

Imaging primaries  
~~Primary~~ and multiples simultaneously ~~imaging~~ with  
^  
depth-focusing

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## ABSTRACT

Seismic imaging ~~value is taken out with~~ <sup>extracted using</sup> the imaging condition  ~~$t = 0$  and  $h = 0$~~   <sup>$t = 0$  and  $h = 0$</sup> , where  $t = 0$  means that the take-off time of ~~upward-coming wave~~ <sup>the upward-coming wave</sup> is zero and  $h = 0$  means that the downward-coming and upward-coming wave ~~meet together~~  <sup>$S$  meet together</sup> during the wavefield extrapolation. However,  $h = 0$  makes no sense for ~~the multiples imaging~~ <sup>the multiples imaging</sup>; here  $h$  is the half-offset between the source and receiver position. This imaging condition is suitable for the primary ~~imaging and~~ <sup>imaging</sup> and the source position must be known. We ~~put forward~~ <sup>introduce</sup> an imaging condition for primary and multiples ~~simultaneously imaging~~ <sup>simultaneously imaging the</sup>. The imaging condition, in essence, is ~~composed with~~ <sup>composed with</sup> that the take-off time of the upcoming wave equals to zero and the radius of the curvature of the wavefront of the upcoming scattering equals to zero. It is known that the primary and multiples ~~scattering wave~~ <sup>scattering wave</sup> will be focused during the wavefield depth extrapolation and the primary and multiples ~~scattering waves~~ <sup>scattering waves</sup> at the same depth focus at different ~~time~~ <sup>times; this is</sup> because the traveltimes ~~from the source to the scattering point~~  <sup>$S$  from the source to the scattering point</sup> are ~~different~~ <sup>for the primary and the multiples,</sup> even for the same scattering point ~~to the primary and multiples~~. The focused scattering wave can be picked out and the image is formed at the focused point. The advantages of the method ~~include that~~ <sup>are several:</sup> the primary and multiples can be imaged simultaneously, only the up-coming wave ~~need to~~ <sup>must</sup> be downward extrapolated, all the scattering wavefield ~~in the different shot gathers~~  <sup>$S$  in the different shot gathers</sup> can be added together and simultaneously extrapolated, and the source position can be unknown. Its disadvantage is that the imaging condition is much more difficult to ~~be used~~ <sup>use</sup>.

[one word]

## INTRODUCTION

Up to now, the multiples in seismic data sets ~~are considered to be noises~~ <sup>have been considered as noise</sup> for the imaging of the primaries (Berkhout and Verschuur, 1997). ~~The reason is that~~ <sup>This is because</sup> it is difficult to ~~put~~ <sup>assign</sup> the multiples ~~onto its~~ <sup>to their</sup> scattering points ~~because~~ <sup>since</sup> the common-used imaging condition ~~s~~ <sup>ly</sup> can not correctly and simultaneously ~~ly~~ <sup>both</sup> pick up the focused primary and ~~its~~ <sup>its</sup> multiples.

[italics] Schuster et al. (2003) ~~put forward~~ <sup>proposed</sup> that if the source under the surface is unknown, the autocorrelation of each trace can be used to determine a suppositional source on the surface, because the autocorrelation of the direct wave is at the delaying time  $t = 0$  and the direct wave is eliminated in the autocorrelogram. The autocorrelogram can be thought to be acquired with the new suppositional shot-receiver pair. Therefore, to the autocorrelogram, the conventional prestack depth migration can ~~make~~ <sup>cause</sup> the ghost wave to be focused and imaged. ~~The ghost wave imaging is the multiples imaging.~~ <sup>, or the first multiple,</sup> However, the disadvantages of the method are that the autocorrelogram ~~does not hold on~~ <sup>apply for</sup> the wave equation, and the traveltime of the direct wave can not be correctly estimated and cancelled ~~at some time, which makes~~ <sup>later,</sup> the travel time calculation in the integral migration does not match the travel time in the autocorrelogram. ~~The imaging noises occur.~~ <sup>so that</sup> The ~~imaging~~ <sup>result is</sup> noises ~~also cause the~~ <sup>causes</sup> imaging noises. On the other hand, the ghost wave is the first multiples, ~~the~~ <sup>and</sup> imaging of higher-order multiples is ignored.

The imaging condition proposed by Claerbout (1971) should be modified if the primary and multiples are simultaneously imaged ~~in the case of the source position known or unknown.~~ <sup>whether</sup> ~~the~~ <sup>source</sup> ~~is~~ <sup>is</sup> known or unknown. The imaging condition is that the radius of the curvature of the wavefront equals ~~to~~ <sup>to</sup> zero. This is the depth-focusing imaging condition.

MacKay and Abma (1993) ~~takes use of the~~ depth focusing to carry out the velocity ~~nan-~~ <sup>analysis</sup> ~~ysis~~. If the migration velocity is larger than the medium velocity, <sup>then</sup> the focusing depth is less than the reflection depth and the imaging depth is larger than the reflection depth; On the other hand, if the migration velocity is less than the medium velocity, <sup>then</sup> the focusing depth is greater than the reflection depth and the imaging depth is less than the reflection depth. The real reflection depth lies at the <sup>mid-</sup> ~~middle~~ point <sup>between</sup> of the focusing depth and the imaging depth. In the <sup>the authors propose</sup> paper, a method for estimating the radius of the curvature of the wavefront ~~was put forward~~. However the formula is <sup>imaging</sup> ~~only~~ suitable for the primary ~~imaging~~.

[lower case (l.c.)]

<sup>a given</sup> For the same scattering point, the primary and multiples scattering ~~wavefield~~ <sup>are</sup> from it is simultaneously focused at the same depth and at different <sup>times</sup> time with the downward wavefield continuation. The "focusing" means that the received scattered wavefield from a scattering point is collapsed into the scattering point and the radius of ~~the~~ curvature of the wavefront diminish <sup>es</sup> to zero. With the depth-focusing imaging condition, the focused imaging <sup>values</sup> ~~value~~ of the primary and multiples can be simultaneously picked up from the depth extrapolated wavefield, which is expressed in the time domain.

<sup>The following</sup> There are some ~~following~~ advantages <sup>of</sup> ~~about the~~ depth-focusing imaging. The primaries and multiples (including the higher-order multiples) can be simultaneously imaged; the source position can be known (for the primaries) or unknown (for the multiples); All of the scattered wavefield can be added together and the calculation efficiency can be improved.

[l.c.]

The disadvantage is that the depth-focusing imaging condition is difficult to <sup>use,</sup> ~~be used~~, especially for ~~the~~ data with a lot of noise.

[Not clear; how can waves be focused "simultaneously" and yet "at different times?" I'm not sure what you mean here.]

## PRINCIPLES OF FOCUSING

Figure 1 geometrically shows the depth-focusing process of the primary scattering wavefield, and Figure 2 shows the same process for the multiple scattering wavefield. Comparing the two figures, it is clearly seen that the focusing process is the same for a scattering point, whether the scattering wavefield from it is primary scattering or multiple scattering.

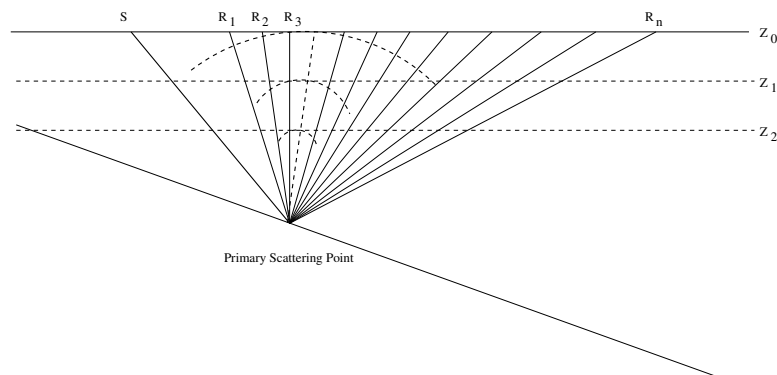


Figure 1: The depth focusing of the primary scattering wavefield with the extrapolation. The radius of the curvature of the wavefront diminishes to zero with the downward wavefield continuation. [Scattering\_primary] [ER]

The imaging condition of prestack migration is that the arrival time of the downgoing wave equals to the take-off time of the upcoming wave (Claerbout, 1971). In principle, this is a prevalent imaging condition. However, conventionally, the downgoing wave means the primary downgoing wave not the multiple downgoing wave. It is difficult to determine the traveltime of the multiple downgoing wave. Therefore, with this imaging condition, it is complex to image the primaries and the multiples simultaneously.

The conventional imaging condition implicitly tell us that the image of a reflector appears

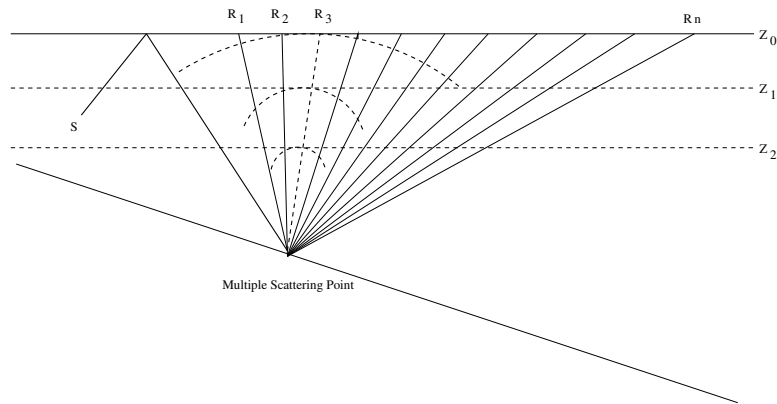


Figure 2: The depth focusing of the multiple scattering wavefield with the extrapolation. The source position can be unknown. The radius of the curvature of the wavefront diminishes to zero with the downward wavefield continuation. The higher-order multiples can be focused also. Scattering\_multiples [ER]

at the point where the received scattering wavefield is collapsed. <sup>In this case,</sup> ~~At the case,~~ the arrival time of the downgoing wave equals <sup>to</sup> ~~the~~ take-off time of the upcoming wave and the radius of the curvature of the scattering wavefield <sup>diminishes</sup> ~~diminish~~ to zero. <sup>[u.c.]</sup> ~~And that the radius of the curvature of the scattering wavefield diminish to zero means the scattering wavefield is focused to the scattering point. It is different with the primary imaging that, to the multiples imaging, the offset equals to zero makes no sense. For the multiples imaging, the offset should be calculated with the "suppositional" source and the receiver position. However, it is not easy to determine the "suppositional" source position for the higher-order multiples.~~

In fact, the statement that the arrival time of the downgoing wave equals ~~to~~ the take-off time of the upcoming wave is equivalent <sup>saying</sup> ~~to the one~~ that the radius of the curvature of the scattering wavefield <sup>diminishes</sup> ~~diminish~~ to zero. However, the latter is much more prevalent than the former. The latter can be used to image the primaries and the multiples, <sup>whether</sup> ~~whatever~~ the source

because the only criterion is position is known or unknown ~~because only~~ whether the scattering wavefield is focused or not ~~is concerned~~. The latter can be called as the depth-focusing imaging condition. The former is a model-driven process; the latter is a hybrid-driven process. ~~wavefield extrapolation is model-driven and the picking the focused amplitude is data-driven.~~

[u.c.]

## IMPLEMENTATION OF DEPTH-FOCUSING

Estimating whether the scattering wavefield is focused or not is a tough work ~~about~~ <sup>dealing with</sup> primaries and multiples <sup>imaged</sup> ~~simultaneously~~ <sup>imaging</sup> with depth-focusing. The wavefield extrapolation is carried out in ~~depth domain~~ <sup>the</sup> and picking the imaging value must be implemented in ~~time domain~~ <sup>the</sup> since the traveltime from the source to the scattering point is not necessarily ~~to be~~ known. Supposing that the ~~macro~~ <sup>large-scale</sup> velocity model is credible, the horizontal positions of the focused scattering points are correct. ~~And the~~ <sup>The</sup> wavefield extrapolation depth determines the focused depth of the scattering ~~which is also correct under the assumption.~~ The apparent method is to ~~take use of~~ the amplitude of the focused scattering wavefield. When a scattering wavefield is focused, the amplitude at a focused point ~~reaches the maximum.~~ <sup>is maximized</sup>

During the process of wavefield extrapolation, the amplitude of the wavefield at every point ~~are fluctuating.~~ <sup>fluctuates</sup> Therefore, the amplitude itself can not be used as an indication. Here, ~~I~~ <sup>we</sup> suggest that the characteristic parameters ~~of~~ depicting the amplitude change should be used, such as the envelope of the amplitude change, the derivative of the envelope, ~~etc.~~ <sup>and so on.</sup> Hence, several extrapolated wavefields should be memorized, ~~which include~~ <sup>including</sup> the current extrapolated layer and its adjacent layers. ~~All of the methodology is in order to avoid picking the wrong focused amplitude.~~ <sup>The purpose of all</sup>

[two authors]

[Do you mean "recorded?"]

Another method is to estimate the radius of the curvature of the wavefront of the scattering wave. MacKay and Abma (1993) put forward an method that, in the CMP geometry, there is a formula uses the following formula:

$$R \approx \frac{(X^2 - \Delta t^2 V_r^2)}{2\Delta t V_r} \quad (1)$$

where  $X$  is the offset,  $V_r$  is the medium velocity, and  $\Delta t$  is the time difference between the two-way vertical traveltimes and the observed traveltimes. However, this formula is not suitable to be used here because the time difference is unknown. For depth-focusing imaging, the source position is not concerned and the traveltimes between the source and the scattering point is not explicitly used.

We propose put forward the following method to estimate the radius of the curvature of the scattered wavefield. Assuming that the large-scale macro velocity is correct and with help of ray-tracing, the radius of the curvature of the scattered wavefield can be estimated with the following formula:

$$R = V_r t_{scatter} \quad (2)$$

where  $t_{scatter}$  is the traveltimes from the scatterer to the receivers,  $V_r$  is the medium velocity. And  $\Delta t = t - t_s = t_{scatter}$  where  $t$  is the observed two-way traveltimes and  $t_s$  is the traveltimes from the source to the scatter point. The value of  $t_s$  may include the traveltimes of the multiples. According to the equation 2, the radius of the curvature of the scattered wavefield can be estimated with the extrapolated wavefield.

Some ideas in Jager et al. (2001) may suggest give us some suggestions about how to estimate the radius of the curvature of the scattered wavefield.



## DISCUSSION AND CONCLUSION

<sup>propose</sup> We ~~put forward~~ a new imaging condition, with which<sup>/</sup> the wavefield extrapolation is carried out in <sup>the</sup> depth domain and the imaging value is <sup>extracted</sup> ~~picked out~~ from the focused scattered wavefield in the time domain if the radius of the curvature of the wavefront <sup>diminishes to</sup> ~~diminish into~~ zero. We call <sup>this</sup> the imaging condition ~~as~~ the depth-focusing imaging condition.

We <sup>assert</sup> ~~think~~ that the statement that the arrival time of the downgoing wave equals to ~~the~~ take-off time of the upcoming wave is equivalent to <sup>saying</sup> ~~the one~~ that the radius of the curvature of the scattering wavefield <sup>diminishes</sup> ~~diminish~~ to zero.

With the imaging condition, the primaries and the multiples can be simultaneously imaged. <sup>The</sup> ~~And the~~ source position can be known or unknown<sup>,</sup> therefore the passive data can be imaged with it. Some shot gathers can be added together according <sup>to</sup> the receiver positions and then the new data set is imaged with the above method <sup>thus</sup> ~~for~~ improving the calculation efficiency. The depth-focusing imaging condition can be used for <sup>the</sup> ~~the~~ imaging <sup>of the</sup> ~~of the~~ multicomponent seismic data.

However, since the focusing of the scattering wave is detected in the time domain, the quality of the data set will affect the use of the imaging condition. <sup>be best suited for processing</sup> It will ~~be better for it to be~~ used <sup>S</sup> ~~in the process of the~~ marine data set.

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