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Interpolating diffracted multiples with prediction-error filters

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ABSTRACT

Diffracted multiples are a persistent problem, especially in the cross-line direction. Methods to remove these multiples typically require dense and complete sampling. I use non-stationary prediction-error filters to interpolate extra shots in the in-line direction and extra receivers in the cross-line direction. The dimensionality of the interpolation does not make a big difference in the in-line direction, but makes a large difference in the cross-line direction. This is shown with several tests on a synthetic dataset.

INTRODUCTION

A common problem with seismic data is the presence of coherent noise, such as multiple reflections. When these multiples are generated by cross-line structure, the lack of sampling along this axis becomes painfully obvious. One example of such noise is out-of-plane diffracted multiples. Methods to remove these multiples, such as 3D surface-related multiple elimination (SRME) (Van Dedem and Verschuur, 2005) require a much greater density of sampling as well as recording aperture. This is particularly true in the cross-line.

A synthetic data set provided by ExxonMobil illustrates an extreme example of out-of-plane diffracted multiples coupled with a modern high-density acquisition. The underlying geology is very simple, excluding a large number of point diffractors which create diffracted multiples that obscure the region of interest. These multiples could be removed by the application of 3D SRME, provided that the input wavefield is adequately sampled, making this synthetic data tailor-made for testing interpolation methods.

Many different interpolation methods exist, including Fourier-based methods (Duijndam and Schonewille, 1999; Liu and Sacchi, 2004; Xu et al., 2005), radon-based methods (Trad, 2003), imaging-operator based methods (Biondi and Vlad, 2001) as well as prediction-error filter based methods (Spitz, 1991). Non-stationary prediction-error based methods (Crawley, 2000) are used throughout this paper to interpolate missing data.

Most tests for interpolation are limited to an increase in source sampling by a factor of two, which corresponds to interpolating missing shots in a flip-flop shooting configuration by using information from the surrounding shots in the same sail line. In addition to missing shots, additional cross-line offsets are needed in the form of additional receiver cables. Both extra shots as well as extra receiver cables are created, using multi-dimensional non-stationary prediction-error filters. The number of dimensions in the filters is varied for both cases, and it is shown that the receiver cable interpolation benefits much more from a higher-dimensional filter than the shot interpolation does. This is in part due to the small number of points sampled in the cross-line.

While this method appears to work well for both in-line shot interpolation as well as cross-line receiver interpolation, still more data needs to be created. The cross-line receivers need to be extrapolated and extra sail lines also need to be interpolated. The measure of the success of these methods needs to be gauged by the end result of the 3D SRME.

BACKGROUND

Interpolation can be posed as a two-stage problem, where in the first stage some statistics of the data are gathered, and in the second stage this information is applied to fill in the missing data. In the case of transform-based interpolation methods, the initial transform corresponds to the gathering of information on existing data, and the second stage is the transform back to the original, more densely-sampled space.

In terms of the prediction-error filter based interpolation used in this paper, in the first stage an estimate of the data is made by creating a non-stationary prediction-error filter (PEF) from the data by solving a linear least-squares inverse problem,

$$\begin{aligned} \mathbf{W}(\mathbf{DKf}+\mathbf{d}) &\approx \mathbf{0} \\ \epsilon \mathbf{Af} &\approx \mathbf{0}, \end{aligned} \tag{1}$$

where $\mathbf{D}$ represents non-stationary convolution with the data, $\mathbf{f}$ is a non-stationary PEF, $\mathbf{K}$ (a selector matrix) constrains the value of the first filter coefficient to 1, $\mathbf{d}$ is a copy of the data, $\mathbf{A}$ is a regularization operator (a Laplacian operating over space) and ϵ is a trade-off parameter for the regularization. Solving this system will create a smoothly varying non-stationary PEF that, when convolved with the data, will ideally remove all coherent energy from the input data.

Once the PEF has been estimated, we can use it to constrain the missing data by solving a second linear least-squares inverse problem,

$$\begin{aligned} \mathbf{S}(\mathbf{m}-\mathbf{d}) &\approx \mathbf{0} \\ \epsilon \mathbf{Fm} &\approx \mathbf{0}, \end{aligned} \tag{2}$$

where $\mathbf{S}$ is a selector matrix which is 1 where data is present and 0 where it is not, $\mathbf{F}$ represents convolution with the non-stationary PEF, ϵ is now a trade-off parameter and $\mathbf{m}$ is the desired model.

In order to interpolate by a factor of two, the coefficients of the PEF are expanded so that the filter coefficients fall on known data. Once the PEF is estimated, the filter is shrunk down to its original size and then used to interpolate.

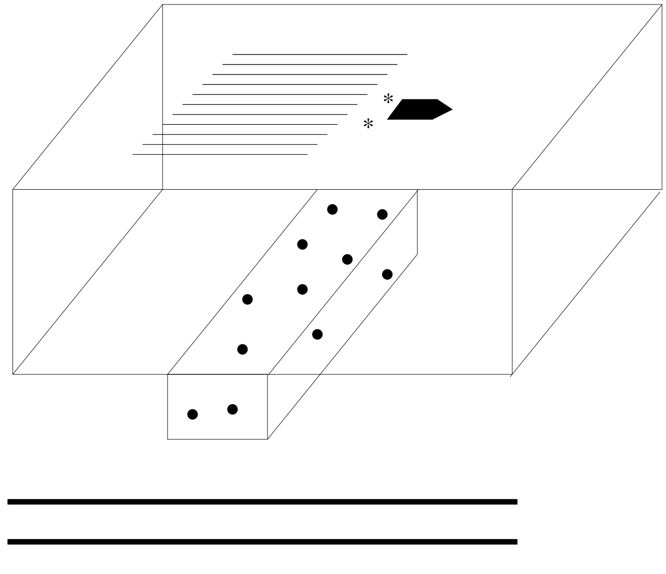

Figure 1: The model for the synthetic data: a prism containting diffractors below the sea floor, with reflectors underneath. bill2-geom [NR]

DATA DESCRIPTION

The dataset used in this paper (provided by ExxonMobil) is designed as a test for interpolation algorithms used for 3D surface-related multiple elimination. The model is constant-velocity, with multiple reflectors and 500 point diffractors contained in a prism near the water bottom that is oriented perpendicular to the acquisition direction. A schematic of the model is shown in Figure 1.

The data coverage is that of a modern marine 3D survey, acquired in a racetrack fashion. There are 12 streamers spaced at 50m, with 400 receivers in each streamer spaced at 12.5m, with a near offset of 100m for a maximum in-line offset of 5100m. There are two sources, spaced at 25m in the cross-line that are fired in a flip-flop fashion so that a single shot is fired every 37.5m. The total cross-line aperture is 550m. There are 10 sail lines with a spacing of 200m. This is very dense for a 3D survey, but the data density requirements of performing 3D SRME are much greater.

Ideal geometry for 3D SRME would include a source at every point where there is a receiver. In this case, a bin size of 12.5m would be ideal, meaning that in terms of interpolation, the following increase in data density would have to occur:

- Receiver spacing (in-line): none,
- Source spacing (in-line): factor of 3,
- Receiver spacing (cross-line): factor of 4,
- Source spacing (cross-line): factor of 8 or 16.

As can be seen, the combined ratio of needed data to sampled data is 96, meaning that the current sampling of the desired output cube is incredibly sparse. This is neglecting the extrapolation that is also required for receivers in the in-line direction to zero offset, which is minor, as well as for the extrapolation of receivers which is required in the cross-line direction, which would add another factor of 4 to this problem.

Most interpolation algorithms are tested by a factor of 2 in the in-line direction, meaning the infill of flip-flop shooting. This will be tested first, and afterward the infill of receiver cables.

FLIP-FLOP INTERPOLATION

The most tractable problem for this data and also the most commonly shown interpolation result is increasing the number of shots by a factor of 2. This will result in a source distribution where the flip and flop shots for a given in-line position are both present. While the desired goal for this data is to increase the in-line source sampling by a factor of 3, interpolating by a factor of 2 does have the benefit of decreasing the cross-line density requirement by a factor of 2, and also results in having pairs of shots with relatively dense (25m) sampling in the cross-line direction. This is a big step up from the 200m between sail lines.

Results for an increase in shots by a factor of two are shown in Figure 2. The panel on the left corresponds to the input data, which is in this case a common receiver gather. The missing traces correspond to missing "flip" shots. The panel in the center of the figure corresponds to a 2D PEF-based interpolation, while the panel on the right corresponds to a 3D PEF-based interpolation, where the PEF is estimated along the receiver cable as well as across the source and time axes. There are only minimal differences between the 2D and 3D results. This is because the 2D result is actually quite good, and shows that an overly computationally intensive method is not necessary for this problem.

Now that the factor of 2 shot interpolation has been shown to be relatively insensitive to the dimensionality of the interpolation, this same test is repeated for increasing the shot sampling by a factor of 3, which would correspond to the ideal output. The results are shown in Figure 3. The interpolation is not nearly as good as for the case with the factor of 2. This is in part due to the increased expansion of the PEF coefficients that is required, and thus the approximation that the filter is scale-invariant becomes less valid.

Increasing the dimensionality of the interpolation in this case appears to have more effect than in the previous factor of 2 case.

RECEIVER CABLE INTERPOLATION

The interpolation of flip-flop shots has the benefit of many in-line samples on which a PEF can be estimated. For receiver cable interpolation, the samples are fewer and farther between. For this data, an ideal interpolation would be a factor of 4. Since there are only 12 cables on which to estimate a PEF, the size of the PEF in the cross-line is much more limited, since increasing the size of the PEF decreases the number of fitting equations in the inverse problem due to

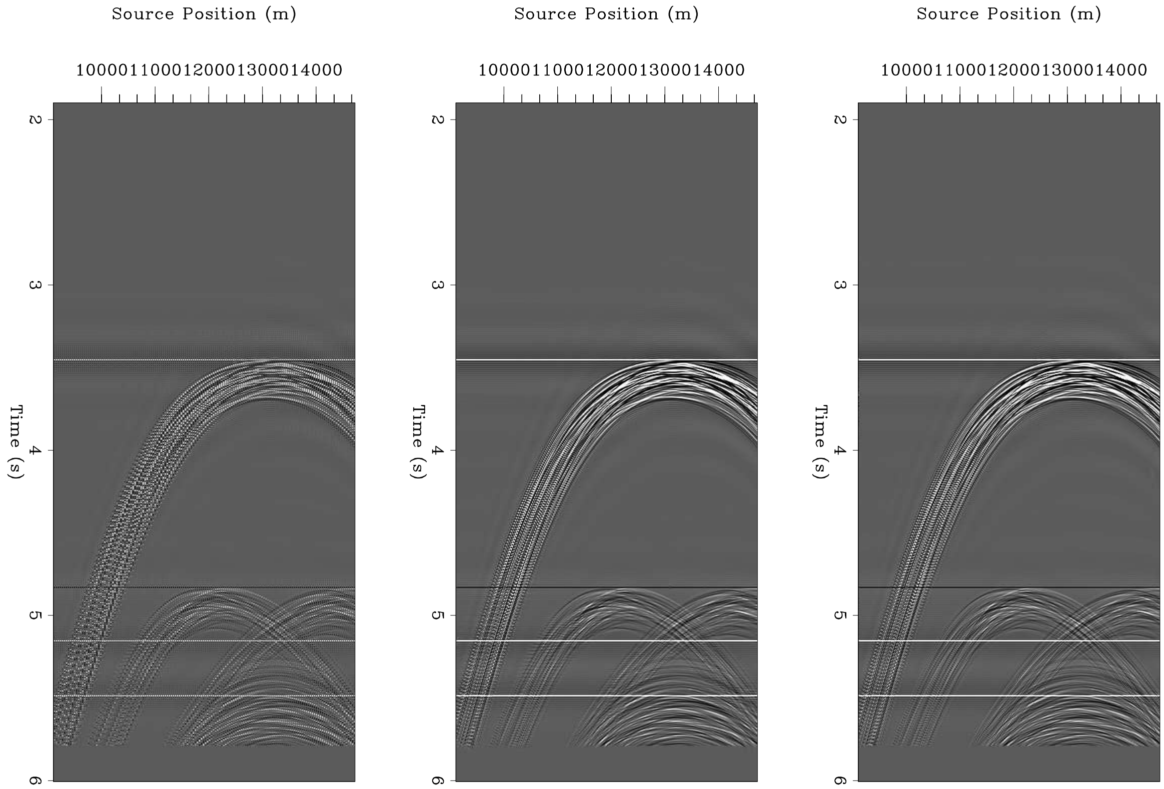

Figure 2: Flip-flop interpolation of a common-receiver-section (offset=1250m). From left to right: original input common-receiver gather; result of 2D interpolation; result of 3D interpolation. bill2-flipflopx2 [CR,M]

increased edge effects.

As we can see in Figure 4, here interpolating in 2D is noticeably worse than in 3D. In particular, the water-bottom multiple, the diffractions at near offset, and the diffracted multiples at far offset were not correctly interpolated with the 2D filter, and were correctly interpolated in 3D. This is because the 2D interpolation acted over receiver-cable and time, so no trends along the receiver cable axis were captured. The added in-line receiver axis in the 3D PEF interpolation allowed much more information to be gathered from the data.

Once the number of receiver cables has been doubled by interpolation, it can be doubled again to get to the desired factor of 4. This is shown in Figure 5. The interpolation differences are even more pronounced between the 2D and 3D interpolations. The end result of the cross-line interpolation in 3D is quite promising, and the poor performance of the 2D result shows that extra dimensionality is a much more important factor when the axis of interpolation is poorly sampled.

Twelve receiver cables is not typical for a marine acquisition. Many 3D surveys use a smaller number of cables, which can be as low as 4 in a speculative survey. Figure 6 shows the results of interpolating from 6 receiver cables up to 24. While the result is not poor, it is not nearly as good as with the original 12 cables. This is most noticeable at the near offsets.

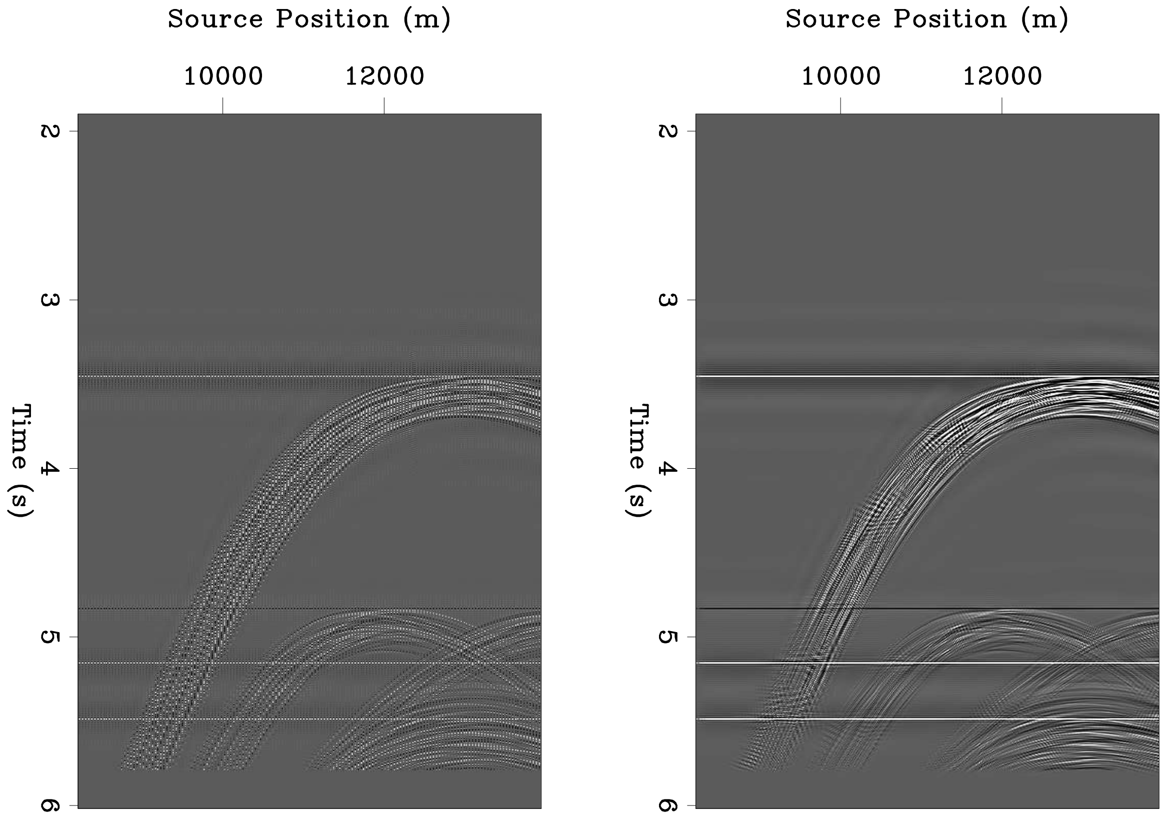

Figure 3: Flip-flop interpolation, factor of 3. From left to right: original input common-receiver gather; result of 2D interpolation. bill2-flipflopx3 [CR,M]

SAIL LINE INTERPOLATION

The most difficult of the interpolation tasks required for 3D SRME is the interpolation of sail lines. This was not attempted in this paper for several reasons. The first of which is that the idealized geometry in this synthetic example is far from the reality of marine acquisition. The sail lines are not parallel, and the receiver cables feather differently from sail line to sail line.

In addition to the issues of inconsistency in the geometry from sail line to sail line, there is also the issue of the sheer amount of additional data needed. In order to reach the desired density of data, a factor of 16 is needed in the interpolation, or a factor of 8 if the flip-flops have been previously interpolated.

CONCLUSIONS AND FUTURE WORK

PEF-based Interpolation methods of varying dimensionality have been tested on synthetic data that mimics modern acquisition and problems. The in-line interpolation of missing shots was very successful, and was not drastically effected by the dimensionality of the filters. However, interpolating by a factor of 3 appears to be much more difficult than by a factor of 2.

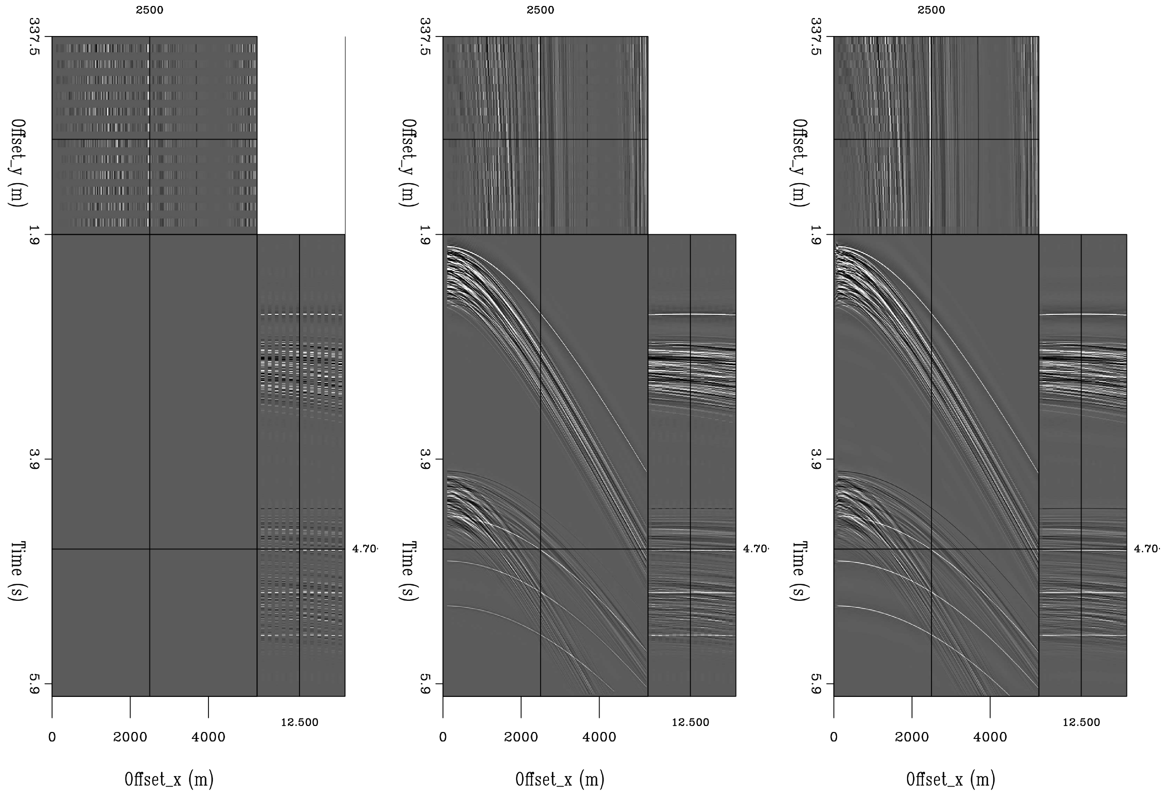

Figure 4: Receiver cable interpolation, factor of 2. From left to right: original input shot gather; result of 2D interpolation; result of 3D interpolation. bill2-cablex2 [CR,M]

For the more difficult task of interpolating missing receiver cables, the added dimensionality is crucial. A simple 2D interpolation is insufficient due to the small number of points in the cross-line. The added third dimension of in-line receiver number gave substantially better results. When the number of recorded receiver cables was reduced from 12 to 6, the results were substantially worse, implying that the additional receiver cables are very valuable.

Interpolation of the sail lines still needs to be performed, but there are more geometrical complications between sail lines as well as the amount of interpolation required.

Finally, the end interpolation results need to be measured in terms of the end result on 3D SRME results.

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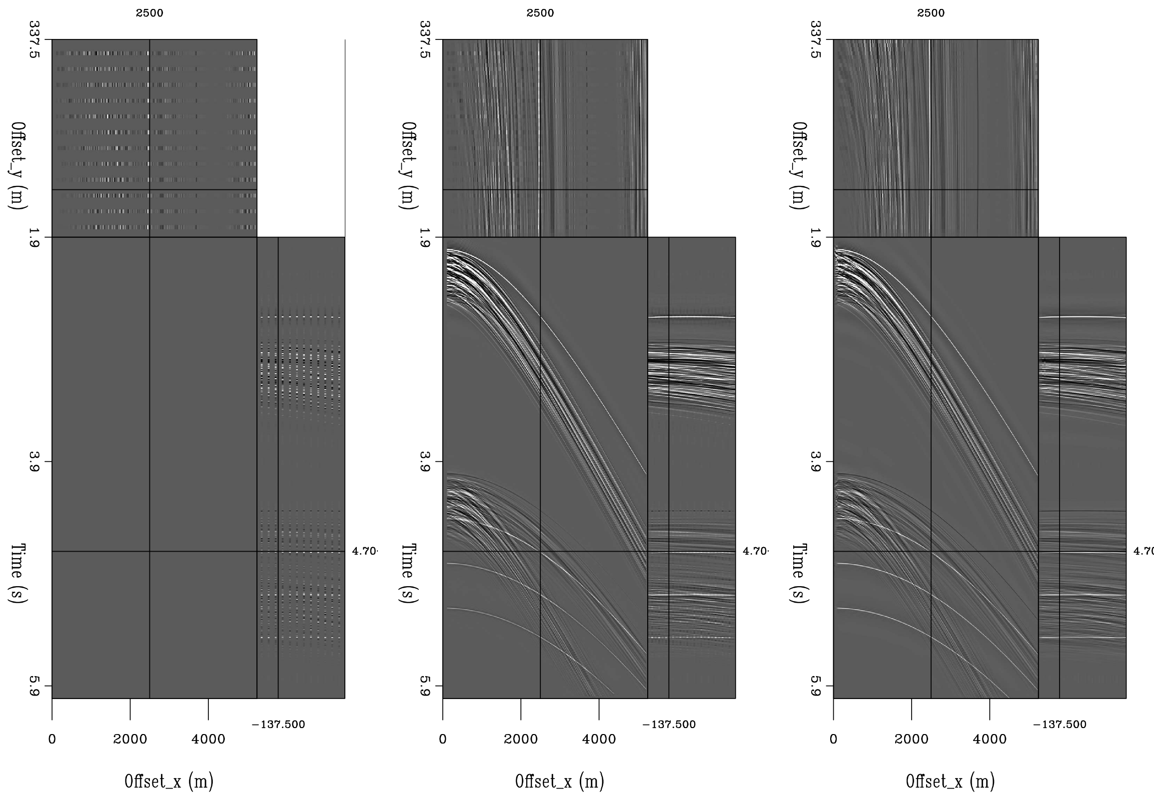

Figure 5: Receiver cable interpolation, factor of 4. From left to right: original input shot gather; result of 2D interpolation; result of 3D interpolation. bill2-cablex4 [CR,M]

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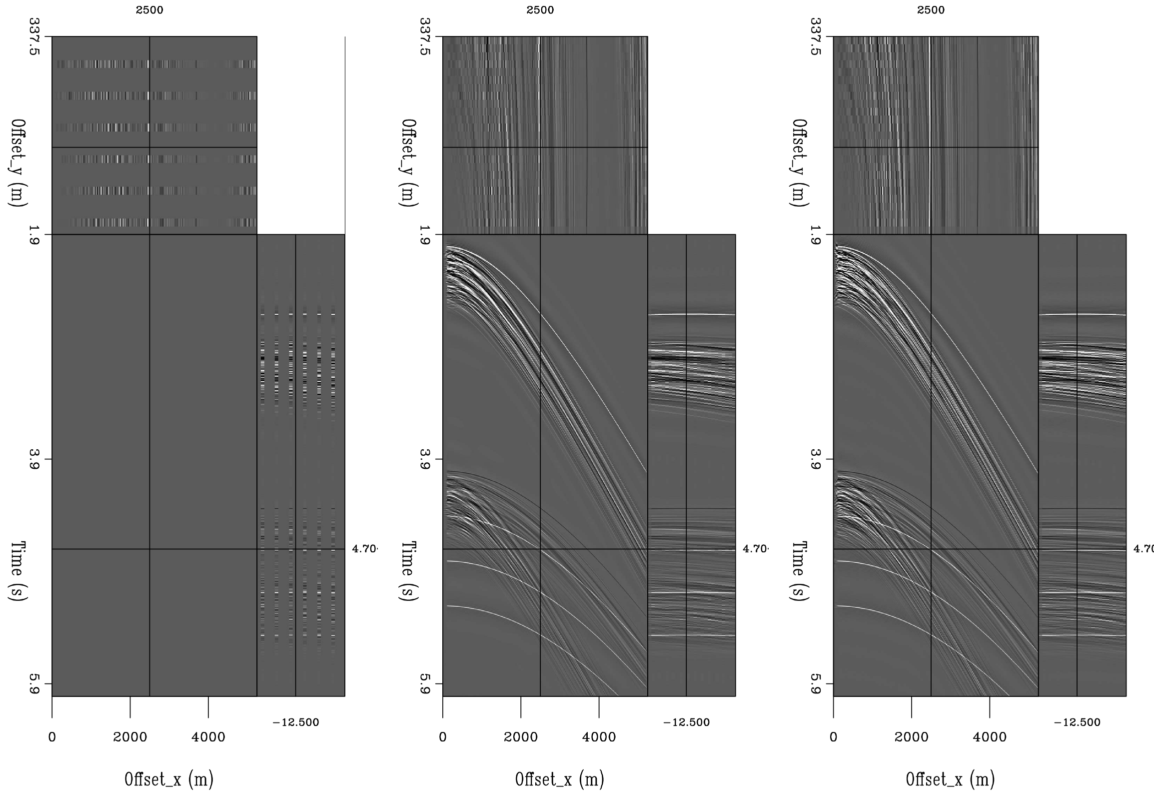

Figure 6: Receiver cable interpolation, factor of 2. Only 6 cables were used for the left and center images. From left to right: original input shot gather with 6 cables; result of 3D interpolation; result of 3D interpolation with 12 cables as input. bill2-6cable [CR,M]

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