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Attenuation of 2D specularly-reflected multiples in image space

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ABSTRACT

I propose a new method to attenuate specularly-reflected multiples in the image space. The method is based on the difference in mapping between primaries and multiples in Subsurface Offset Domain Common Image Gathers (SODCIGs). I migrate the data with a velocity slower than that of the primaries but faster than that of the multiples. The primaries are therefore undermigrated whereas the multiples are overmigrated. For positive data offsets, the primaries are mapped to positive subsurface offsets and the multiples to negative subsurface offsets in SODCIGs. I then apply a tapered mute to eliminate the primaries and do adjoint migration on the multiples with the same velocity model to get an estimate of the multiples in data space. Similarly, a tapered mute is applied to eliminate the multiples and adjoint migration used to obtain an estimate of the primaries in data space. The estimate of the multiples is adaptively matched to the data with the estimate of the primaries used as a weight function to prevent matching the primaries. I illustrate the method with a 2D synthetic dataset and show that the primaries can be well recovered although some residual from the water bottom multiple remains.

INTRODUCTION

Specularly-reflected multiples, such as water-bottom multiples, peg-leg multiples and internal multiples contaminate the seismic data to varying degrees making it more difficult to process and interpret. Water-bottom multiples are relatively easy to eliminate since their moveout is completely predictable. Peg-leg multiples and internal multiples, on the other hand, are difficult to eliminate if the subsurface is complex.

Surface-Related Multiple Elimination (SRME) (Berhout and Verschuur, 1997; Verschuur and Berkhout, 1997; Weglein et al., 1997; Dragoset and Jericevic, 1998; Dragoset, 1999) can be used to eliminate all multiples with at least one bounce at the water surface. SRME has been proven to be very effective when the data is finely and regularly sampled in the space coordinates and when the data aperture is sufficient to capture all surface bounces of the multiples. In many practical situations this is difficult to achieve and a large amount of data interpolation and extrapolation is required. Even when all the sampling conditions are met, SRME cannot be used to eliminate internal multiples unless the data is downward-continued to the multiple-generating layer, which in general is difficult to do for all layers.

Alvarez (2005) showed that specularly-reflected multiples have kinematics equivalent to that of primaries and therefore, when migrated with the faster velocities of the sediments, they are mapped to negative subsurface offsets in Subsurface Offset Domain Common Image Gathers (SODCIGs), if the surface offset itself is positive. Since the primaries are migrated to positive subsurface offsets when migrated with velocities slower than their exact velocities, we can in principle separate primaries from multiples in SODCIGs provided that we choose a migration velocity that is faster than the velocity of the multiples but slower than the velocity of the primaries. I explore this idea here and apply it to a simple 2D synthetic model.

The next section describes the synthetic data, the following section details the methodology, and the next section shows the results and points out some practical issues with the application of the method.

SYNTHETIC DATASET

I created a simple velocity model with three flat layers as shown in Figure 1. The top layer represents the water layer and the other two represent the sediments. With this velocity model

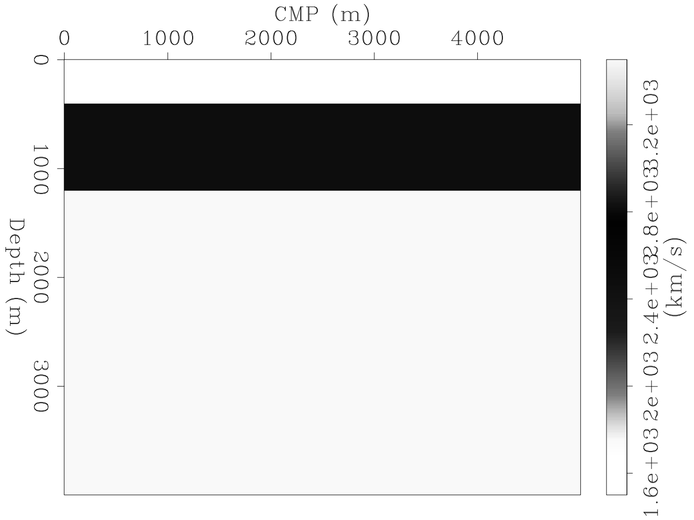


Figure 1: Velocity model for the synthetic dataset. gabriel1-vel_model [ER]

I analytically computed the traveltimes of the two primaries and three multiples whose raypaths are shown in Figure 2. Figure 3 shows a CMP gather with 100 offsets. The first and third events are the primaries and the other are the multiples as evidenced by their reverse polarity. The second event is the water-bottom multiple, the fourth event is the internal multiple, the next one is the peg-leg multiple and the last one is the surface multiple of the deeper reflector.

METHODOLOGY FOR ATTENUATING THE MULTIPLES

The methodology to attenuate the multiples consists of four basic steps: prestack migration of the data with a migration velocity V_{mig} such that $V_{\text{mul}} < V_{\text{mig}} < V_{\text{prim}}$. Estimation of the multiples and the primaries by tapered mute of the SODCIGs to eliminate the primaries

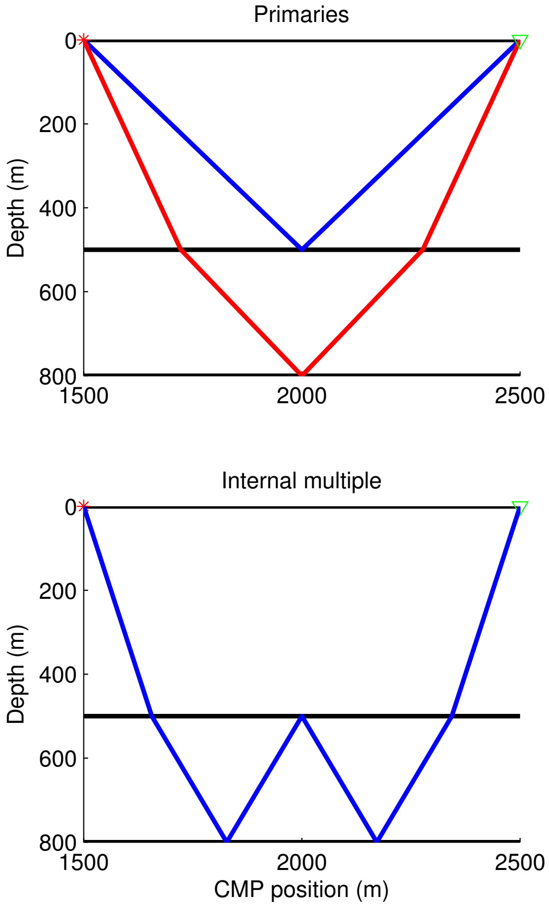


Figure 2: Raypath for the primaries and the multiples. gabriel1-rays [CR]

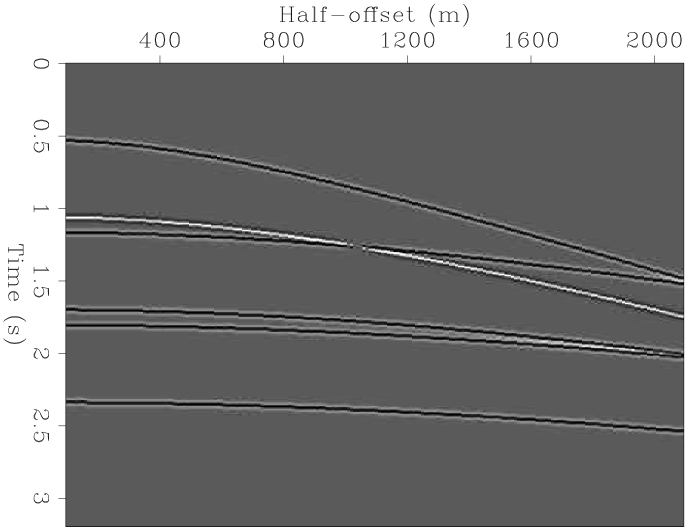


Figure 3: Common Midpoint gather showing the primaries (shallow two reflections) and the multiples. gabriel1-cmp [ER]

and the multiples, respectively. Adjoint migration of the estimated primaries and multiples and adaptive subtraction of the estimated multiples from the original data with the estimated primaries used as a weighting function to restrict the adaptive subtraction to the multiples only. I will now illustrate each of these steps with one gather of the synthetic dataset.

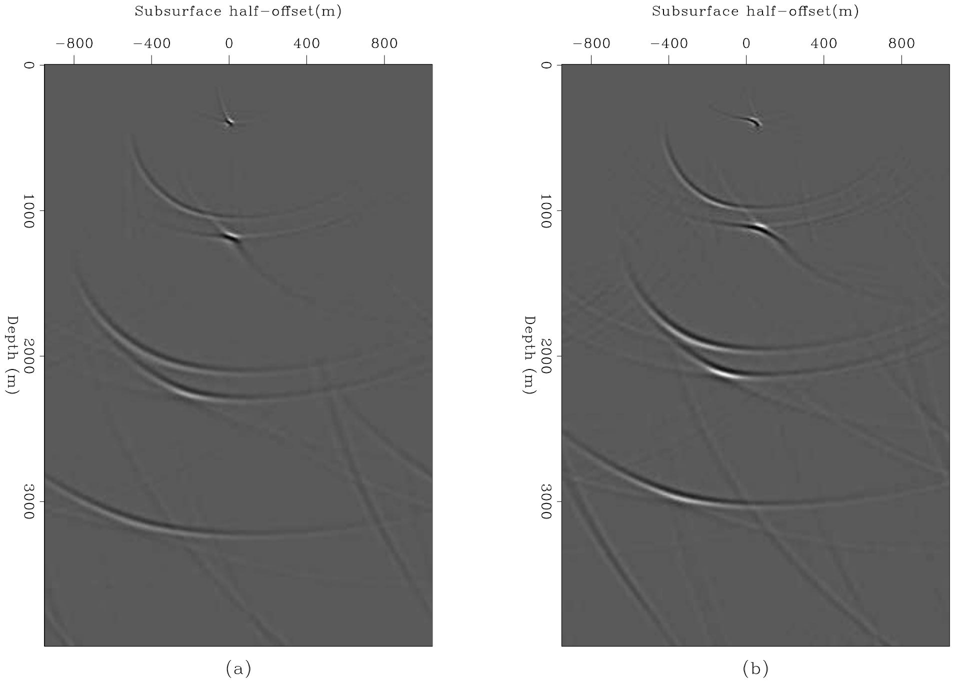


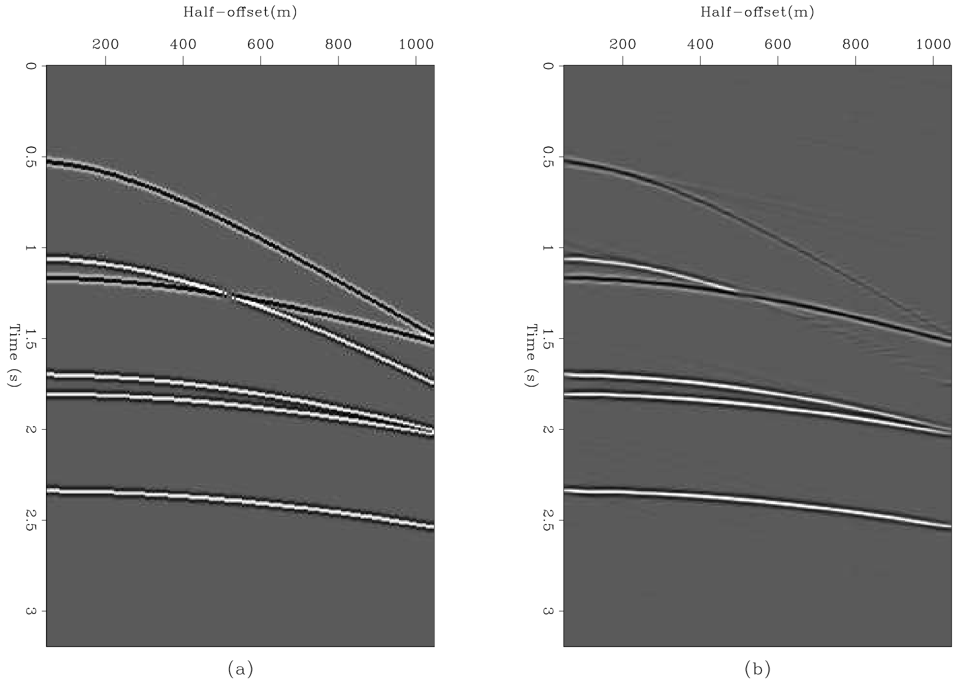
Figure 4: SODCIG migrated with the correct velocity (a) and with 90% of the correct velocity (b). gabriel1-sodcigs1 [CR]

Prestack depth migration

The data was migrated with source-receiver migration. The depth step was 10 m and 400 depth steps were computed in total. Figure 4 shows one SODCIG migrated with the correct velocity (left panel) and with 90% of the correct velocity (right panel). Obviously, with the correct velocity both primaries are well focused around zero subsurface offset, whereas the multiples are mapped to the negative subsurface offsets. With the slower velocities, the primaries are now mapped shallower, with downward moveout and toward the positive subsurface offsets. The multiples are still mapped to the negative subsurface offsets although they are somewhat closer to the zero subsurface offsets.

Muting the primary and multiple reflections

In order to estimate the multiples we need to eliminate the primaries as much as possible, even if that implies underestimating the multiples. On the other hand, since the estimate of the primaries is only used as a weight function for the adaptive matching of the multiples, in estimating the primaries we would prefer to overestimate the multiples. The choice of the mute pattern is obviously important and requires careful examination of the SODCIGs.

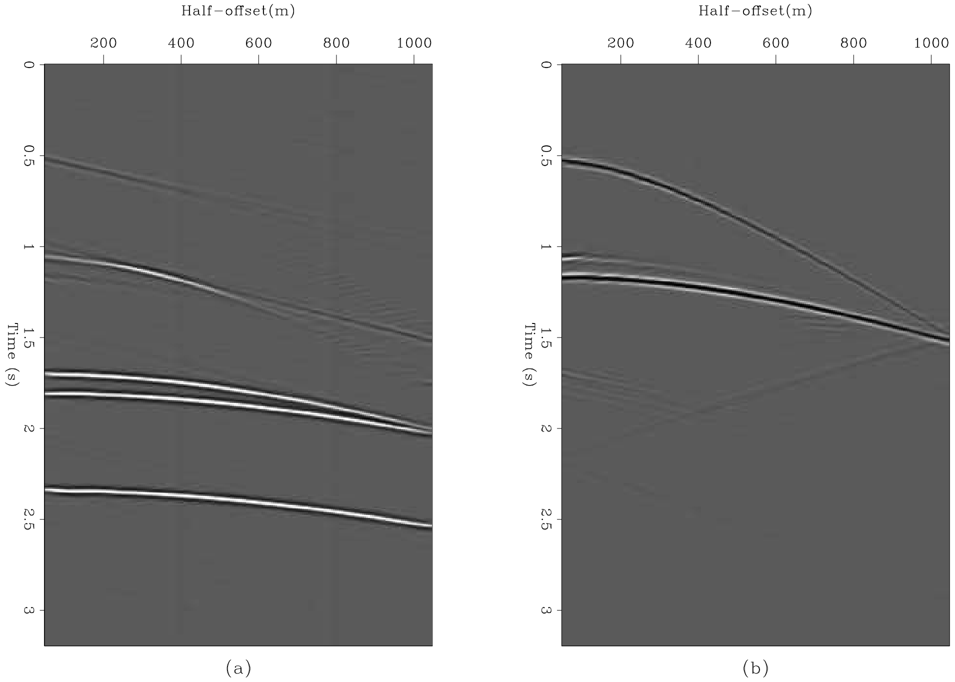


(a) (b)

Figure 5: Comparison between a CMP of the original data (a) and the corresponding CMP after migration-adjoint migration with the exact velocity (b). gabriel1-cmp_comp [CR]

Adjoint migration

To obtain the estimates of the primaries and multiples in data space we need to apply the inverse of the migration operator to the muted SODCIGs of the primaries and the multiples. Here I used the adjoint of the migration in lieu of the inverse. To assess just how good the adjoint migration is in recovering the kinematics of the data, I first applied the adjoint migration to the unmuted migrated SODCIGs (with the correct velocity) and I show the comparison between a CMP gather of the original dataset and the migration-adjoint migration result in Figure 5. Clearly, the kinematics of the reflections have been recovered fairly well, except at the large offsets of the water-bottom primary for which the subsurface offset sampling in the SODCIGs was a little coarse given its steep moveout as seen in Figure 4 (below -400 m). In the next subsection I show that adaptive subtraction recovers these amplitudes very well. Figure 6 shows the result of applying adjoint migration to the muted SODCIGs. The left panel corresponds to the estimated multiples whereas the right panel corresponds to the estimated primaries. Obviously, some primary energy remains in the estimated multiples and some energy from the water-bottom primary remain in the estimated primaries.



(a) (b)

Figure 6: Estimated multiples (a) and estimated primaries (b). Some multiples leaked into the estimate of the primaries and some primaries leaked into the estimate of the multiples. gabriel1-cmp_inv [CR]

Adaptive subtraction

This step is subdivided into two tasks: adaptive matching of the multiples to the original data and subtraction of the matched multiples to get the primaries (Guitton, 2005). To assess the ability of adaptive subtraction to restore the amplitudes lost by the migration-adjoint migration process, I applied it to the adjoint-migrated, unmuted SODCIGs (panel (b) of Figure 5). Figure 7 shows the original [panel (a)] and the matched [panel (b)] CMP. Figure 8 shows the difference between the two panels in Figure 7 plotted with 100% clip and with the clip of the original data. The adaptive pattern matching has recovered the the original data remarkably well despite the loss of energy on the large offsets of the water-bottom primary and multiple after the migration-adjoint migration process (see panel (b) of Figure 5). We would like to similarly match the estimated multiples to the original data but we cannot, because nothing prevents the pattern-matching algorithm from attempting to match the primaries as well. This is obviously undesirable because, as much as possible, we want to keep the primaries as they are in the original data. Here is where the estimate of the primaries will help. We can estimate filters to simultaneously match both the estimated primaries and the estimated multiples to the data as in Guitton's thesis (2005) thus preventing the primaries to be matched to the same dataset as the multiples. An easier approach is to use the estimated primaries to compute a mask such that where the primaries are present the mask is zero and therefore the primaries are not matched when attempting to match the multiples (Claerbout and Fomel, 2002) . This is the approach I used here although in practice the other approach is likely to be better. In order

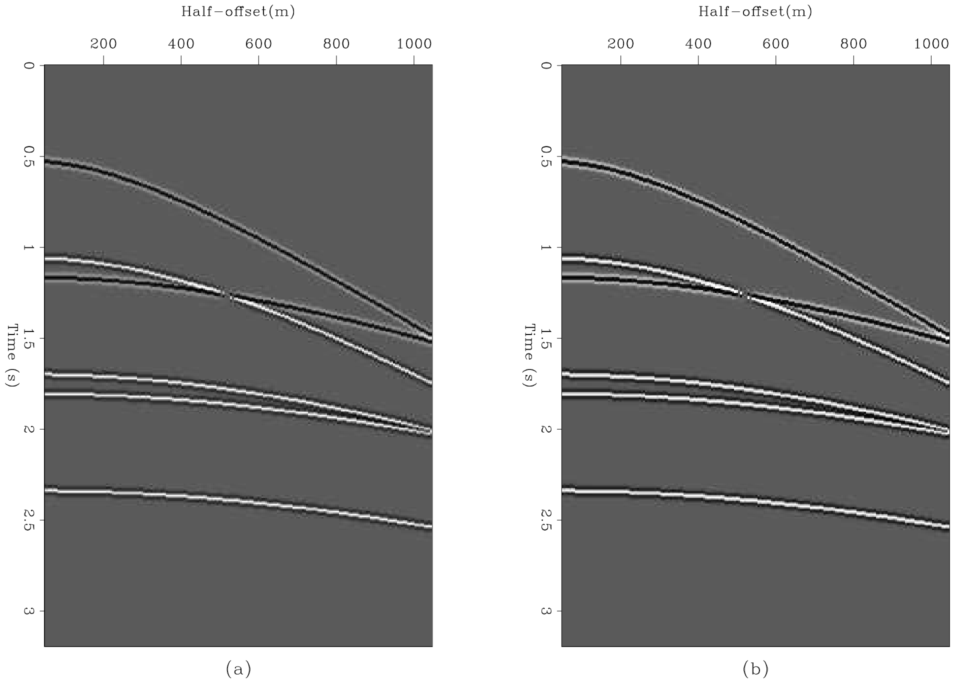


Figure 7: Comparison between a CMP of the original data (a) and the same CMP after migration, adjoint migration and adaptive matching (b). gabriel1-cmp_comp2 [CR]

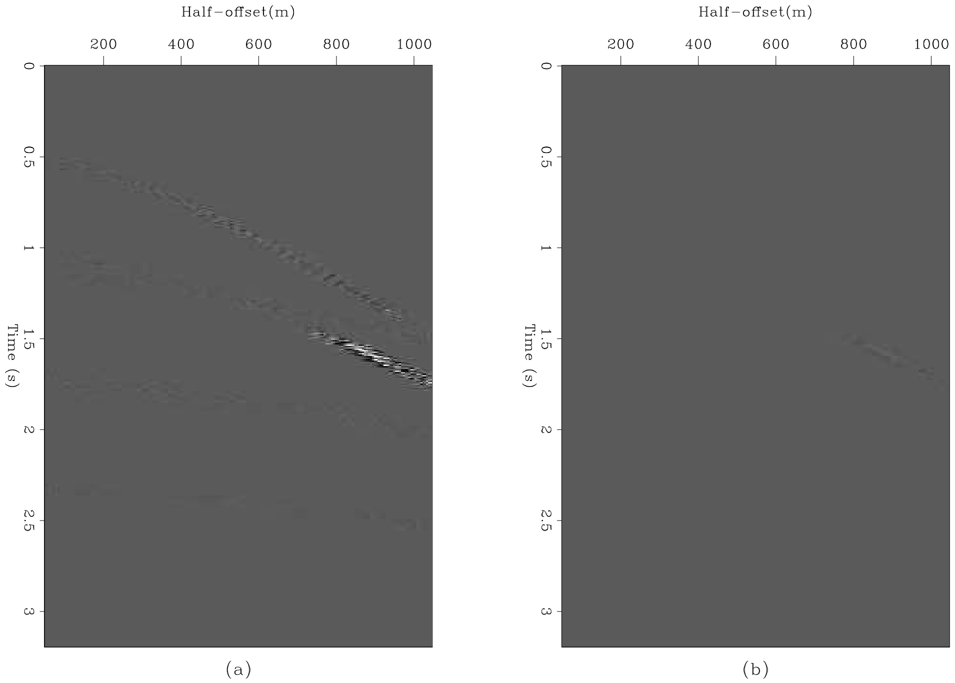


Figure 8: Residual energy after subtracting the matched CMP from the original, plotted at 100% clip (a) and at the clip of the original CMP (b). gabriel1-resid1 [CR]

to compute the mask I first computed the envelope of the estimated primaries, and then chose a threshold amplitude above which all samples were set to zero and below which all samples were set to one. This mask was then smoothed with a triangular filter both in offset and in time. Figure 9 shows the envelope of the estimated primaries and the mask. The result of applying the weighted adaptive matching to the estimated multiples is shown in Figure 10. The water-bottom multiple has not been well recovered, in contrast to the other multiples. The primaries didn't leak much into the multiples, which is a very satisfactory result. Figure 11 shows the residual, obtained by subtracting the matched estimated multiples from the data. Here we see that although the result is not perfect, most of the multiple energy has been attenuated except for the water-bottom multiple. In particular, the primaries have been well-recovered.

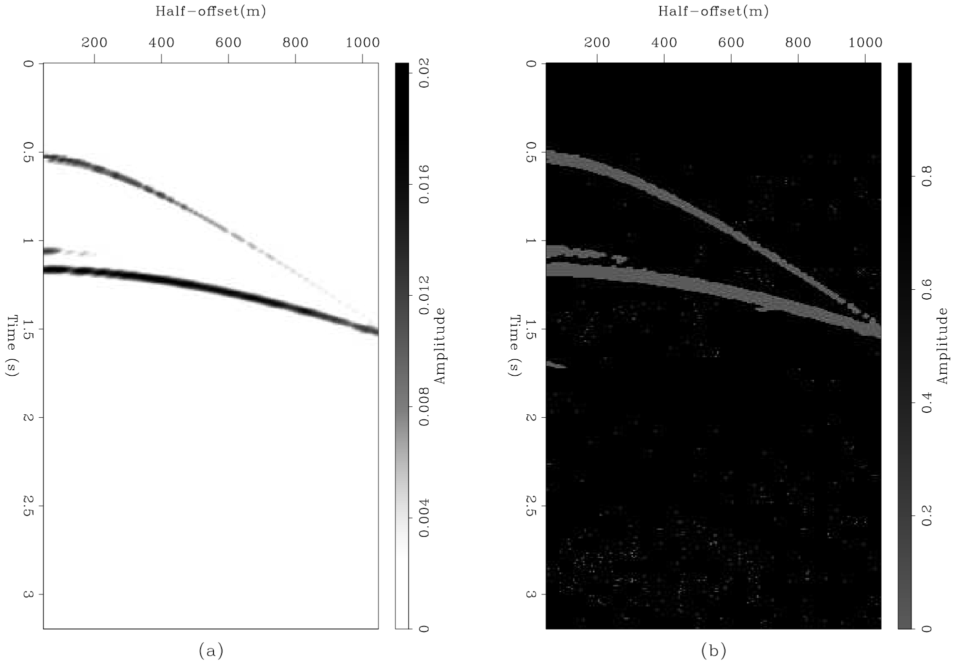
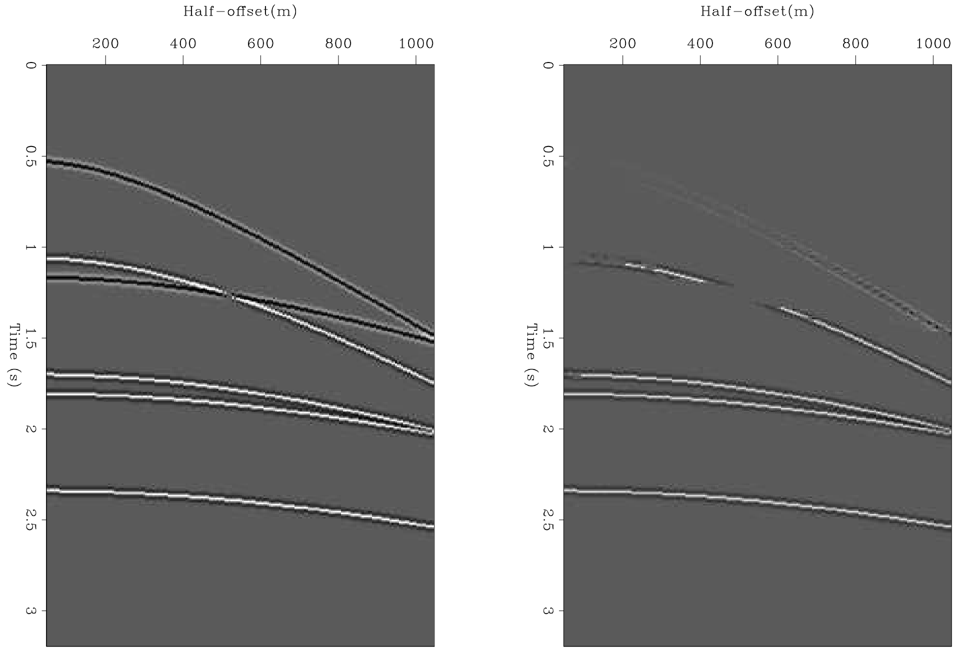


Figure 9: Envelope of the estimated primaries (a) and mask of ones and zeros derived from it (b). gabriel1-mask [CR]

DISCUSSION

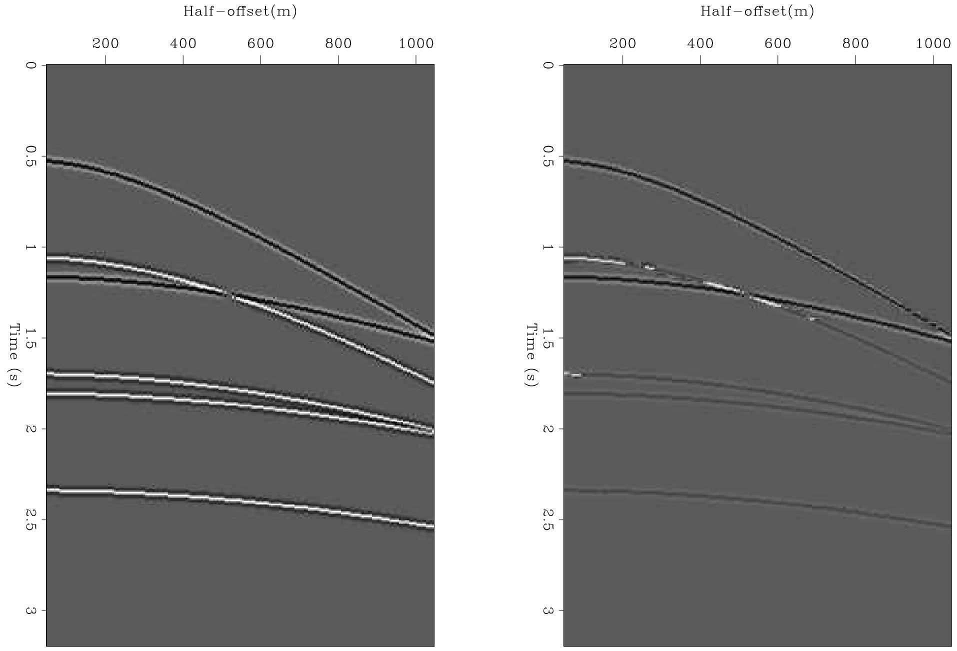
There are several important issues with the above methodology that can impact the quality of the results. Making sure that the adjoint migration operator is a good approximation to the inverse of the migration operator can probably provide the biggest improvement. Here I used a standard wave-equation migration without any compensation for the migration Jacobian. Other important factors that are relatively easy to control are: choosing the right migration velocity, avoiding aliasing for the adjoint migration of the SODCIGs, choosing the appropriate tapered mutes for the estimation of the primaries and the multiples, choosing the threshold amplitude to compute the weight mask and muting the direct wave and the water-bottom primary reflection. I will discuss my current understanding of these issues now:



(a)

(b)

Figure 10: Comparison between a CMP of the original data (a) and the same CMP after migration, muting of the primaries, adjoint migration and adaptive matching (b). Some residual energy from the large offset of the water-bottom primary remains and the water-bottom multiple has been imperfectly recovered. gabriel1-cmp_comp3 [CR]



(a)

(b)

Figure 11: Residual energy after subtracting the matched CMP of the multiples from the original, plotted at 100% clip (a) and at the clip of the original CMP (b). gabriel1-resid2 [CR]

Choosing the migration velocity

In principle, we don't need to migrate the data with the correct velocity for the primaries, just a velocity that satisfies the inequality $V_{\text{mul}} \leq V_{\text{mig}} \leq V_{\text{prim}}$. This will guarantee that the multiples are overmigrated and the primaries undermigrated. Since we are more willing to tolerate some leftover multiple energy rather than to lose primary energy, the second part of the inequality is more critical. Unless there are very strong lateral velocity variations, a simple $V(z)$ will probably suffice since both migration and adjoint-migration are performed with the same velocity model. Migrating with a $V(z)$ model has the obvious advantage that the migration and adjoint-migration will run faster.

Sampling of Offsets in SODCIGs

In order for the migration-demigration process to yield useful results, the offset sampling of the CMPs, as well as the subsurface offset sampling of the SODCIGs must satisfy the well-known aliasing condition: $|p| \leq \frac{K_N}{\omega_d}$ with $K_N = \frac{\pi}{\Delta x}$. This condition is easy to satisfy for the CMPs but is much more difficult for the SODCIGs, in particular for the water-bottom multiple which has a steep moveout as shown in Figure 4. Therefore, if we want to attenuate the water-bottom multiples we may need to interpolate the subsurface-offsets before the adjoint migration. This will obviously increase the cost of the adjoint migration. It may be better to attenuate the water-bottom multiples in the time domain since there moveout is predictable and apply the procedure described here to the data without the water-bottom multiples.

Muting in CMPs

Besides muting the direct arrival, it may be desirable to mute the large offsets of the water-bottom primary because of aliasing and wavelet stretching.

Muting in SODCIGs

Choosing the right mute pattern in SODCIGs in order to obtain the estimate of the primaries and the multiples is obviously an important issue. In theory, if the migration velocity satisfies the inequality mentioned before ($V_{\text{mul}} < V_{\text{mig}} < V_{\text{prim}}$), the multiples would map entirely to the negative subsurface offsets while the primaries would map entirely to the positive subsurface offsets. In practice, however, some overlap is unavoidable. It is safer to allow some residual primaries in the estimate of the multiples and some residual primary in the estimate of the multiples. Appropriate tapers should be applied to avoid noises in the adjoint migration.

CONCLUSIONS

The method presented here can attenuate any specularly-reflected multiple including internal multiples. In contrast to SRME, it is robust in the presence of limited data aperture and missing shots but requires some knowledge of the velocity field. There are still several key practical issues that require more research, but the basic idea is attractive for its simplicity and generality.

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