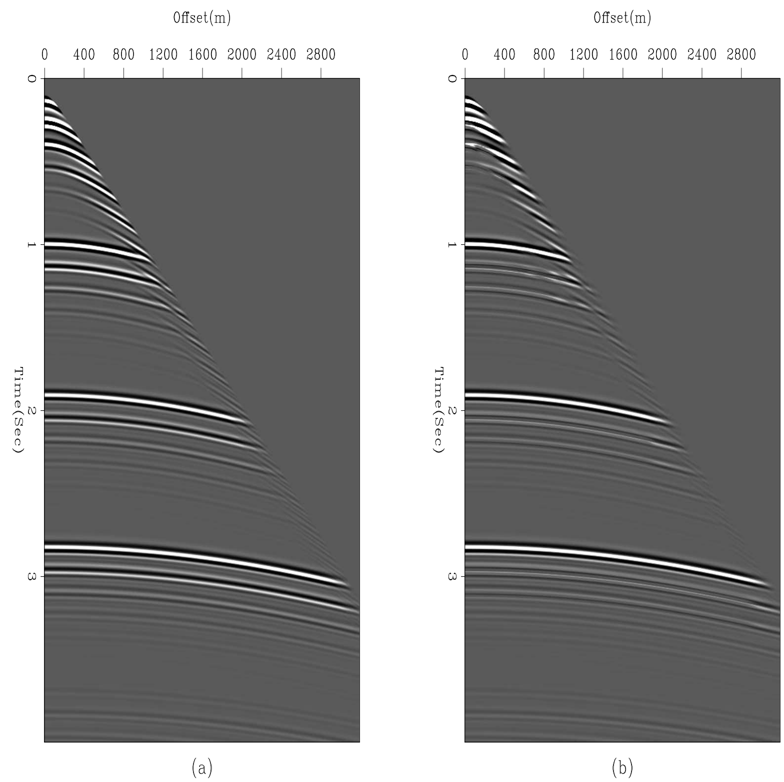

Figure 4: Comparing (a) raw data (SYN-1) and (b) data generated by applying forward-modeling operator on primary image generated by LSJIMP madhav1-syn1c1 [ER]

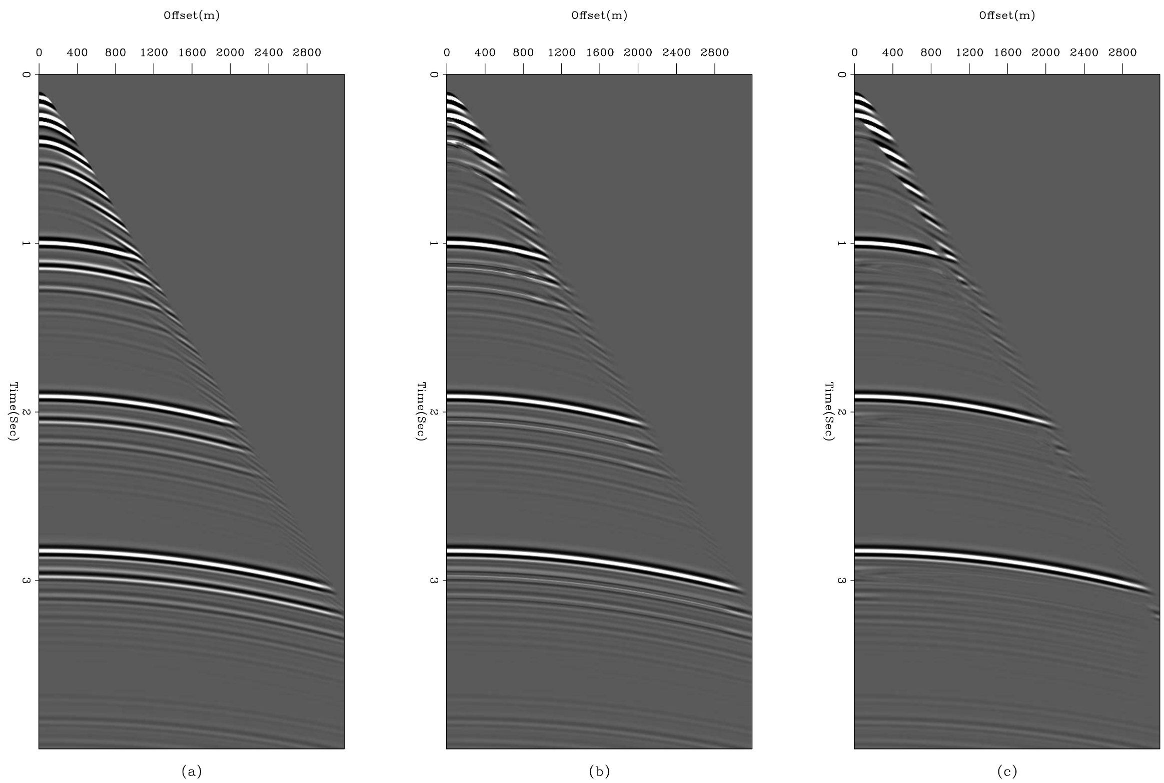

Figure 5: Comparing (a) raw data (SYN-1) and (b) data generated by applying forward-modeling operator on primary image generated by LSJIMP, and (c) LSJIMP with the modified crosstalk model. Notice the improvement in multiple suppression with modified LSJIMP. madhav1-syn1c2 [ER]

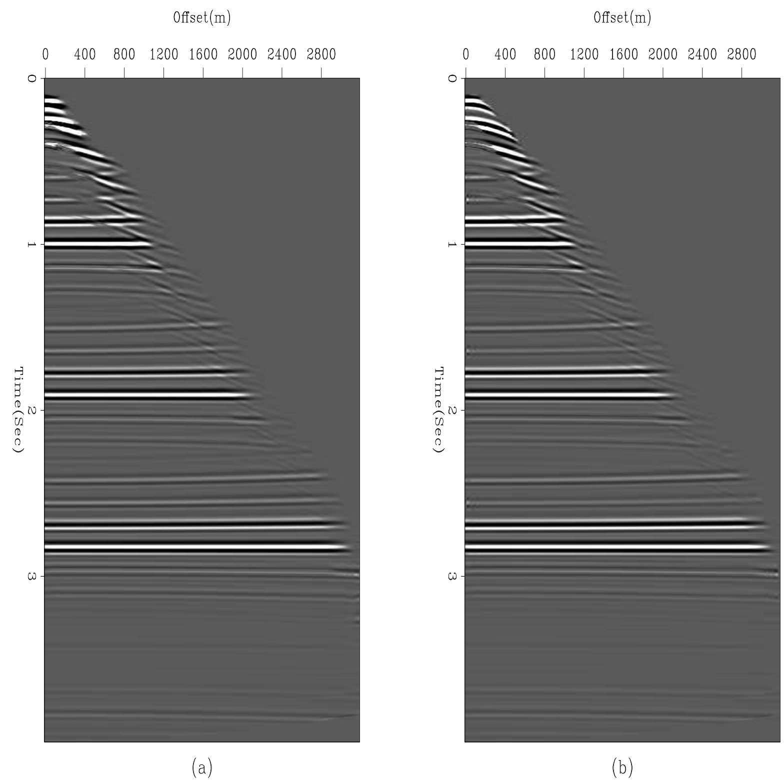

Figure 6: Illustrating (a) primary image and (b) first-order multiple image with a revised regularization scheme. Notice the amount of ringing, and spurious events in primary image. madhav1-regmod [ER]

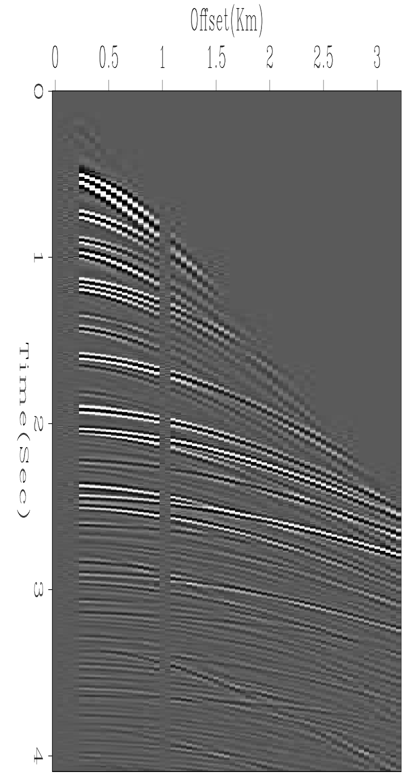

Figure 7: Hask Data Set madhav1-hdata [ER]

idea of the updating scheme is to compute a scalar update to the reflection coefficient of the m^{th} multiple generator, $\Delta\alpha_m$, such that

$$\|\mathbf{r}_d - \Delta\alpha_m \, \mathbf{d}_{i,k,m}\|^2 \tag{14}$$

is minimized. We could not see any noticable difference with non-linear updates, as demonstrated in Figure 10.

CONCLUSIONS AND FUTURE WORK

In this paper we discussed application of LSJIMP on shallow water-bottom data-sets. At first, results did not look promising, but they motivated us to probe into fine details of the method and to come up with improvements that would make it work better. Crosstalk models and regularization operators proposed under LSJIMP seem to break down for extremely shallow water-bottom data sets like SYN-1, as shown through equations (12) and (13). Discriminating between signal and crosstalk on the basis of moveout did not work very well for this case. We also proposed and tested a new scheme for generating crosstalk, which seems to be a slight improvement over the previous one.

As was evident in Figure 3, we were not able to suppress primary crosstalk effectively on multiple images. Derivative between images could suppress it but its convergence was

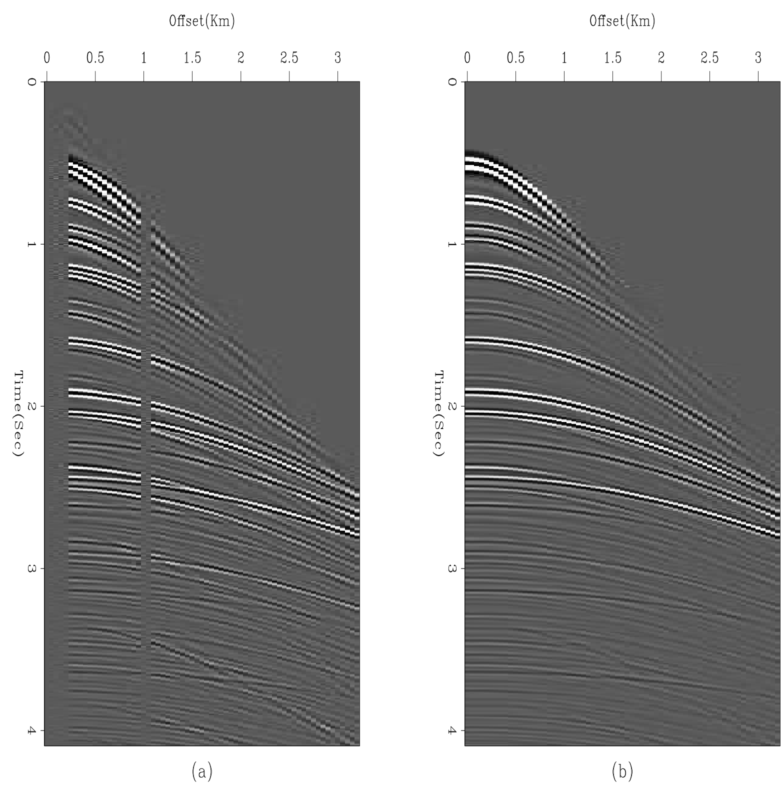

Figure 8: Comparison of (a) raw data and (b) results from first run of LSJIMP. madhav1-hask1 [ER]

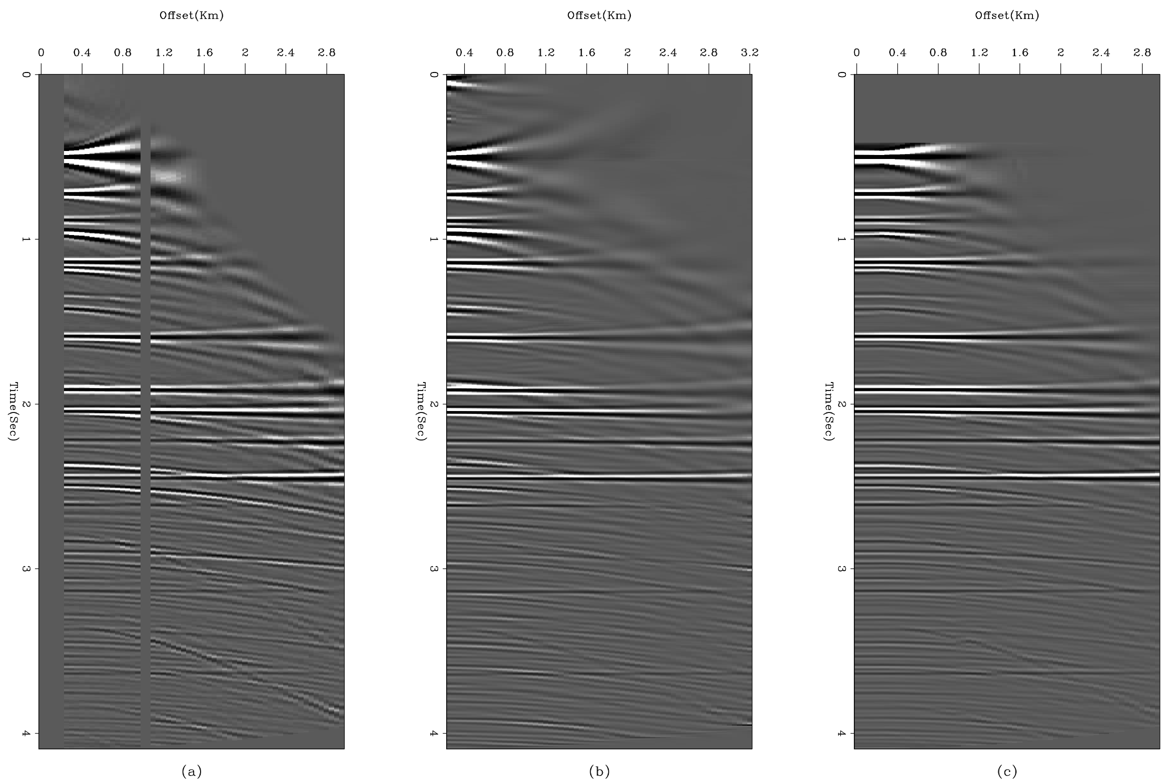

Figure 9: Comparison of (a) raw data, (b) results presented in SEP-111 and (c) present results. Notice that a lot of multiple energy in the shallow parts is eliminated in our present results. madhav1-hask-comp [ER]

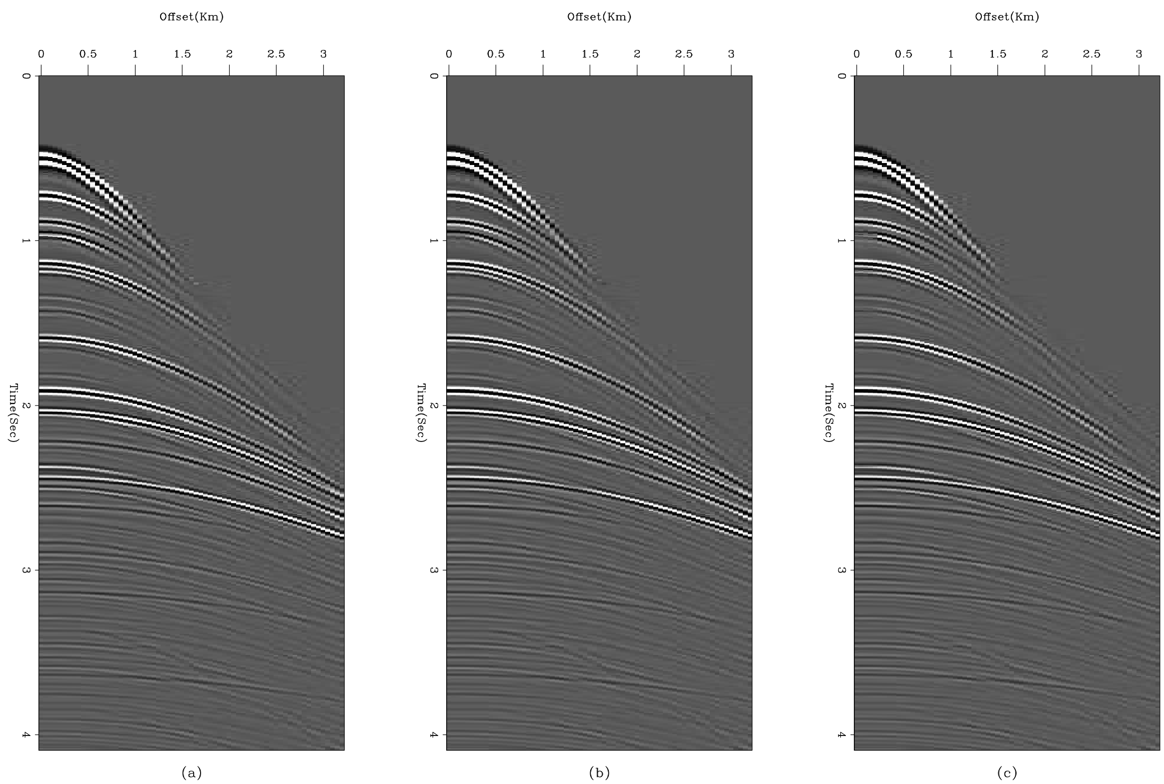

Figure 10: Comparison of results from (a) LSJIMP, (b) LSJIMP(modified) and (c) LSJIMP after non-linear update. madhav1-hask2 [ER]

really slow and our new proposed scheme for difference did not work. Possible reason for slow convergence might be presence of bad eigen value spectrum, that results in appearence of smooth components at the very end. Preconditioning might handle the problem of slow convergence.

There lies a lot of potential in designing appropriate weighting functions and crosstalk-modeling operators that can make the method of Least-Squares Joint Imaging more effective. We would also like to see how our method works if, instead of the NMO operator, we use a migration operator for imaging. This might yield better results.

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