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Residual moveout of 2D multiples in Angle-Domain Common-Image Gathers

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ABSTRACT

I show that, for specularly-reflected multiples, the constant velocity straight-ray approximation of the residual moveout in Angle-Domain Common-Image Gathers (ADCIGs) is only appropriate for small aperture angles. The approximation is good for the primaries because the difference between the migration velocity and the true velocity is likely to be small. For the multiples, however, this difference may be large and correcting for ray bending produces a better approximation that leads to better focusing of the multiples in the Radon domain. This in turn allows a more accurate muting of the multiples. I show results with two ADCIGs, one synthetic and one real.

INTRODUCTION

When primary reflections are depth migrated with the exact velocity of the medium, their moveout in Angle-Domain Common-Image Gathers (ADCIGs) is flat (Biondi, 2005). When they are migrated with the wrong velocity, their residual moveout in ADCIGs can be approximated, to first order, by the equations given in Biondi and Symes (2004). For a flat reflector, their approximation reduces the residual moveout of the primaries as a function of aperture angle, to a tangent squared.

Specularly-reflected multiples, when migrated with the velocity of the primaries, behave as primaries migrated with too slow velocity (Alvarez, 2005). The tangent-squared approximation can be used to design a Radon transform that focuses the energy of the primaries and the multiples in and ADCIG according to their residual curvature and so can be used to attenuate the multiples in image space (Sava and Guitton, 2003). This approximation is robust enough that it can even be used to approximate the residual moveout of diffracted multiples, provided that another dimension is added to the Radon transform to account for the shift of the apex of these multiples (Alvarez et al., 2004).

Here I show that the approximation of Alvarez (2005) for the residual moveout of the multiples is better than the straight-ray approximation, because it takes into account the non-negligible ray bending of the multiples at the water-bottom interface and by extension any interface in which the velocity of propagation of the primaries and the multiples is substantial, for example at a salt boundary. I show, with both a synthetic and a real ADCIG that the new approximation is more accurate and that it focuses better the multiples in the Radon domain. I then show that this results in an improvement of the estimation of the multiples and therefore in their attenuation.

The first section briefly reviews the theory and shows a comparison of the two residual moveout curves for a given ADCIG. The next section compares both approximations in the Radon domain and the following section compares the results of attenuating the multiples with both approximations on a synthetic and a real ADCIG.

THEORY OVERVIEW

The residual moveout of primaries in ADCIGs, under the approximation of stationarity of the rays (local constant velocity) is given by (Biondi and Symes, 2004):

$$\Delta\mathbf{n}_{\mathrm{RMO}} = \frac{\rho-1}{\rho}\frac{\sin^2\gamma}{(\cos\alpha-\sin^2\gamma)\cos\alpha}\bar{z}\mathbf{n}, \tag{1}$$

where $\Delta\mathbf{n}_{\mathrm{RMO}}$ is the residual moveout function with respect to the aperture angle γ, ρ is the ration between the migration and the true slowness, α is the reflector dip, $\bar{z}$ is the true (unknown) depth of the reflector and $\mathbf{n}$ is the unit normal vector to the reflector in the direction of decreasing depth. For a flat reflector ($\alpha = 0$) equation 1 reduces to

$$\Delta\mathbf{n}_{\mathrm{RMO}} = \frac{\rho-1}{\rho}\tan^2\gamma\,\bar{z}\mathbf{n}. \tag{2}$$

For primaries, we can estimate the true depth $\bar{z}$ using the migration depth at normal incidence z_0 (Biondi and Symes, 2004) as

$$z_0 = \frac{\bar{z}}{\rho} \tag{3}$$

which leads to the simple result

$$\Delta\mathbf{n}_{\mathrm{RMO}} = (\rho-1)\tan^2\gamma\, z_0\mathbf{n}. \tag{4}$$

For specularly-reflected multiples, Alvarez (2005) showed that, for a flat reflector, the functional dependence between the image depth and the aperture angle is given by

$$z_{\xi_\gamma} = \frac{z_{\xi_\gamma}(0)}{1+\rho)}\left[1+\frac{\cos\gamma(\rho^2-(1-\rho^2)\tan^2\gamma)}{\sqrt{\rho^2-\sin^2\gamma}}\right], \tag{5}$$

where $z_{\xi_\gamma}(0)$ is the normal-incidence migrated-depth, (*i.e.* z_0) in the previous equations. There is an important and unfortunate difference in notation, however, because ρ in equations 1 through 4 is the ratio of the migration to the true *slowness* whereas ρ in equation 5 is the ratio of the migration to the true *velocity*. Therefore, in order to get a better idea of how the approximation for the RMO of the multiples (accounting for ray bending at the reflector interface) relates to that of the primaries (neglecting ray bending), I rewrite equation 5 replacing ρ by $1/\rho$ and $z_{\xi_\gamma}(0)$ with z_0 to get:

$$z_{\xi_\gamma} = \left[1 + \frac{\cos\gamma(\rho^2 - (\rho^2 - 1)\tan^2\gamma)}{\sqrt{1-\rho^2\sin^2\gamma}} \right] \frac{z_0}{1+\rho}. \tag{6}$$

Finally, since $\Delta\mathbf{n}_{\text{RMO}} = z_0 - z_{\xi_\gamma}$ we get:

$$\Delta\mathbf{n}_{\text{RMO}} = \left[1 - \frac{\cos\gamma(1 - (\rho^2 - 1)\tan^2\gamma}{\sqrt{1-\rho^2\sin^2\gamma}} \right] \frac{z_0}{1+\rho}\mathbf{n}, \tag{7}$$

which, for small γ, reduces to

$$\Delta\mathbf{n}_{\text{RMO}} = (\rho^2 - 1)\tan^2\gamma \frac{z_0}{1+\rho} = (\rho - 1)\tan^2\gamma z_0\mathbf{n}. \tag{8}$$

This is the same as equation 4. This result is intuitively appealing because it shows that the approximation of neglecting the ray bending at the reflecting interface deteriorates as the aperture angle increases which is when the ray bending is larger.

Figure 1 shows a comparison of the residual moveout curves for an ADCIG computed with equations 7 (ray-bendinga pproximation) and 8 (straight-ray approximation). The residual moveouts correspond to a water-bottom multiple from the flat interface of a two layer model where the top layer is water and the second layer is a half space. The velocity of the water layer is 1500 m/s and its thickness is 500 m. The velocity of the half space is 2500 m/s. The migration was done with the true velocity model. Therefore, there is significant ray bending of the multiple at the reflecting interface. Figure 2 shows the actual ADCIG with the depth moveout as a function of angle superimposed for both approximations. For large aperture angles the departure of the straight ray approximation can be significant.

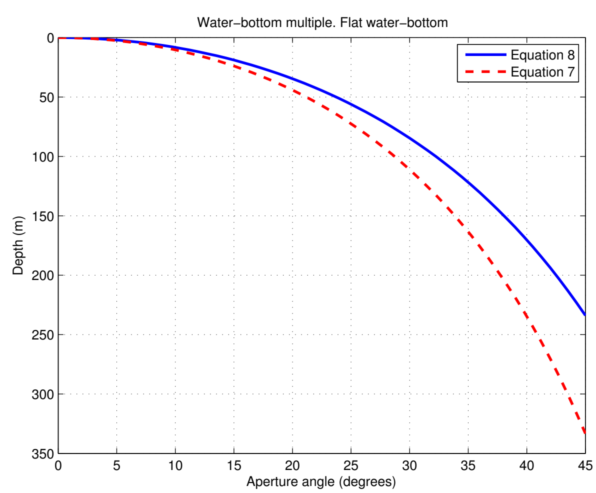

Figure 1: Residual moveout curves for an ADCIG from a two flat-layer model. The curves correspond to straight ray and the ray-bending approximations to a water-bottom multiple. gabriel3-rmos2 [CR]

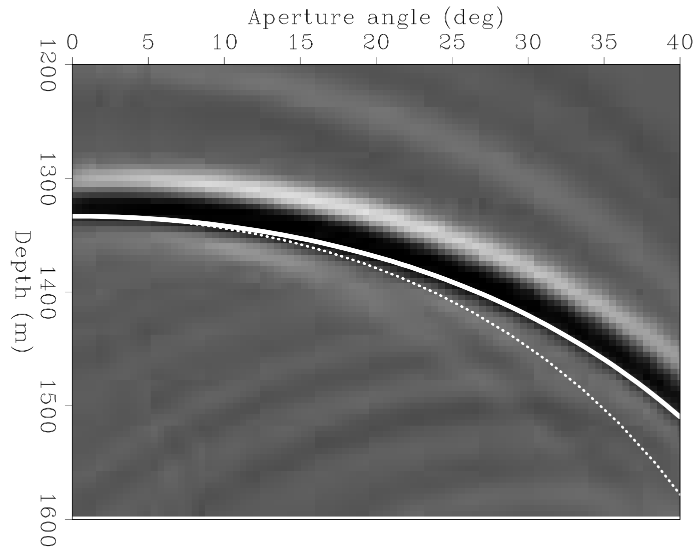

Figure 2: ADCIG for a water-bottom multiple from a two flat-layer model. The dotted curve corresponds to the straight ray approximation whereas the solid curve corresponds to the ray-bending approximation. gabriel3-adcig1 [CR]

COMPARISON OF RADON TRANSFORMS

The better fit of the ray bending approximation to the actual moveout of the multiple, as shown in Figure 2, suggests that a better focusing may be achieved for the multiples in the Radon domain by using equation 6 as the kernel of the Radon transform. To assess validity of this claim, I used the same synthetic data presented in Sava and Guitton (2003). Figure 3 is their figure 1 and shows a CMP and an ADCIG contaminated with multiples. Clearly, the primaries are flat in the ADCIGs (above about 350 m), whereas the multiples show the expected overmigrated residual moveout. The general expression for the Radon transform in the angle domain is (Sava and Guitton, 2003)

$$z(q,\gamma) = z_0 + q\ g(\gamma). \tag{9}$$

The straight-ray approximation uses

$$g(\gamma) = \tan^2\gamma. \tag{10}$$

The ray-bending approximation uses

$$g(\gamma) = \frac{1}{1+\rho}\left[\frac{\cos\gamma(\rho^2-(1-\rho^2)\tan^2\gamma)}{\sqrt{\rho^2-\sin^2\gamma}} - \rho\right]. \tag{11}$$

Figure 4 shows a comparison of both Radon transforms for the ADCIG shown in the left panel of Figure 3. Notice that the focusing of the primaries does not change since their moveout is zero. The multiples, on the other hand, are better focused with the new transform since the curvature more closely represents their residual moveout in the ADCIGs. In order to assess the improvement in focusing power of the new transform with real data, I applied both Radon transforms to an ADCIG from a real dataset (Sava and Guitton, 2003). Figure 5 shows the ADCIG and the transforms computed with the straight-ray and the ray-bending approximations. Again, the ray-bending approximation improves the focusing of the primaries. This may be better seen in the envelopes of the two transforms.

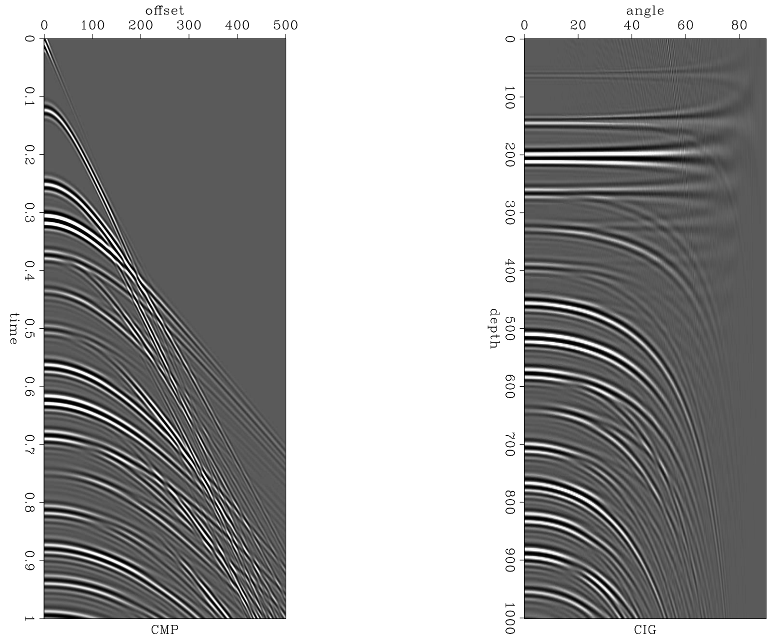

Figure 3: A data CMP and an ADCIG from a simple synthetic model. Figure taken from Sava and Guitton (2003). gabriel3-synth [NR]

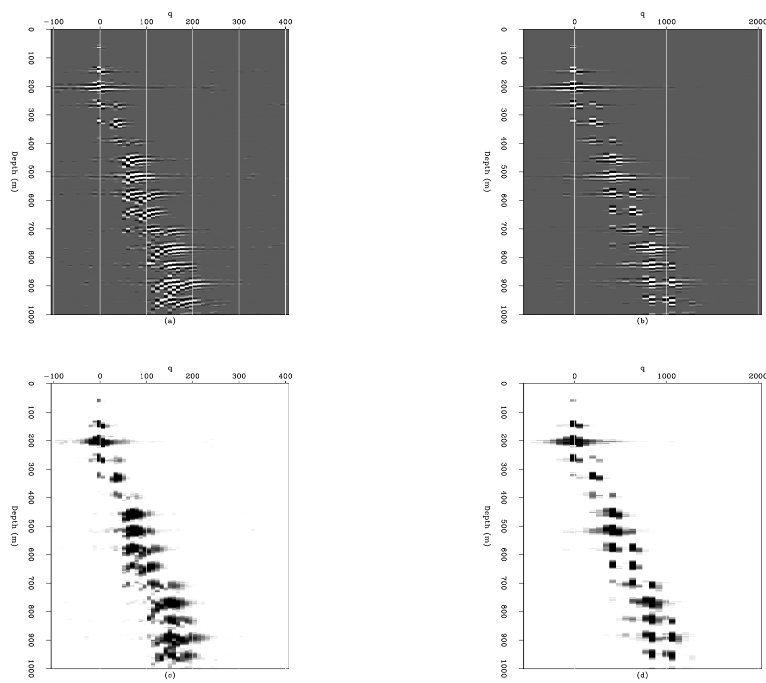

Figure 4: Comparison of Radon transforms of the ADCIG shown in panel (a) of Figure 3. Panel (a) corresponds to the straight-ray approximation whereas panel (b) corresponds to the ray-bending approximation. Panel (c) and (d) are the envelopes of panels (a) and (b) respectively. gabriel3-radon1 [CR]

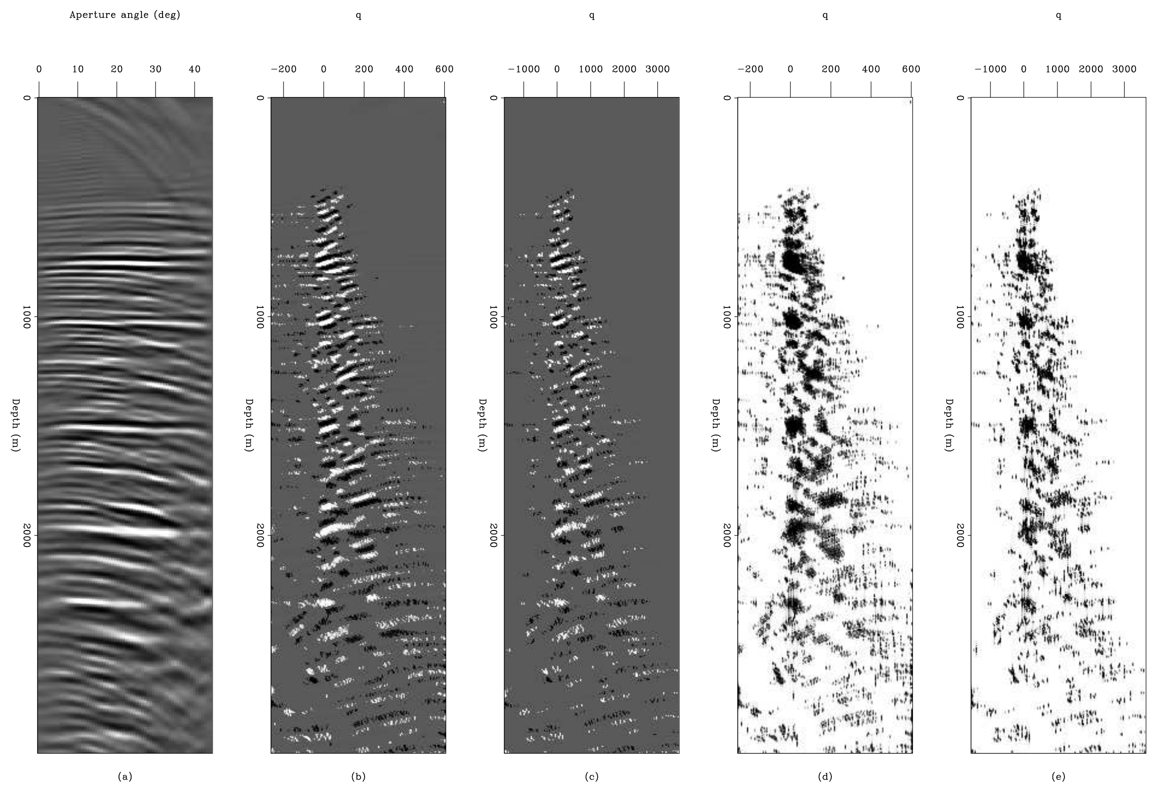

Figure 5: ADCIG from a real dataset (a), the Radon transform corresponding to the straight-ray approximation (b), and the Radon transform with the ray-bending approximation (c). Panels (d) and (e) are the envelopes of panels (b) and (c). gabriel3-radon2 [CR]

MULTIPLE ATTENUATION

In this section I examine the difference between the estimate of the multiples and primaries obtained with the Radon transforms computed with both approximations. I use the same ADCIGs from the previous section.

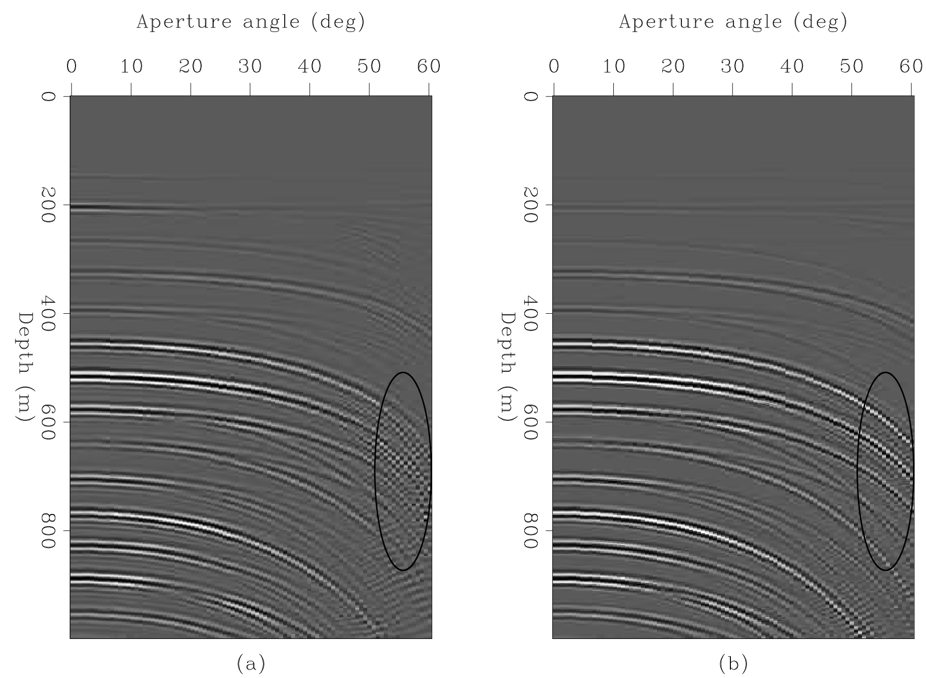

Figure 6: Comparison of the multiple model for the synthetic ADCIG of Figure 3. Panel (a) shows the multiples obtained with the straight-ray approximation and panel (b) the multiples obtained with the ray-bending approximation. gabriel3-mul_comp1 [CR]

Figure 6 shows the comparison of the multiple model estimated with both transforms for the synthetic data shown in Figure 3. The better focusing of the multiples in the Radon domain with the ray-bending approximation translates into a slightly better estimate of the multiples, especially at the large aperture angles. Some weak residual primary still leaks into the multiples, although with higher amplitude amplitude with the straight-ray approximation (see the primary at 200 m). Figure 7 shows the comparison of the primary estimates. Again, a better result is obtained with the ray-bending approximation. In particular, less energy from the multiples leaks into the primaries.

Figure 8 shows the comparison of the multiple model for the real dataset of Figure 5. Notice that again, the new transform [panel (b)], recovers a little better the multiple energy on the large aperture angles, especially above 1000 m. This energy will otherwise leak into the estimate of the primaries. Finally, Figure 9 shows the comparison of the primary estimate with both transforms. Although the two panels look similar, careful examination of the large aperture angles specially above 1000 m shows that the new transform [panel (b)] has recovered the primaries better.

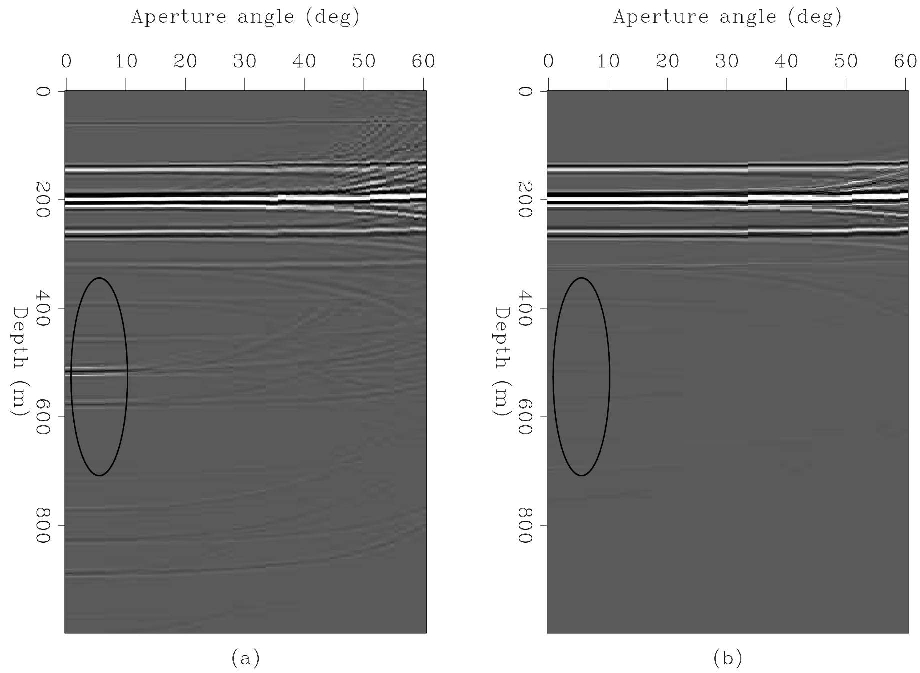

Figure 7: Comparison of the primary model for the synthetic ADCIG of Figure 3. Panel (a) shows the primaries obtained with the straight-ray approximation and panel (b) the primaries obtained with the ray-bending approximation. gabriel3-prim_comp1 [CR]

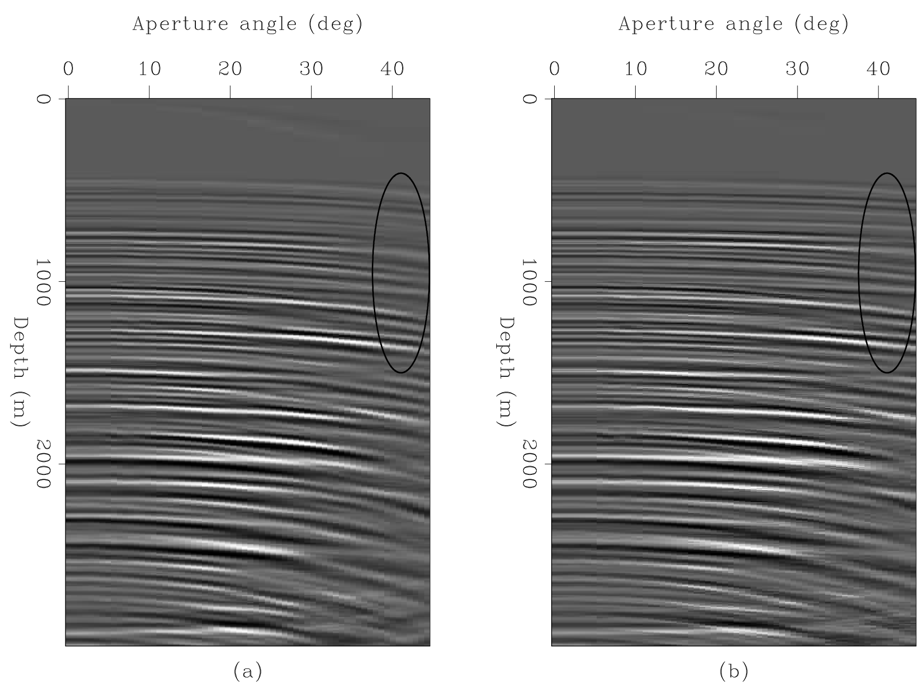

Figure 8: Comparison of the multiple model for the real ADCIG of Figure 4. Panel (a) shows the multiples obtained with the straight-ray approximation and panel (b) the multiples obtained with the ray-bending approximation. gabriel3-mul_comp2 [CR]

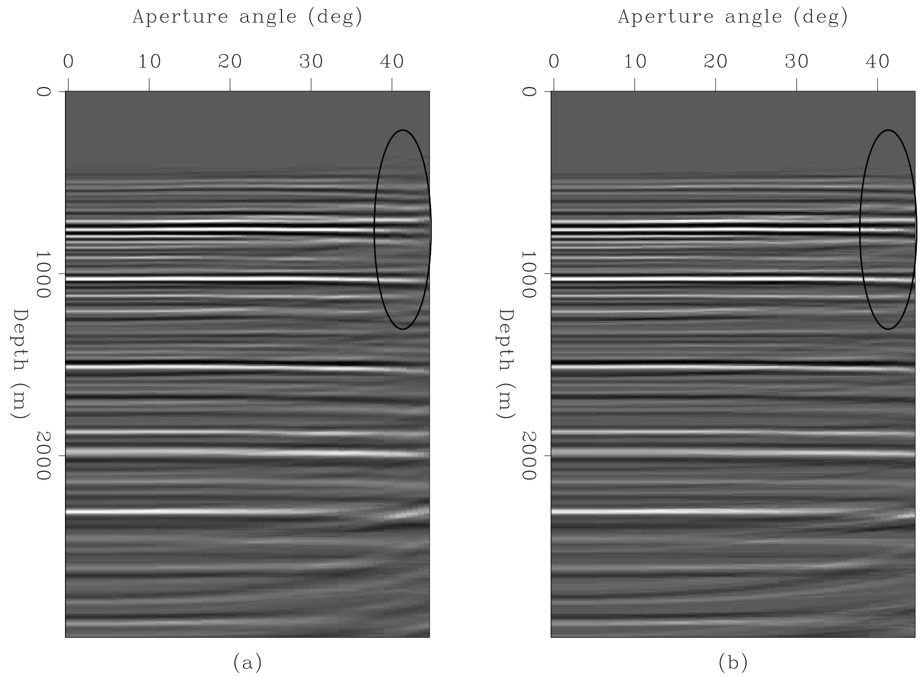

Figure 9: Comparison of the primary model for the real ADCIG of Figure 4. Panel (a) shows the primaries obtained with the straight-ray approximation and panel (b) the primaries obtained with the ray-bending approximation. gabriel3-prim_comp2 [CR]

DISCUSSION AND CONCLUSIONS

Increasing the focusing power of the Radon transform is critical in real situations in which the primaries and the multiples may map close together in the Radon domain. Therefore, taking into account the ray bending of the multiples, at least to first order, is an improvement. The cost of the new transform is essentially the same and no additional information is required. Although the new transform explicitly depends on the ratio (ρ) between the multiple velocity and the migration velocity, in practice this ratio can be fixed to something reasonable like 1.5 and the results are good. The new transform may also be advantageous in the implementation of the apex-shifted Radon transform for the attenuation of diffracted multiples (Alvarez et al., 2004).

Taking into account the ray bending of the multiple raypaths at the multiple generating interface improves the focusing power of the Radon transform when applied to ADCIGs. This in turn improves our ability to separate the primaries from the multiples and, therefore, allows a better estimate of the multiple model to be computed. The new transform can be implemented at essentially no extra cost compared with the tangent-squared approximation designed to treat the primaries ignoring ray bending.

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