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Interpolation with pseudo-primaries

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ABSTRACT

Large gaps exist in marine data, particularly at near offsets. I generate pseudo-primaries by cross-correlating a multiple model with the original data. These pseudo-primaries are used as training data for a non-stationary prediction- error filter, which is then used to interpolate the missing near offsets. This method yields good results, and also provides a quality control measure to judge the usefulness of the pseudo-primaries.

INTRODUCTION

Interpolation has become of more importance recently, largely due to increased reliance on algorithms that require dense and regular data sampling, such as wave-equation migration and 3D surface-related multiple elimination (SRME) (Van Dedem and Verschuur, 2005). Examples of current methods include Fourier (Duijndam and Schonewille, 1999; Liu and Sacchi, 2004; Xu et al., 2005), Radon transform (Trad, 2003), and prediction-error filter (PEF) based methods (Spitz, 1991). Other methods that rely on the underlying physics (and typically also a velocity model) include migration/demigration (**?**), DMO-based methods (Biondi and Vlad, 2001), and the focal transform (**?**), which requires an input focal operator instead of velocity.

In this paper, I describe a hybrid approach that combines both non-stationary prediction-error filters (Crawley, 2000) and pseudo-primaries generated from surface-related multiples (Shan and Guitton, 2004) in order to interpolate missing near offsets. I generate pseudo-primaries by a surface-consistent cross-correlation of a multiple model with the input data. Once the pseudo-primaries have been generated, I estimate a non-stationary PEF on the pseudo-primaries by solving a least-squares problem. I then solve a second least-squares problem where the newly found PEF is used to interpolate the missing data (Claerbout, 1999).

The data used in this example is from the Sigsbee2B synthetic dataset where the first 2000 feet of offset were removed. Near-offset data is typically missing from marine data, and large near-offset gaps can exist when undershooting obstacles such as drilling platforms. I estimate a PEF on the original data (with the missing offsets) and can produce an ideal reconstruction. Estimating a PEF on the pseudo-primaries, which are generated without the near offset data, gives promising results, which can be quality-controlled with the output of the convolution of the pseudo-primary-derived PEF with the recorded data.

GENERATION OF PSEUDO-PRIMARIES

Pseudo-primaries can be generated by computing (Shan and Guitton, 2004)

$$W(x_p, x_m, \omega) = \sum_{x_s} M(x_s, x_m, \omega) \bar{P}(x_s, x_p, \omega), \tag{1}$$

where W is the pseudo-primary data, ω is frequency, x_s is the shot location, x_p is the surface location, $\bar{P}(x_s, x_p, \omega)$ is the complex conjugate of the original trace at (x_s, x_p) and M are the multiple reflections recorded at x_m. In this equation, the cross-correlation of the first-order multiples in M with the primaries and first-order multiples in P produces primaries and zero-lag components, respectively. Cross-correlation of the second-order multiples in M with the primaries, first-order, and second-order multiple reflections in P produces first-order multiples, primaries, and zero-lag components, respectively. With higher orders of multiples this trend continues.

Pseudo-primaries generated in this fashion contain subsurface information that would not be recorded with a non-zero minimum offset. One example of this is a first-order multiple that reflects at the free surface within the recording array, resulting in near offsets being recorded when that wave returns to the surface. An example of this is shown in Figure 1, where (a) is a single Sigsbee2B shot (including the negative offsets predicted by reciprocity) but with offsets less than 2000 feet removed, and (b) is the corresponding pseudo-primaries for the same area, which is generated in part with (a).

We can see in Figure 1 where the first and second-order multiples in P map to in the zero-lag at the top of the image. We can also see a lot of near-offset information present in the pseudo-primaries that is not present in the recorded primaries. However, simply replacing the missing near offsets of the primaries with the corresponding pseudo-primaries would not yield a satisfactory result due to the crosstalk and noise in the pseudo-primary shot.

INTERPOLATION WITH NON-STATIONARY PEFS

Interpolation can be cast as a series of two inverse problems where a prediction-error filter is estimated on known data and is then used to interpolate missing data. A prediction-error filter (PEF) can be estimated by minimizing the output of convolution of known data with an unknown filter (except for the leading 1), which can be written in matrix form as

$$\mathbf{0} \quad \approx \quad \mathbf{r} = \begin{bmatrix} d_2 & d_1 & d_0 \\ d_3 & d_2 & d_1 \\ d_4 & d_3 & d_2 \\ d_5 & d_4 & d_3 \\ d_6 & d_5 & d_4 \end{bmatrix} \begin{bmatrix} 0 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} 1 \\ f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix}, \tag{2}$$

where f_i are unknown filter values and d_i are known data values.

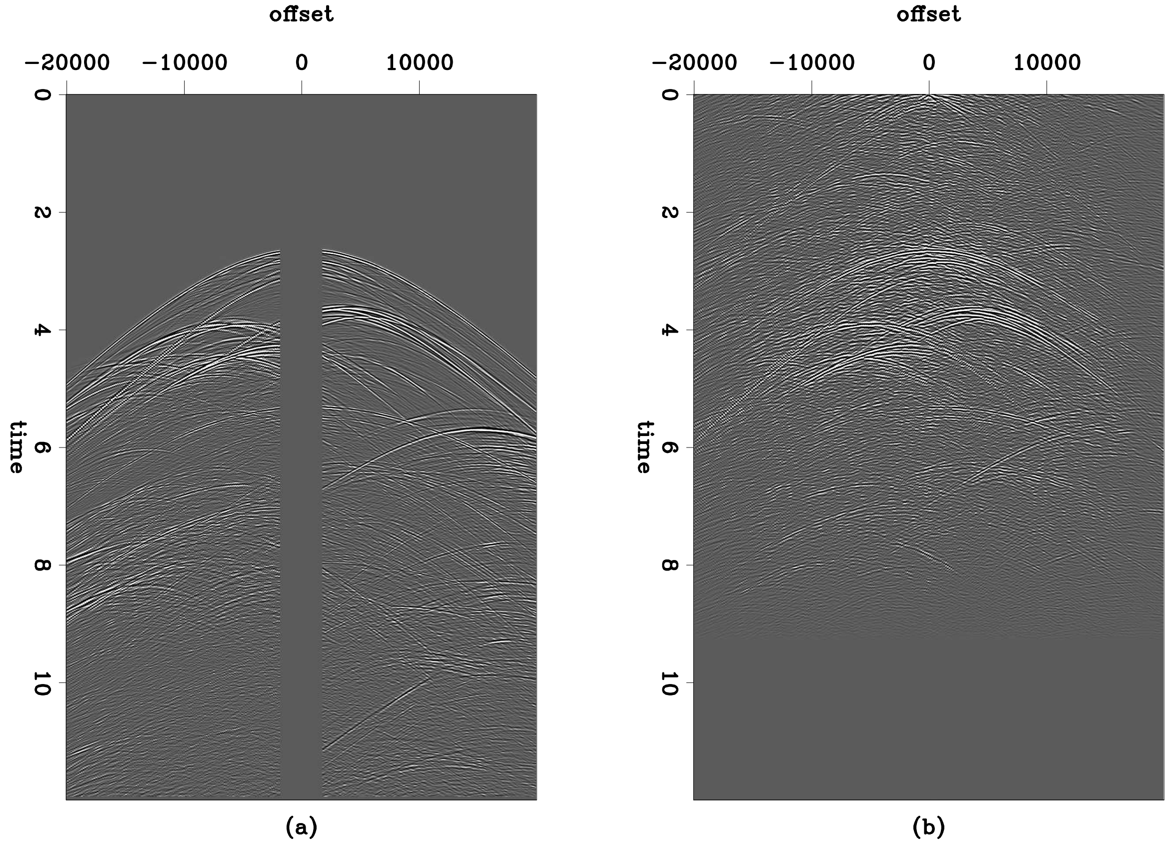

Figure 1: (a) Original shot record and (b) pseudo-primaries for the same area from the Sigsbee2B dataset. bill1-shot [CR]

The filters used in this paper are all multidimensional, which are computed with the helical coordinate. In the case of a stationary multidimensional PEF, this is an over-determined least-squares problem with a unique solution.

Seismic data is non-stationary in nature, so a single stationary PEF is not adequate for the many changing dips present. We estimate a single spatially-variable nonstationary PEF and solve a global optimization problem (Guitton, 2003). In that case the problem is now under-determined, and a regularization operator is introduced to the least-squares problem (in matrix notation) so that,

$$\begin{aligned} \mathbf{W}(\mathbf{DKf}+\mathbf{d}) &\approx \mathbf{0} \\ \epsilon\mathbf{Af} &\approx \mathbf{0}, \end{aligned} \tag{3}$$

where $\mathbf{D}$ represents non-stationary convolution with the data, $\mathbf{f}$ is now a non-stationary PEF, $\mathbf{K}$ (a selector matrix) and $\mathbf{d}$ (a copy of the data) both constrain the value of the first filter coefficient to 1, $\mathbf{A}$ is a regularization operator (a Laplacian operating over space) and ϵ is a trade-off parameter for the regularization. Solving this system will create a smoothly non-stationary PEF.

Once the PEF has been estimated, it can be used in a second least squares problem that

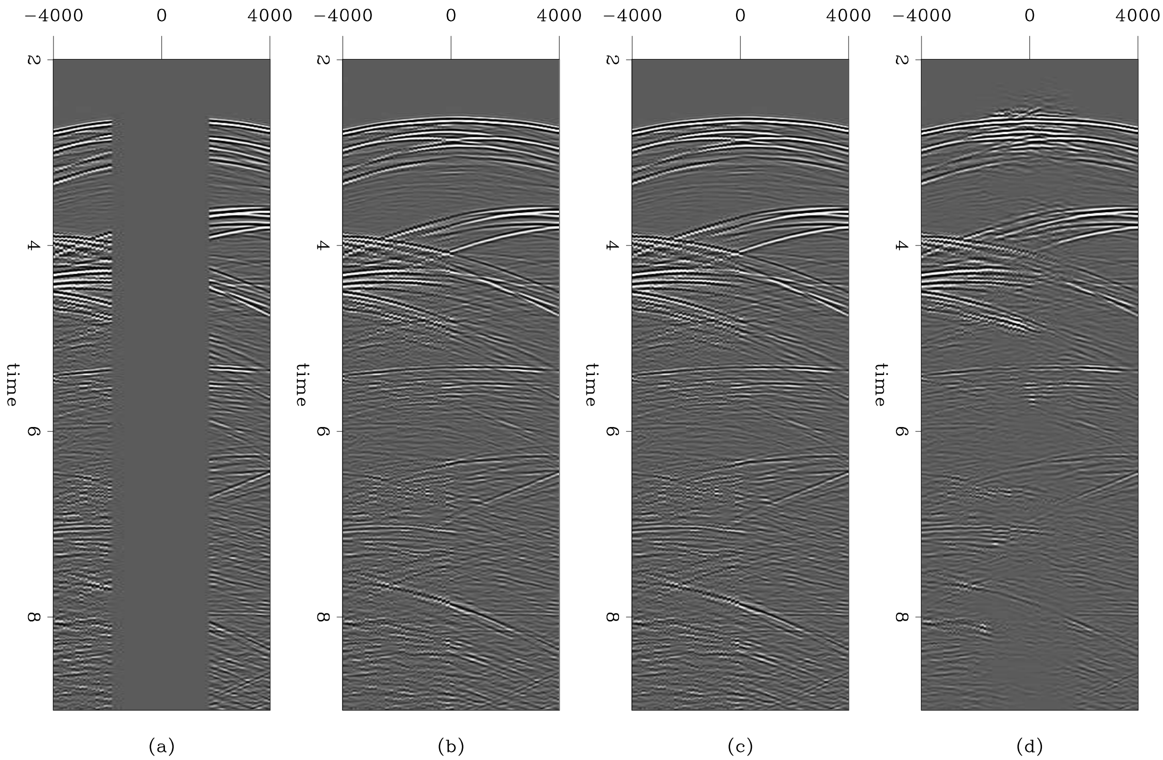

Figure 2: (a) Original data with near offsets (<2000 feet) missing. (b) Original complete data. (c) Interpolation with PEF based upon complete data. (d) Interpolation with PEF based upon pseudo-primaries. bill1-interped [CR]

matches the output model to the known data while simultaneously regularizing the model with the newly found PEF,

$$\begin{aligned} \mathbf{S}(\mathbf{m} - \mathbf{d}) &\approx \mathbf{0} \\ \epsilon \mathbf{F}\mathbf{m} &\approx \mathbf{0}, \end{aligned} \tag{4}$$

where $\mathbf{S}$ is a selector matrix which is 1 where data is present and 0 where it is not, $\mathbf{F}$ represents convolution with the non-stationary PEF, ϵ is now a trade-off parameter and $\mathbf{m}$ is the desired model.

RESULTS

To increase the sampling by an integer factor, a PEF is typically estimated on the input data. In this example with a large gap, this will not suffice. Instead, we estimate the PEF on the pseudo-primaries generated by equation 1 using equation 3 and then use that PEF to interpolate the recorded data with equation 4. The results of this experiment are shown in Figure 2.

The near offset gap is 4000 feet or 53 traces, as shown in Figure 2(a). The complete data in Figure 2(b) is used as input to equation 3 with Figure 2(a) as input to equation 4, which

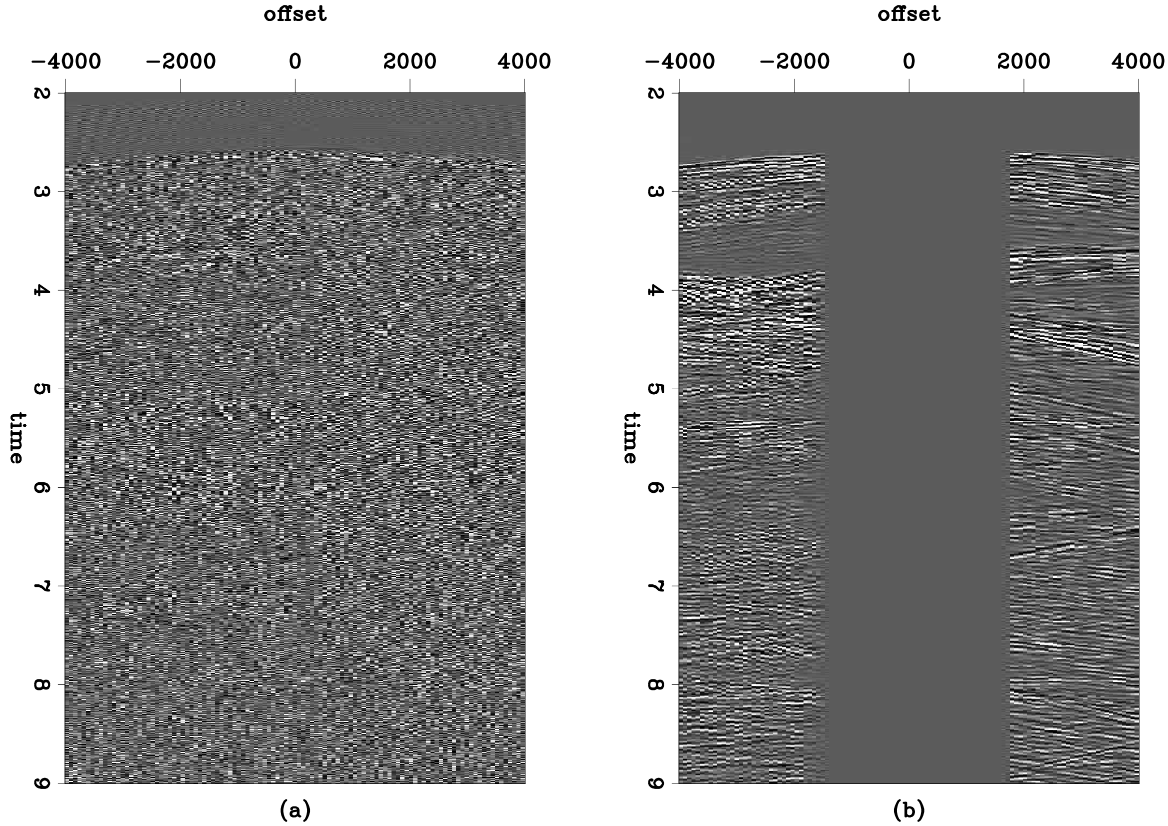

Figure 3: (a) PEF estimated on pseudo-primaries convolved with pseudo-primaries. (b) PEF after convolution with input data. bill1-resid [CR]

produces Figure 2(c), which illustrates that if we estimate the PEF on the answer we can perfectly recreate the data. Figure 2(d) shows the main result of this paper, which is when the pseudo-primaries are used as input to equation 3.

The results in Figure 2(d) are promising, but not ideal. Most of the events are successfully continued through the data, but some interference is present. One method to quality control the PEF estimation is to examine the residual of the estimation of equation 3. If this result is uncorrelated and low in amplitude, the PEF has captured all of the useful information in the pseudo-primaries. Similarly, the PEF can then be convolved with the recorded data, with the output shown in Figure 3.

Figure 3(b) shows that while the PEF estimated on the pseudo-primaries does a good job of whitening the recorded data, the result is not ideal, unlike Figure 3(a) where the PEF is convolved with the pseudo-primaries. Differences in spectral content of the data are the most obvious cause, and any adjustments to this algorithm can be quality controlled by looking at this intermediate result.

CONCLUSIONS AND FUTURE WORK

Incorporating pseudo-primary data into a non-stationary prediction-error filter based interpolation method gives promising results for large gaps in the near offset. While most interpolation algorithms suffer from an objective measure of the quality of interpolation in practical applications, the usefulness of the pseudo-primaries can be judged in a relatively objective manner by looking at the convolution of the pseudo-primary based PEF with the recorded data. Future work includes reducing this difference and examining the final results after SRME compared to a high-resolution parabolic radon transform.

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