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Seismic waves in rocks with fluids and fractures

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ABSTRACT

Seismic wave propagation through the earth is often strongly affected by the presence of fractures. When these fractures are filled with fluids (oil, gas, water, CO_2, etc.), the type and state of the fluid (liquid or gas) can make a large difference in the response of the seismic waves. This paper will summarize some early work of the author on methods of deconstructing the effects of fractures, and any fluids within these fractures, on seismic wave propagation as observed in reflection seismic data. Methods to be explored here include Thomsen's anisotropy parameters for wave moveout (since fractures often induce elastic anisotropy), and some very convenient fracture parameters introduced by Sayers and Kachanov that permit a relatively simple deconstruction of the elastic behavior in terms of fracture parameters (whenever this is appropriate).

INTRODUCTION

Fractures can play a key role in many reservoirs. Cracks and/or fractures increase rock compliance (reduce stiffness), and thus lower wave speeds. If the distribution in space and/or orientation of fractures present is not isotropic, then significant anisotropy can be observed in seismic data. Aligned vertical fractures in particular are important as they can lead to azimuthal dependence (*i.e.*, so that results depend on the direction in which any linear surface seismic array has been emplaced). Fracture-induced effects are also sensitive to fluids within the fractures. In particular, gas or air will have little effect, while liquids in fractures can stiffen them so much that liquid-saturated fractures are nearly as stiff as the surrounding rock. For partially saturated cracks and fractures, the fracture will be almost as compliant as a gas-filled fracture until 90 to 95% or more of the fracture volume is filled with liquid. Fractures having patchy saturation (separate and distinct pockets of gas and liquid) (White, 1975; Berryman *et al.*, 1988; Endres and Knight, 1989; Mavko and Nolen-Hoeksema, 1994; Dvorkin and Nur, 1998; Johnson, 2001; Berryman *et al.*, 2002a; Berryman, 2004) can also behave differently from any of the other cases mentioned.

Fluids such as oil, gas, water, or CO_2 are often involved in many of the problems of most practical interest. Resolution of various practical and scientific issues in the earth sciences (Wawersik *et al.*, 2001) depends on knowledge of fluid properties underground, and also how the fluids move. In environmental cleanup applications, the contaminants to be removed from the earth are typically liquids such as gasoline or oil, or ground water contaminated with traces of harmful chemicals. In commercial oil and gas exploration, the fluids of interest are hydrocarbons in liquid or gaseous form. In analysis of the earth structure, partially melted rock is key to determining temperature and local changes of structure in the Earth's mantle. In all these cases the tool most commonly used to analyze the fluid content is measurements of seismic (compressional and shear) wave velocities in the earth. Depending on the application, the sources of these waves may be naturally occurring such as earthquakes, or man-made such as reflection seismic surveys at the surface of the earth, or ship-based survey methods over the ocean, vertical seismic profiling from boreholes to surface, or still more direct (but higher frequency) measurements using logging tools in either shallow or deep boreholes.

In many of the cases mentioned a variety of possible explanations for the observed wave velocity and attenuation discrepancies between theory and experiment have been put forward, including viscoelastic effects (velocity decrement due to frequency-dependent attenuation), fluid-enhanced softening of intragranular cementing materials, chemical changes in wet clays that alter mechanical properties, etc. Providing some of the analytical and computational tools needed for treating these difficult problems as well as others for various applications is one of the goals of the work presented here.

A review article by Berryman (1995) summarized the state of the art in effective medium theories as applied to heterogeneous rocks and rock/fluid mixtures. I assume throughout that this material is available to or already known to the reader and will, therefore, not attempt to repeat this review already covered in the AGU Handbook. Then, I can concentrate on more recent developments that are the specific focus of the paper.

ANISOTROPY DUE TO FRACTURES

Prior work on effective medium theory (Berryman, 1995) and double-porosity dual-permeability modeling (Berryman and Wang, 2000; Berryman and Pride, 2002a; Pride and Berryman, 2003a,b) has most often involved calculation of isotropic properties. In almost all cases, it is much harder to estimate anisotropic properties because the first step in such a calculation requires knowledge of both the effects of an oriented inclusion, and knowledge of a distribution (both in space and in orientation) of such inclusions. Then an additional calculation of the overall properties based on microdistribution information is needed. Unfortunately, we will seldom know the microdistribution of the inclusions, and so we are immediately limited in what we can do scientifically along these lines in most cases. However, there is one exception to this that arises in the case of flat cracks in otherwise elastic media. This problem was originally studied in some detail by O'Connell and Budiansky (1976) and Budiansky and O'Connell (1977). They showed in particular that, in the flat crack limit, a single parameter — the crack density ρ — was sufficient to describe the behavior of isotropic systems. This analysis was good for representing the behavior at very low crack densities. In order to arrive at higher crack densities, these authors made use of an older effective medium theory sometimes called "self-consistent", and sometimes more accurately described as "asymmetric self-consistent." This approach had the drawback that it overpredicted the effect of cracks on reducing elastic compliance, and therefore gave a relatively low value $\rho_c \simeq 9/16 \simeq 0.56$ at which the cracked medium would fail. But it is known that failure does not usually occur at such small crack densities, so these overall predictions are often criticized on this basis. [See Henyey and Pomphrey, 1982; Zimmerman, 1991.] Hudson (1980; 1996) used a different method, the so-called "method of smoothing" first introduced in the mathematics literature, for the crack problem. Keeping density corrections just to first order in the Hudson approach gives an improvement over the previously mentioned scheme. Hudson also introduced a second order correction, but Sayers and Kachanov (1991) point out that this approach then violates rigorous Hashin-Shtrikman upper bounds on the moduli for this problem. They recommend instead using a differential scheme [see Zimmerman (1991) for an excellent review of the DS], because the DS tracks Hudson's first order model at low concentration of cracks, but never violates the HS bounds at high concentrations.

Elastic energy and the crack density tensor

Sayers and Kachanov (1991) also introduce a very interesting and useful scheme in the same paper that permits the calculation of constants for anisotropic cracked media from estimates of the behavior (such as that predicted by DS) for the isotropic case. This approach is a tremendous simplification of an otherwise very difficult technical problem. The key idea they use is to introduce an elastic potential energy quadratic in the stress tensor that can be expressed in terms of invariants of the stress tensor in various combinations involving the "crack density tensor." This approach results in a fairly complicated energy potential function involving nine distinct terms. But this function has the advantage that, upon linearization in the crack density, it reduces to only four terms. Two of these terms are the standard ones for the pure (uncracked) medium and the remaining two terms contain the linear contributions due to the cracks. Now it is not obvious that linearization is permissible in the crack density ranges of interest, but Sayers and Kachanov (1995) showed in later work that the remaining contributions from the fourth rank crack-density tensor are always small — and therefore negligible in most situations of practical interest. The neglect of these terms nevertheless implies a certain amount of error in any calculation made based on their neglect, but — if this error is of the size of our measurement error or less — it should not be a serious impediment to studies and analysis of these systems.

To give one example, we find that the corrections to the compliance matrix S_{ij} due to the presence of an isotropic crack distribution take the form:

$$\Delta S_{ij} = \rho \begin{pmatrix} 2(\eta_1+\eta_2)/3 & 2\eta_1/3 & 2\eta_1/3 & & & \\ 2\eta_1/3 & 2(\eta_1+\eta_2)/3 & 2\eta_1/3 & & & \\ 2\eta_1/3 & 2\eta_1/3 & 2(\eta_1+\eta_2)/3 & & & \\ & & & 4\eta_2/3 & & \\ & & & & 4\eta_2/3 & \\ & & & & & 4\eta_2/3 \end{pmatrix}, \quad (1)$$

where η_1 and η_2 are the two coefficients appearing in the Sayers and Kachanov (1991) theory that depend on the presence of cracks, and $\rho = Nr^3/V$ is the crack density (where N/V is the number desnity and r is the radius of the flat cracks when they are penny-shaped as assumed here). These two coefficients can be determined for any crack density by computing the bulk and shear moduli from the compliance matrix $S^*_{ij} = S_{ij} + \Delta S_{ij}$ and comparing the results one-to-one with the results from any effective medium theory one trusts. For these purposes, the differential scheme (DS) is the one that Sayers and Kachanov (1991) recommend, but I have shown elsewhere that another scheme — a symmetric self-consistent scheme that is sometimes called the CPA (for coherent potential approximation) – gives very comparable results. The results can also be compared to rigorous bounds (this work is in progress) and, therefore, also used to obtain rigorous upper bounds on both $|\eta_1|$ and η_2. I have done some initial studies of this type and found that the value of $|\eta_1|$ is generally much smaller in magnitude that of η_2. In particular, $|\eta_1/\eta_2| \leq 0.01$ is typical of the observed results for both DS and CPA.

The real advantage of this approach can now be shown very simply using a couple of examples.

First, consider the situation in which all the cracks in the system have the same vertical (z-)axis of symmetry. Then, the cracked/fractured system is not isotropic, and we have the compliance correction matrix

$$\Delta S_{ij} = \rho \begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1+\eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}. \tag{2}$$

Now it is also not difficult to see that, if the cracks were oriented instead so that all their normals were pointed horizontally along the x-axis, then I would have one permutation of this matrix and, if instead they were all pointed horizontally along the y-axis, then I would have a third permutation of the matrix. If I then want to understand the isotropic correction matrix in (1), I can average these three permutations: just add the three ΔS's together and then divide by three. Having done that, I exactly recover the isotropic compliance corrections matrix displayed in (1). This construction shows in part both the power and the simplicity of the Sayers and Kachanov (1991) approach.

Next, consider the case when all cracks have their normals lying randomly in parallel planes. Then, if the parallel planes are taken to be horizontal, the cracks are all vertically aligned as in Figure 1. So, I immediately find the anisotropic (*i.e.*, vertical transverse isotropy or VTI) result that

$$\Delta S_{ij} = \rho \begin{pmatrix} (\eta_1+\eta_2) & \eta_1 & \eta_1/2 & & & \\ \eta_1 & (\eta_1+\eta_2) & \eta_1/2 & & & \\ \eta_1/2 & \eta_1/2 & 0 & & & \\ & & & \eta_2 & & \\ & & & & \eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}. \tag{3}$$

The reader should check that adding two-thirds of (3) to one-third of (2) recovers (1), since this combination also represents an isotropic ensemble of fractures.

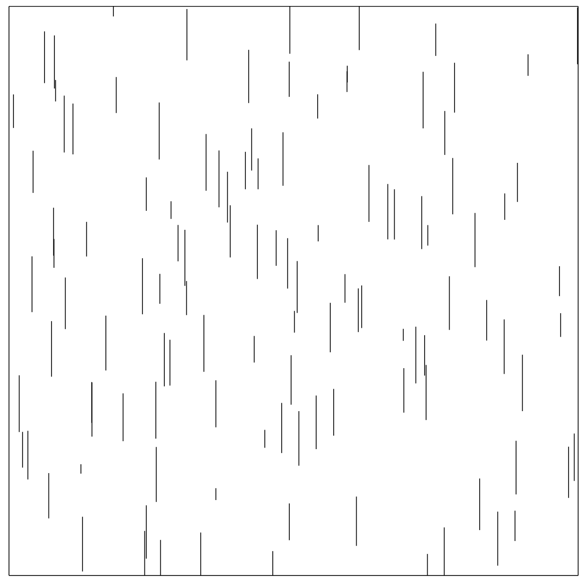

Figure 1: Example of a vertical cross-section (xz-plane) through a medium having penny-shaped cracks with radius $r/L = 0.05$, where $L = 1.0$ is the length on each side of a cube in 3D. This image was produced by randomly placing 2000 crack centers in the box of volume $= L^3$ (so crack density $\rho = 0.25$), and testing to see if the center is within a distance $r \leq 0.05$ of the central square at $y = 0.5$. If so, then a random angle is chosen for the crack. If this crack orientation results in an intersection with the plane $y = 0.5$, the line of intersection is plotted here. The resulting lines can have any length from $2r = 0.1$ to zero. The number of intersections found for this realization was 114, whereas the expected value for any particular realization is approximately $(2/\pi) \times 200 \simeq 127$. jim1-ranpc2vert [NR]

This same basic concept then works very well for any assumed symmetry that we might like to model. There is no additional work to be done once (*i*) the isotropic results are known (for some EMT) and (*ii*) the layout of the two η's in the correction matrix ΔS have been determined once and for all for a given elastic symmetry resulting from a specific choice of crack orientation distribution. Sayers and Kachanov (1991) give a precise prescription for this. Although I make use of this prescription here, I will not show the details in order to avoid some of the mathematical complications inherent in their tensorial expressions.

There are interesting and important questions of uniqueness related to the inverse problem (*i.e.*, deducing the η's from seismic wave observations) since more than one type of distribution can give rise, for example, to vertical transverse isotropy (VTI). Then, the question is whether quantities such as the Thomsen parameters of anisotropy can help us to remove some of these possible ambiguities from the interpretations of field measurements.

Thomsen parameters

If we have the compliance correction matrix ΔS_{ij}, then we can quickly find expressions for the Thomsen seismic wave parameters for weak anisotropy (Thomsen, 1986; 1995; 2002; Rathore *et al.*, 1995; Rüger, 1998; 2002; Grechka, 2005). Clearly, a weak anisotropy assumption is also consistent with the small crack density assumption that was needed above to justify the use of the Sayers and Kachanov (1991) method.

There are three Thomsen parameters: γ, ϵ, and δ. Parameter γ is essentially the fractional difference between the SH-wave velocities in the horizontal and vertical directions for a VTI medium. Similarly, parameter ϵ is essentially the difference between the P-wave velocities in the horizontal and vertical directions. Parameter δ is more difficult to interpret, but contributes in an essential way both to near vertical P-wave speed variations, and also to the angular dependence of the SV-wave speed. There are a great many steps that go into Thomsen parameter calculations since the crack density effects are most conveniently expressed in terms of the compliance matrix while the Thomsen parameters are usually defined instead in terms of the stiffness matrix (inverse of the compliance matrix). I will not show my work here, but merely quote the final result for the case of randomly oriented vertical fractures considered in the previous subsection.

For present purposes, I just want to show in a quick way how this method works, so I will concentrate on the easiest two parameters which are γ and ϵ. For these two parameters, I have the following results:

$$\gamma = \frac{c_{66} - c_{44}}{2c_{44}} = -\eta_2\rho\frac{E}{4(1+\nu)} = -\eta_2\rho\frac{G}{2}, \tag{4}$$

and

$$\epsilon = \frac{c_{11} - c_{33}}{2c_{33}} = -[(1+\nu)\eta_1 + \eta_2]\rho\frac{E}{2(1-\nu^2)} \simeq -\eta_2\rho\frac{G}{1-\nu}, \tag{5}$$

where $\nu = (K - 2G/3)/2(K + G/3)$ is Poisson's ratio, E is related to the host medium's bulk (K) and shear (G) moduli by $1/E = 1/9K + 1/3G$, and $G = E/2(1+\nu)$. In the second expression of (5), I have neglected the term proportional to $\eta_1\rho$ as this term is normally very small (on the order of 1% of the term retained). It can also be shown that for this model the remaining Thomsen parameter δ takes exactly the same value as ϵ to lowest order in the crack density parameter.

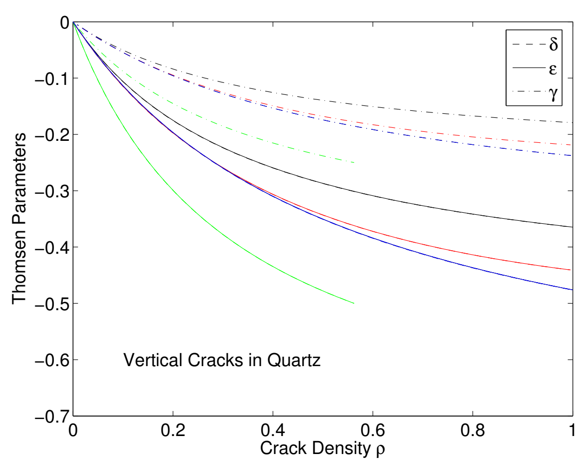

Figure 2: Computed values of the Thomsen parameters δ, ϵ, γ, for four distinct EMT models: noninteracting (black), CPA (red), DS (blue) and the Budiansky-O'Connell self consistent (green). The parameter δ is not seen separately here because for this choice of crack microstructure (randomly oriented vertical cracks) $\delta = \epsilon$ to the order to which we are working, for small crack densities. jim1-thomsenALLv [NR]

Examples of Thomsen's parameters for various choices of EMT are displayed in Figure 2. The results illustrate how estimates of η_1 and η_2 obtained from four different isotropic estimation schemes [noninteracting, DS (Zimmerman, 1991), CPA (Berryman, 1980), and nonsymmetric self-consistent scheme of Budiansky and O'Connell (1976), and O'Connell and Budiansky (1977)] can then be used to predict what values Thomsen's parameters should take in field data.

Some judgment is required then as to the most appropriate EMT to use, and prior work shows that some knowledge of microstructure can serve as a very useful guide when making this choice (Berge *et al.*, 1993a; 1995).

Rayleigh wave speed

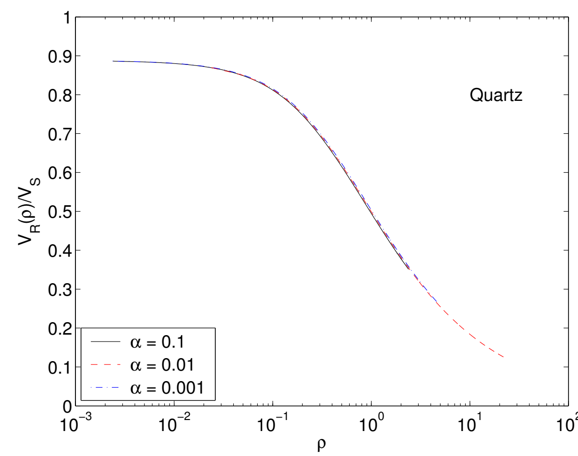

Figure 3: Computation of Rayleigh wave speed in quartz with horizontally aligned cracks for three choices of penny-crack aspect ratio, and a range of values of the crack density ρ. If the assumption/prediction that aspect ratio is does not really matter, just crack density, then all three curves should overlap, as they do here at the lower crack densities. jim1-Qrtzsemilogvrvs3b_rho [NR]

Now, to provide one simple illustration of the use of what has been presented so far, consider the well-known formula for the Rayleigh wave speed $v = v_R$ in an isotropic medium [see Ewing *et al.* (1957), Al-Husseini *et al.* (1981), Weertman (1996)]:

$$\beta_S \beta_P = \beta_{2S}^4, \tag{6}$$

where $\beta_S = \sqrt{1 - v^2/v_s^2}$, $\beta_P = \sqrt{1 - v^2/v_p^2}$, $\beta_{2S} = \sqrt{1 - v^2/2v_s^2}$, with v_s being the (isotropic) shear wave speed and v_p being the (isotropic) compressional wave speed in the host medium. For an anisotropic medium having the same transversely isotropic (VTI) symmetry that I have been considering (for the case of randomly oriented vertical cracks), Musgrave (1959) shows that the equivalent result for the Rayleigh wave speed $v = v_R$ in the plane perpendicular to the VTI axis of symmetry is determined by the cubic equation

$$\frac{1}{16}q^3 - \frac{1}{2}q^2 + \left(\frac{3}{2} - \frac{c_{66}}{c_{11}}\right)q + \left(\frac{c_{66}}{c_{11}} - 1\right) = 0, \tag{7}$$

where $q \equiv \rho_0 v^2/c_{66}$ and ρ_0 is the density of the medium (which is assumed to be the same as that of the pure material without cracks, since the cracks are very flat and are not introducing any significant amount of porosity). [It is not difficult to check that (6) and (7) are equivalent when the elastic medium is isotropic.]

From the definitions of γ and ϵ, it is now straightforward to see that

$$c_{66} = c_{44}(1+2\gamma) \tag{8}$$

and

$$c_{11} = c_{33}(1+2\epsilon). \tag{9}$$

Shear modulus $c_{44} = G/(1+\eta_2 G\rho)$, while c_{33} is found by inverting $S_{ij} + \Delta S_{ij}$ for the 33 component of the stiffness matrix. So we can now easily compute the Rayleigh wave speed by solving the cubic equation (7). Some results of this type are displayed in Figure 3. In particular, we find that the crack density is indeed a good parameter to use, as all these plots for different choices of crack aspect ratio clearly overlap to numerical accuracy in the low crack density range.

GASSMANN'S EQUATIONS AND FRACTURED MEDIA

It is also important to make a connection between the fracture results quoted above and Gassmann's results (Gassmann, 1951; Berryman, 1999) for fluid saturated and undrained porous media. Even very flat cracks could harbor some fluid at various times and the question is how this fluid affects the response of the cracks.

Recall first that Gassmann's result on the effect of fluids in a porous medium in anisotropic media can be expressed by using a compliance correction matrix of the form:

$$\Delta S_{ij} = -\gamma^{-1}\begin{pmatrix} \beta_1^2 & \beta_1\beta_2 & \beta_1\beta_3 & & & \\ \beta_1\beta_2 & \beta_2^2 & \beta_2\beta_3 & & & \\ \beta_1\beta_3 & \beta_2\beta_3 & \beta_3^2 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}, \tag{10}$$

where the fluid bulk modulus appears only in the factor γ, and the coefficients β_1,β_2,β_3 satisfy a sumrule of the form $\sum_{i=1}^3 \beta_i = \alpha/K_d = 1/K_d - 1/K_m$, and $\alpha = 1 - K_d/K_m$ is the Biot-Willis coefficient (Biot and Willis, 1957). The bulk moduli K_d and K_m are, respectively, the drained (porous) bulk modulus of the overall system and the mineral modulus whenever there is only one mineral present in the system (as I will assume here for the present). [The scalar drained modulus K_d for an anisotropic system is identical to the Reuss average for the bulk modulus of the compliance matrix.] When the system is responding anisotropically as in the case of a set of cracks having vertical symmetry axis in the example (3), we can easily make (10) compatible with the structure of (3) by first ignoring the coefficient η_1 (which is known to be quite small anyway), and then setting $\beta_1 = \beta_2 = 0$. No other possibiities are available. This means that Gassmann's results are introduced into the anisotropic problem by making a fluid-dependent perturbation to $\eta_2\rho$ and ignoring $\eta_1\rho$, since its value is two orders of magnitude smaller.

Gassmann's formula for an isotropic medium can be written in the form:

$$K_u = K_d + \frac{\alpha^2}{(\alpha - \phi)/K_m + \phi/K_f}, \tag{11}$$

where K_u is the undrained bulk modulus, K_d is the drained bulk modulus, K_m is the mineral or solid modulus, K_f is the bulk modulus of the pore fluid, ϕ is the porosity and $\alpha = 1 - K_d/K_m$. This result can be rearranged in order to express it in terms of compliances, instead of stiffnesses, as

$$\frac{1}{K_u} - \frac{1}{K_d} = -\frac{\alpha}{K_d}\left[1 + \frac{K_d\phi}{K_f\alpha}\left(1 - \frac{K_f}{K_m}\right)\right]^{-1}. \tag{12}$$

Now for fractured media having no other porosity ϕ except the fractures themselves, I have

$$\phi = \frac{4\pi}{3}b^3\left(\frac{a}{b}\right)\frac{N}{V} = \frac{4\pi}{3}\left(\frac{a}{b}\right)\rho, \tag{13}$$

where the crack density is $\rho = Nb^3/V$, N/V is the number of cracks per unit volume, b is the radius of the (assumed) penny-shaped crack, and a/b is its aspect ratio. Substituting (13) into (12) shows that crack density ρ is always multiplied by the factor $(1 - K_f/K_m)$ in Gassmann's formula, and this result thereby provides a convenient means of introducing the fluid effects into the formulas for compliance in the presence of distributions of cracks.

The preceding review of Gassmann's original derivation shows that it is not appropriate to replace all occurrences of ρ in (1) by $\rho(1 - K_f/K_m)$. Only those terms that determine the strain response to the principle stresses need to be considered. Furthermore, the analysis of the symmetry conditions has shown that only those terms involving η_2 need to be modified. If we neglect η_1 for the remainder of this argument (it is small anyway — a 1% effect — as stated previously), then we find that, for an isotropic system,

$$\Delta S_{ij} \simeq \frac{2\eta_2\rho}{3}\begin{pmatrix} (1-K_f/K_m) & 0 & 0 & & & \\ 0 & (1-K_f/K_m) & 0 & & & \\ 0 & 0 & (1-K_f/K_m) & & & \\ & & & 2 & & \\ & & & & 2 & \\ & & & & & 2 \end{pmatrix}. \tag{14}$$

Changes in the pertinent compliances are therefore given by

$$\Delta\left(\frac{1}{K_u}\right) = 2\eta_2\rho(1 - K_f/K_m) \tag{15}$$

for the undrained bulk modulus K_u, and

$$\Delta\left(\frac{1}{G^r_{eff}}\right) = 4\eta_2\rho(1 - K_f/K_m)/3, \tag{16}$$

which is the change in the uniaxial shear modulus due the presence of cracks containing fluids. Then, I recover the result of Mavko and Jizba (1991) easily by noting first that the change in the undrained shear modulus for the isotropic system is

$$\Delta\left(\frac{1}{G_u}\right) = \frac{2}{5}\Delta\left(\frac{1}{G^r_{eff}}\right) + \frac{3}{5}\times 0, \tag{17}$$

since there are two equal shear contributions from the upper left hand 3×3 submatrix of (14), while the three remaining contributions to shear compliance exhibit no fluid dependence and so do not contribute here. Then, finally, I have

$$\Delta\left(\frac{1}{G_u}\right) = \frac{4}{15}\Delta\left(\frac{1}{K_u}\right), \tag{18}$$

in agreement with previous results of this type (Berryman *et al.*, 2002b). So this formalism provides an efficient means of correctly deriving both old and new results.

Berryman *et al.* (2002b) show that the factor $\frac{4}{15}$ in (18) holds strictly only for very flat cracks, and that the appropriate factor in other situations can be either higher or lower than $\frac{4}{15}$, depending on details. The neglected terms depending on the Sayers and Kachanov parameter η_1 provide very small corrections to the drained moduli, but actually have no dependence on fluid saturation, and so have no influence on the relationship between undrained (fluid saturated) moduli shown in (18).

DECONSTRUCTION

The ultimate goal of the work presented has been to enable some approaches to the problem of characterizing reservoirs, especially reservoirs containing fractures, using seismic data. This idea is obviously not a new one (Lynn *et al.*, 1995). But some of the consequences of the Sayers and Kachanov (1991) method need more detailed exploration and explication, and I will provide some of that discussion in the following two examples. I first treat a fairly typical reflection seismic example, and then show how to use similar ideas in a different way for very shallow imaging and characterization.

Reflection seismic example

Assume for the sake of argument that all three Thomsen parameters, γ, ϵ, and δ, have been determined for a given reservoir and that the reservoir exhibits VTI characteristics. If the reservoir does not exhibit VTI symmetry, then I might need to consider HTI (horizontal transverse isotropy) or some still more complicated type of anisotropy. But, for VTI, we need to know something about the variety of behaviors that are possible in the presence of fractures. Equations (4) and (5) show the results expected if the fractures are vertical and randomly oriented. But there are obviously other possibilities as well, and to have a better chance of making a valid interpretation of the observed behavior, we need to know more about the range of possibilities for the Thomsen parameters. I will not attempt to be exhaustive here, but just present one other result that can clearly be distinguished by such data.

Consider the case of horizontal fractures. Then, the axis of symmetry is vertical, and so the reservoir would exhibit VTI symmetry again, just as in the case of vertical fractures randomly oriented. But the resulting expressions for the Thomsen parameters in terms of the Sayers and Kachanov (1991) parameters are quite different.

I find

$$\gamma = \frac{c_{66} - c_{44}}{2c_{44}} = \eta_2 \rho G, \tag{19}$$

and

$$\epsilon = \frac{c_{11} - c_{33}}{2c_{33}} = [(1+\nu)\eta_1 + \eta_2]\rho \frac{E}{(1-\nu^2)} \simeq \eta_2 \rho \frac{2G}{1-\nu}. \tag{20}$$

The background shear modulus is G, and corresponding Poisson ratio is ν. Again, I find that $\delta = \epsilon$ to the lowest order in the crack density parameter. Also, I have neglected the term in η_1 in the final expression as this is on the order 1% of the term retained. So I find that the magnitude of the coefficients in this case differs by a factor of 2 from those of randomly oriented vertical fractures as in Eqs. (4) and (5). But more importantly, the sign of these expressions is opposite that of this other case.

Equations (19) and (20) could have been found in a very simple way from the results of previous sections by using the following argument: We know that an isotropic distribution of fractures is represented in the Sayers and Kachanov (1991) formalism by the correction matrix (1) and, furthermore, that (1) is also the weighted sum of (2) and (3) — specifically, $2/3\times$ Eq. (3) plus $1/3\times$ Eq. (2). So the similarly weighted averages of the individual Thomsen parameters for these two cases must add up to zero (since Thomsen's parameters vanish for isotropic media). In fact, this is exactly what we have found to be true. We could have used this fact to provide a quicker (and much more elegant) derivation of (19) and (20) than the method I actually used. Of course, the utility of this type of argument is limited to the linear contributions of crack density to the Thomsen parameters that I have been considering here.

In both examples, the Thomsen parameter measurements may be used to estimate the magnitude of the $\eta_2\rho$ product assuming the background shear modulus G and the background Poisson ratio ν are known, or can be estimated. But, horizontally fractured systems can also be easily distinguished from vertically fractured systems, since the sign of the constants is opposite in these two scenarios.

It would be helpful for interpretation purposes to enumerate other related scenarios that could be distinguished from these two by using the anisotropy parameter data. I will leave such problems, especially those involving azimuthal dependence (and therefore not VTI), to future work. There is no fundamental problem with computing the relations between the Thomsen parameters and the Sayers and Kachanov parameters for arbitrarily complicated choices of fractured reservoir scenarios. All can be studied, but making good choices about which of the necessarily limited number of scenarios time permits us to consider are also the most fruitful ones will be one of the key steps in the process.

Shallow example

The preceding example assumed that typical reflection seismic data collection could be performed at the site of interest. But suppose instead that the region of interest is quite shallow, possibly very soft and/or compliant sediments or soils, and that, in particular, it is not possible to obtain shear wave data directly. Then what can be done?

One of the most common problems with traditional compressional wave surveys is ground roll. Ground roll is typically composed of Rayleigh and/or Love waves, and usually the Rayleigh wave component is the one we need to eliminate because it is contaminating the P-wave data near the shot point. The Rayleigh wave speed depends on both the compressional and shear wave speeds of the medium, and – being a surface wave – it is most strongly influenced by the topmost layers of the earth (usually those within about one wavelength from the surface). So for shallow imaging and analysis, why not consider using Rayleigh wave speed measurements together with P-wave speed measurements to infer the S-wave speed.? The pertinent S-wave speed in an anisotropic (VTI) medium is the shear wave speed in the symmetry plane (perpendicular to the axis of symmetry). So the formula shown previously (7) is pertinent, but it needs to be used in a different way to find the shear wave speed $v_s = \sqrt{c_{66}/\rho_0}$, when $v_p = \sqrt{c_{11}/\rho_0}$ and v_R are known.

To accomplish this goal, I first square (6). The result is a quartic equation for $q = (v_R/v_s)^2$. In this case, v_R is known, but v_s is unknown (opposite of the earlier case). But this difference does not cause any difficulty in the analysis. The equation can be rearranged into the form:

$$\frac{1}{16}q^4 - \frac{1}{2}q^3 + \frac{3}{2}q^2 - \left(1 + \frac{v_R^2}{v_p^2}\right) q + \frac{v_R^2}{v_p^2} = 0. \tag{21}$$

Equation (21) is straightforward to solve by iteration using a simple Newton-Raphson scheme (Hildebrand, 1956; Press *et al.*, 1988). Generally a good starting value for the scheme will be $q \simeq 0.8$ as this corresponds roughly to a trial value of $v_R = 0.9v_s$.

Having once determined the value of $v_s = \sqrt{c_{11}/\rho_0}$ — using the measured Rayleigh wave speed and the compressionial wave speed v_p — in the symmetry plane, Thomsen parameter analysis can be combined with the Sayers and Kachanov (1991) method in order to deduce useful information about the nature of the heterogeneities causing the anisotropic at the macroscale. Once these wave speeds are known, the analysis for interpretation can proceed in essentially the same manner as in the previous example.

TABLE 1. Examples of Sayers and Kachanov (1991) parameters $\eta_1(\rho)$ and $\eta_2(\rho)$ when crack density $\rho << 1$ for penny-shaped cracks. Four choices of effective medium theory are considered: NI (non-inteacting), DS, (differential scheme), CPA (coherent potential approximation), and SC (the Budiansky and O'Connell self-consistent scheme). Note that crack density is defined here as $\rho = Nr^3/V$, where N/V is number density of cracks, and $A = \pi r^2$ is the area of the circular crack face.

	η_1 (GPa^{-1})	η_2 (GPa^{-1})
NI	-0.000216	0.0287
DS	-0.000216	0.0290
CPA	-0.000258	0.0290
SC	-0.0000207	0.0290

DISCUSSION AND CONCLUSIONS

Sayers and Kachanov (1991) introduced a convenient method of analyzing fractured (but otherwise) elastic systems. I showed here that their method can be successfully generalized to fluid-saturated fractures. Furthermore, when their method is used in conjunction with Thomsen's anisotropy parameters (Thomsen, 1986), we find not only analytical results that aid our intuition about these complex problems, but also a means to deconstruct velocity data and then to interpret the nature (approximate crack density) of fractures in the system being studied. The magnitudes of the parameters η_1 and η_2 can be determined in a straightforward way using any effective medium theory we trust (Kachanov, 1994; Prat and Bažant, 1997; Grechka, 2005); and also this calculation can be done just for the isotropic (and, therefore, the simplest) case. For examples, see TABLE 1. These parameter values do not change. Only the crack density parameter, the crack orientation distributions, and possibly the crack shapes, etc., change. For very dilute fracture systems, any of the standard effective medium theories will actually produce virtually the same values of the parameters η_1 and η_2. The only variable is the crack shape, which I have assumed here (as is most commonly done) to be penny-shaped cracks with small aspect ratios. Values of η_1 and η_2 can vary with changes in the assumed microstructure (*i.e.*, other choices of crack shapes), but values could be tabulated once and for all for the low density limit with any choices of crack shape we might ever want to consider and then the numbers would be universally available. Users would not need to be experts in effective medium theory to make use of these results — although they would, of course, still need to be experts in the interpretation of seismic data and, in particular, of the Thomsen parameters themselves.

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