Stanford Exploration Project, Report 120, May 3, 2005, pages 1–??

Geomechanical constants of heterogeneous reservoirs: pore fluid effects on shear modulus

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ABSTRACT

To provide quantitative measures of the importance of fluid effects on shear waves in the heterogeneous reservoirs, a model material called a "random polycrystal of porous laminates" is introduced. This model poroelastic material has constituent grains that are layered (or laminated), and each layer is an isotropic, microhomogeneous porous medium. All grains are composed of exactly the same porous constituents, and have the same relative volume fractions. But the order of lamination is not important because the up-scaling method used to determine the transversely isotropic (hexagonal) properties of the grains is Backus averaging, which — for quasi-static or long-wavelength behavior — depends only on the volume fractions and layer properties. Grains are then jumbled together totally at random, filling the reservoir, and producing an overall isotropic poroelastic medium. The poroelastic behavior of this medium is then analyzed using the Peselnick-Meister-Watt bounds (of Hashin-Shtrikman type). We study the dependence of the shear modulus on pore fluid properties and determine the expected range of behavior. In particular we compare and contrast these results with those anticipated from Gassmann's fluid substitution formulas, and to the predictions of Mavko and Jizba for very low porosity rocks with flat cracks. This approach also permits the study of arbitrary numbers of constituents, but for simplicity the numerical examples are restricted here to just two constituents. This restriction also permits the use of some special exact results available for computing the overall effective stress coefficient in any two-component porous medium. The bounds making use of polycrystalline microstructure are very tight. Results for shear modulus demonstrate that the ratio of compliance differences R (*i.e.*, shear compliance changes over bulk compliance changes) is usually nonzero and can take a wide range of values, both above and below the value $R = 4/15$ for low porosity, very low aspect ratio flat cracks. Results show the overall shear modulus in this model can depend relatively strongly on mechanical properties of the pore fluids, sometimes (but rarely) more strongly than the dependence of the overall bulk modulus on the fluids.

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INTRODUCTION

Heterogeneity of the earth plays a significant role in determining geophysical and geomechanical constants such as the bulk and shear moduli and the elastic and/or poroelastic wave speeds. The heterogeneities of importance may be due to fine layering (Postma, 1955; Backus, 1962) [layers being thin compared to seismic wavelength], due to partial or patchy saturation of pore fluids (White, 1975; Knight and Nolen-Hoeksema, 1990; Dvorkin *et al.*, 1999; Johnson, 2001; Li *et al.*, 2001), due to random positioning of joints and fractures (Berryman and Wang, 1995; Pride and Berryman, 2003; Pride *et al.*, 2004), due to anisotropic stress distribution, etc. There have been many attempts to attack all of these problems, and the up-scaling methods employed have ranged from *ad hoc* to mathematically rigorous, and have had varying degrees of success in modeling field and laboratory data.

One of the main purposes of the present paper is to introduce a semi-analytical model of the earth, and especially of fluid-bearing earth reservoirs, that provides well-controlled estimates of the properties of most interest such as elastic/poroelastic constants, fluid permeability, etc. The concept is based on "random polycrystals of porous laminates." Locally layered regions are treated as laminates and the poroelastic and other constants can be computed essentially exactly (*i.e.*, within the assumed long wavelength limit and perfect layering of the laminate model) using Backus (1962) averaging for poroelastic constants (and similar methods for other parameters), in the long-wavelength or quasi-static limits. Then, since such layered materials are typically anisotropic (having hexagonal symmetry when the layers are isotropic), I assume that the earth is composed of a statistically isotropic jumble of such layered regions. The locally layered, anisotropic regions may be termed "grains" or "crystals." Then, the overall behavior of this system can be determined/estimated using another method from the theory of composites: the well-known Hashin-Shtrikman bounds (Hashin and Shtrikman, 1962). In this case the bounds of interest for the types of crystal symmetry that arise are those first obtained by Peselnick and Meister (1965) and later refined by Watt and Peselnick (1980). These bounds have been refined further recently by the author (Berryman, 2004b; 2005). In particular, these recent refinements provide sufficient insight into the resulting equations that self-consistent estimates (lying between the rigorous bounds) of the elastic constants can be formulated and very easily computed. I find that the Peselnick-Meister-Watt upper and lower bounds are already quite close together for this model material, so the resulting self-consistent estimates are very well constrained. The bounds then serve as error bars on the self-consistent model estimates.

The method being introduced can be applied to a wide variety of difficult technical issues concerning geomechanical constants of earth reservoirs. The one issue that will be addressed at length here is the question of how shear moduli in fully saturated, partially saturated, and/or patchy saturated porous earth may or may not depend on mechanical properties of the pore fluids. The well-known fluid substitution formulas of Gassmann (1951) [also see Berryman (1999)] show that — for isotropic, microhomogeneous (single solid constituent) porous media — the undrained bulk modulus depends strongly on a pore-liquid's bulk modulus, but the undrained shear modulus is not at all affected by changes in the pore-liquid modulus. Since the system we are considering violates Gassmann's microhomogeneity constraint as well as

the the isotropy constraint in the vicinity of layer interfaces, I expect that the shear modulus will in fact depend on the fluid properties in this model (Mavko and Jizba, 1991; Berryman and Wang, 2001; Berryman *et al.*, 2002b). The semi-analytical model presented here allows me to explore this issue in some detail, to show that overall shear modulus does depend on pore-fluid mechanics, and to quantify these effects.

The next section introduces the basic tools used later in the layer analysis. The third section reviews the Peselnick-Meister-Watt bounds and presents the new formulation of them. The fourth section summarizes the results needed from poroelastic analysis. The fifth section presents the main new results of the paper, including four distinct scenarios that help elucidate the behavior of the overall shear modulus and compare it to that of the bulk modulus. The final section summarizes our conclusions. Appendix A provides a brief proof of one of the results used in the text concerning the behavior of the effective stress coefficient for patchy saturation. Appendix B shows that Hill's equation should be used cautiously in analysis of heterogeneous reservoirs.

ELASTICITY OF LAYERED MATERIALS

We assume that a typical building block of the random system is a small grain of laminate material whose elastic response for such a transversely isotropic (hexagonal) system can be described locally by:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & \\ c_{12} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & 2c_{44} & & \\ & & & & 2c_{44} & \\ & & & & & 2c_{66} \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix}, \tag{1}$$

where σ_{ij} are the usual stress components for $i, j = 1-3$ in Cartesian coordinates, with 3 (or z) being the axis of symmetry (the lamination direction for such a layered material). Displacement u_i is then related to strain component e_{ij} by $e_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$. This choice of definition introduces some convenient factors of two into the $44, 55, 66$ components of the stiffness matrix shown in (1).

For definiteness we also assume that this stiffness matrix in (1) arises from the lamination of N isotropic constituents having bulk and shear moduli K_n, μ_n, in the $N > 1$ layers present in each building block. It is important that the thicknesses d_n always be in the same proportion in each of these laminated blocks, so that $f_n = d_n / \sum_{n'} d_{n'}$. But the order in which layers were added to the blocks in unimportant, as Backus's formulas (Backus, 1962) for the constants show. For the overall behavior for the quasistatic (long wavelength) behavior of the system we are studying, Backus's results [also see Postma (1955), Berryman (1998,2004a,b), Milton

(2002)] state that

$$\begin{aligned} c_{33} &= \left\langle \tfrac{1}{K+4\mu/3} \right\rangle^{-1}, & c_{13} &= c_{33}\left\langle \tfrac{K-2\mu/3}{K+4\mu/3} \right\rangle, \\ c_{44} &= \left\langle \tfrac{1}{\mu} \right\rangle^{-1}, & c_{66} &= \langle \mu \rangle, \\ c_{11} &= \tfrac{c_{13}^2}{c_{33}} + 4c_{66} - 4\left\langle \tfrac{\mu^2}{K+4\mu/3} \right\rangle, & c_{12} &= c_{11} - 2c_{66}. \end{aligned} \tag{2}$$

This bracket notation can be correctly viewed: (a) as a volume average, (b) as a line integral along the symmetry axis x_3, or (c) as a weighted summation $\langle Q \rangle = \sum_n f_n Q_n$ over any relevant physical quantity Q taking a constant value Q_n in the n-th layer.

The bulk modulus for each laminated grain is that given by the compressional Reuss average K_R of the corresponding compliance matrix s_{ij} [the inverse of the usual stiffness matrix c_{ij}, whose nonzero components are shown in (1)]. The well-known result is given by $e = e_{11} + e_{22} + e_{33} = \sigma / K_{\rm eff}$, where $1/K_{\rm eff} = 1/K_R = 2s_{11} + 2s_{12} + 4s_{13} + s_{33}$. When $\mu_n = \mu$ is constant in a layered grain, the definition of K_R implies Hill's equation (Hill, 1963, 1964; Milton, 2002), which is given by

$$K^* = \left[\sum_{n=1}^{N} \frac{f_n}{K_n + 4\mu/3} \right]^{-1} - 4\mu/3. \tag{3}$$

Here the bulk modulus of the n-th constituent is K_n, the shear modulus takes the same value $\mu_n = \mu$ for all $n = 1, \ldots, N$, and the overall effective bulk modulus is K^*. The volume fractions f_n are all nonnegative, and sum to unity.

Even though $K_{\rm eff} = K_R$ is the same for every grain, since the grains themselves are not isotropic, the overall bulk modulus K^* of the random polycrystal does not necessarily have the same value as K_R for the individual grains (Hill, 1952). Hashin-Shtrikman bounds on K^* for random polycrystals whose grains have hexagonal symmetry (Peselnick and Meister, 1965; Watt and Peselnick, 1980) show in fact that the K_R value lies outside the bounds in many situations (Berryman, 2004b).

BOUNDS ON ELASTIC CONSTANTS FOR RANDOM POLYCRYSTALS

Voigt and Reuss Bounds

For hexagonal symmetry, the nonzero stiffness constants are: c_{11}, c_{12}, $c_{13} = c_{23}$, c_{33}, $c_{44} = c_{55}$, and $c_{66} = (c_{11} - c_{12})/2$.

The Voigt average (Voigt, 1928) for bulk modulus of hexagonal systems is well-known to be

$$K_V = [2(c_{11} + c_{12}) + 4c_{13} + c_{33}]/9. \tag{4}$$

Similarly, for the shear modulus we have

$$\mu_V = \frac{1}{5}\left(G_{\rm eff}^{v} + 2c_{44} + 2c_{66}\right), \tag{5}$$

where the new term appearing here is essentially defined by (5) and given explicitly by

$$G^v_{\text{eff}} = (c_{11} + c_{33} - 2c_{13} - c_{66})/3. \tag{6}$$

The quantity G^v_{eff} is the energy per unit volume in a grain when a pure uniaxial shear *strain* of unit magnitude [*i.e.*, $(e_{11}, e_{22}, e_{33}) = (1, 1, -2)/\sqrt{6}$], whose main compressive strain is applied to the grain along its axis of symmetry (Berryman, 2004a,b).

The Reuss average (Reuss, 1929) K_R for bulk modulus can also be written in terms of stiffness coefficients as

$$\frac{1}{K_R - c_{13}} = \frac{1}{c_{11} - c_{66} - c_{13}} + \frac{1}{c_{33} - c_{13}}. \tag{7}$$

The Reuss average for shear is

$$\mu_R = \left[\frac{1}{5}\left(\frac{1}{G^r_{\text{eff}}} + \frac{2}{c_{44}} + \frac{2}{c_{66}}\right)\right]^{-1}, \tag{8}$$

which again may be taken as the definition of G^r_{eff} – *i.e.*, the energy per unit volume in a grain when a pure uniaxial shear *stress* of unit magnitude [*i.e.*, $(\sigma_{11}, \sigma_{22}, \sigma_{33}) = (1, 1, -2)/\sqrt{6}$], whose main compressive pressure is applied to a grain along its axis of symmetry.

For each grain having hexagonal symmetry, two product formulas hold (Berryman, 2004a): $3K_R G^v_{\text{eff}} = 3K_V G^r_{\text{eff}} = \omega_+ \omega_-/2 = c_{33}(c_{11} - c_{66}) - c_{13}^2$. The symbols $\omega_\pm$ stand for the quasi-compressional and quasi-uniaxial-shear eigenvalues for the crystalline grains. Thus, it follows that

$$G^r_{\text{eff}} = K_R G^v_{\text{eff}}/K_V \tag{9}$$

is a general formula, valid for hexagonal symmetry. We can choose to treat (5) and (8) as the fundamental defining equations for G^v_{eff} and G^r_{eff}, respectively. Equivalently, we can use (9) as the definition of G^r_{eff}.

Hashin-Shtrikman Bounds

It has been shown elsewhere (Berryman, 2004a,b) that the Peselnick-Meister-Watt bounds for bulk modulus of a random polycrystal composed of hexagonal (or transversely isotropic) grains are given by

$$K^\pm_{PM} = \frac{K_V(G^r_{\text{eff}} + \zeta_\pm)}{(G^v_{\text{eff}} + \zeta_\pm)} = \frac{K_R G^v_{\text{eff}} + K_V \zeta_\pm}{G^v_{\text{eff}} + \zeta_\pm}, \tag{10}$$

where G^v_{eff} (G^v_{eff}) is the uniaxial shear energy per unit volume for a unit applied shear strain (stress). The second equality follows directly from the product formula (9). Parameters $\zeta_\pm$ are defined by

$$\zeta_\pm = \frac{G_\pm}{6}\left(\frac{9K_\pm + 8G_\pm}{K_\pm + 2G_\pm}\right). \tag{11}$$

In (11), values of $G_\pm$ (shear moduli of isotropic comparison materials) are determined by inequalities

$$0 \le G_- \le \min(c_{44}, G^r_{\rm eff}, c_{66}), \tag{12}$$

and

$$\max(c_{44}, G^v_{\rm eff}, c_{66}) \le G_+ \le \infty. \tag{13}$$

The values of $K_\pm$ (bulk moduli of isotropic comparison materials) are then determined by equalities

$$K_\pm = \frac{K_V(G^r_{\rm eff} - G_\pm)}{(G^v_{\rm eff} - G_\pm)}, \tag{14}$$

given by Peselnick and Meister (1965) and Watt and Peselnick (1980). Also see Berryman, 2004b).

Bounds on the shear moduli are then given by

$$\frac{1}{\mu^\pm_{\rm hex}+\zeta_\pm} = \frac{1}{5}\Big[\frac{1-\gamma_\pm(K_V-K_\pm)}{G^v_{\rm eff}+\zeta_\pm+\delta_\pm(K_V-K_\pm)} + \frac{2}{c_{44}+\zeta_\pm} + \frac{2}{c_{66}+\zeta_\pm}\Big], \tag{15}$$

where $\gamma_\pm$ and $\delta_\pm$ are given by

$$\gamma_\pm = \frac{-1}{K_\pm + 4G_\pm/3} \qquad \text{and} \qquad \delta_\pm = \left[\frac{4}{15} - \frac{2}{5G_\pm\gamma_\pm}\right]^{-1}. \tag{16}$$

K_V is the Voigt average of the bulk modulus as defined previously.

POROELASTICITY ESTIMATES AND BOUNDS

My main focus here will be the extension of earlier work in elasticity to the case of locally layered poroelastic media (Wang, 2000; Coussy, 2004), where the laminated grains (or crystals) are formed by sequential layering of N porous isotropic layers. Although these grains each have the same quasi-static anisotropic elastic behavior, they do not necessarily have the same shapes or the same orientations of their crystal symmetry axes. Specifically, we want to study the case of isotropic random polycrystals, wherein the individuals can and do take on all possible orientations of their symmetry axes (equiaxed, statistically isotropic polycrystals) so that the overall composite polycrystal has isotropic behavior at the macroscopic level. Furthermore, in some applications, the pores of some grain layers may be filled with different fluids (heterogeneous saturation conditions) than those in other layers. This model may or may not be a realistic one for any given fluid-bearing reservoir whose geomechanics we need to model. My first goal is arrive at a model for which many of the available modern tools of elastic and poroelastic analysis apply, including Hashin-Shtrikman bounds for a reservoir having isotropic constituents (Hashin and Shtrikman, 1962a,b,c; 1963a,b), Peselnick-Meister-Watt bounds for

random polycrystals (Peselnick and Meister, 1965; Watt and Peselnick, 1980), certain exact relationships known for two-component poroelastic media (Berryman and Milton, 1991), and — whenever appropriate — self-consistent or other effective medium estimates of both elastic constants and conductivities (electrical, thermal, and hydraulic). By constructing such a model material, we expect to be able to make estimates of the behavior of the system and at the same time be able to predict the range of variation likely to be observed around these estimates, as well as identifying what material and microgeometry properties control those variations. My further goal is to be able to make fairly precise statements about this model that are then useful to our (both mine and the reader's) intuition and to quantify how much is really known about these complex systems. In particular, the hope is to identify assumptions currently and commonly used in the literature without much apparent justification and to provide a means of either verifying or falsifying these assumptions in the context of this model — if that proves to be possible.

Two distinct results that will be required from poroelasticity theory are: (a) Gassmann's equations and (b) certain relationships that determine the overall effective stress coefficient of a composite poroelastic medium when it is composed of two porous materials satisfying Gassmann's assumptions. Gassmann's results (Gassmann, 1951; Berryman, 1999; Wang, 2000) for the undrained bulk (K) and shear (μ) moduli of microhomogeneous (one solid constituent) porous media are:

$$K_u = K_d + \frac{\alpha^2}{(\alpha-\phi)/K_m + \phi/K_f} = \frac{K_d}{1-\alpha B} \tag{17}$$

and

$$\mu_u = \mu_d. \tag{18}$$

Here, K_u and μ_u are the undrained (pore fluid trapped) constants, while K_d and μ_d are the drained (pore fluid untrapped) constants. Porosity (void volume fraction) is ϕ. Grain bulk and shear moduli of the sole mineral constituent are K_m and μ_m. The bulk modulus of the pore fluid is K_f. The factor α is the Biot-Willis (Biot and Willis, 1957) or volume effective stress coefficient (Nur and Byerlee, 1971; Berryman, 1992; Gurevich, 2004), related to K_m and K_d within each layer by

$$\alpha^{(n)} = 1 - K_d^{(n)}/K_m^{(n)}. \tag{19}$$

Skempton's coefficient (Skempton, 1954) is B in (17).

Although my presentation is based on quasi-static results, my ultimate interest is often applications to seismic wave propagation. In such circumstances a slightly different terminology is used by some authors (Mavko and Jizba, 1991). In particular, for high frequency wave propagation, fluid may be effectively trapped in the pores as it is unable to equilibrate through pore-pressure diffusion on the time scale of wave passage. In this case, the term "unrelaxed" is sometimes used instead of "undrained." We will not make any further issue of this distinction here and stick to the single term "undrained" for both types of applications.

For a porous medium composed of only two constituent porous media, each of which is microhomogeneous and obeys Gassmann's equations, the exact relation (Berryman and

Milton, 1991) that determines the overall effective stress coefficient α^* – assuming only that the constituents are in welded contact (volume fractions and spatial distribution of constituents do not directly affect the result) – is:

$$\frac{\alpha^* - \alpha^{(1)}}{\alpha^{(2)} - \alpha^{(1)}} = \frac{K_d^* - K_d^{(1)}}{K_d^{(2)} - K_d^{(1)}}. \tag{20}$$

Here K_d^* is the overall drained bulk modulus of the composite system, and the superscripts (1) and (2) reference the two distinct components in the composite porous medium.

FOUR SCENARIOS

We now consider four scenarios of progressively greater complexity. For the first pair we assume the mineral K_m and drained K_d bulk moduli of all layers are uniform, and therefore that the effective stress coefficient ($\alpha = 1 - K_d/K_m$) is the same in each layer. Furthermore, the overall volume effective stress coefficient is also the same [a fact that follows from (20)]. Although specific in many respects, this model still permits some flexibility in the choice of pore fluids and their spatial distribution. The other main freedom we have left is to assume that each layer's shear modulus is as sensitive or more sensitive than its bulk modulus to irregularities in the pore space (Makse *et al.*, 1999). So, the shear modulus can vary significantly from layer to layer, which will be important to our main discussion. The second pair of scenarios allow the bulk modulus to vary in the layers, and again study both uniform and patchy pore-fluid saturation.

Constant Drained Bulk Modulus, Uniform Fluid Saturation

For assumed constant isotropic drained bulk modulus, we have $K_d^* = K_d^{(n)} \equiv K_d$ for all N layers as well as the overall medium, and when $N = 2$ we can prove easily using (20) that $\alpha^* = \alpha^{(1)} = \alpha^{(2)}$. When the fluid is uniform throughout the medium, the undrained bulk moduli also satisfy $K_u^* = K_u^{(n)} \equiv K_u$, since Gassmann's equation depends only on constants that are uniform throughout this model material. Now it has been shown previously (Berryman, 2004b) that when the drained bulk modulus is uniform, a general result for $G_{\rm eff}^v = G_{\rm eff}^r$ is

$$G_d^v = \left[\sum_{n=1}^{N} \frac{f_n}{\mu_n + 3K_d/4}\right]^{-1} - 3K_d/4, \tag{21}$$

f_n being the volume fraction of the layers. This result follows easily from the Backus averages presented in (2) and the formula for $G_{\rm eff}^v$ in (6). In the presence of pore fluid and since each layer is a Gassmann material, the shear moduli of the individual porous layers do not change. So, a second result of the same type is available for the undrained uniaxial shear energy per unit volume $G_{\rm eff}^v$ in this medium:

$$G_u^v = \left[\sum_{n=1}^{N} \frac{f_n}{\mu_n + 3K_u/4}\right]^{-1} - 3K_u/4, \tag{22}$$

f_n again being the volume fraction of the layers.

Neither of these two shear contributions is the overall modulus. They are just contributions of the uniaxial shear component (within each laminated grain) as defined earlier. However, they can be substituted for the term $G^v_{\rm eff}$ in the Peselnick-Meister-Watt bounds defined by (15). Note that it is easy to show from the forms of (21) and (22) that $c_{44} \le G^v_d \le c_{66}$, and similarly that $c_{44} \le G^v_u \le c_{66}$. [Furthermore, since $K_d \le K_u$ and the functionals in (21) and (22) vary monotonically with their arguments K_d and K_u, it is easy to see that $G^v_d \le G^v_u$.] Thus, from (12) and (13), the best choices for shear moduli of the comparison materials are always given by $G_- = c_{44}$ and $G_+ = c_{66}$ for this particular model material. So $\zeta_\pm = (G_\pm/6)(9K + 8G_\pm)/(K + 2G_\pm)$ in (15), where K takes the values $K = K_d$ for the drained case and $K = K_u$ for the undrained case. In both cases, $K_R = K_V = K$ since the drained bulk modulus is uniform, so the form of the shear modulus bounds in (15) also simplifies to

$$\frac{1}{\mu^\pm + \zeta_\pm} = \frac{1}{5}\left[\frac{1}{G^v_{\rm eff} + \zeta_\pm} + \frac{2}{c_{44} + \zeta_\pm} + \frac{2}{c_{66} + \zeta_\pm}\right]. \tag{23}$$

We now have upper and lower bounds on the shear modulus in both drained and undrained circumstances by using the appropriate values of $G^v_{\rm eff}$ and $\zeta_\pm$ for each case. It is also possible to generate self-consistent estimates (Berryman, 2004b) for these moduli directly from the form of these bounds by instead making the substitutions $\mu^\pm \to \mu^*$ and $\zeta_\pm \to \zeta^* \equiv (\mu^*/6)(9K + 8\mu^*)/(K + 2\mu^*)$. The results of all these formulas are illustrated in Figure 1.

Another important concept in these analyses will be the ratio of compliance differences defined by

$$R \equiv \frac{1/\mu^*_d - 1/\mu^*_u}{1/K^*_d - 1/K^*_u}. \tag{24}$$

This quantity has been defined and discussed previously by Berryman *et al.* (2002b). It is most useful for determining the extent to which an identity derived by Mavko and Jizba (1991) for very low porosity media containing randomly oriented fractures is either satisfied or violated by other types of porous media. For the case studied by Mavko and Jizba (1991), $R \equiv 4/15$. However, it has been shown that for penny-shaped cracks at finite porosities R can be either higher or lower than $4/15$, and furthermore that the factor R tends to zero when the pores approach spherical shapes (aspect ratio $\simeq 1$) [see Goertz and Knight (1998) and Berryman *et al.* (2002b)]. So this ratio is a sensitive measure of the dependence of μ^*_u on the fluid content of a porous medium, and also to some extent on the microgeometry of the pores.

Figure 1 shows that, for most choices of volume fractions, the drained and undrained values of shear modulus bounds do not overlap. Clearly, as the volume fractions approach zero or unity, the system approaches a pure Gassmann system; but, away from these limiting cases, the results are both qualitatively and quantitatively different from Gassmann's predictions. Graphically speaking, it appears that the lower bound of the undrained constant is always greater than the upper bound on the drained constants, *i.e.*, $\mu^-_u > \mu^+_d$. But, when this figure is magnified, we find there are small regions of volume fraction where this inequality is violated slightly. So there is still little doubt that shear modulus is affected by pore fluids in these systems, and for some ranges of volume fraction there is no doubt. This result is a clear indication

that Gassmann's results for shear are not generally valid for this model – as expected. Figure 2 shows that the maximum value of R for this case occurs around $f_2 \simeq 0.2$. Furthermore, the magnitude of this value is about 0.32, and therefore greater than 4/15. This shows again [as was shown previously by Berryman *et al.* (2002b)] that $R = 4/15$ is also *not* in general an upper bound on R.

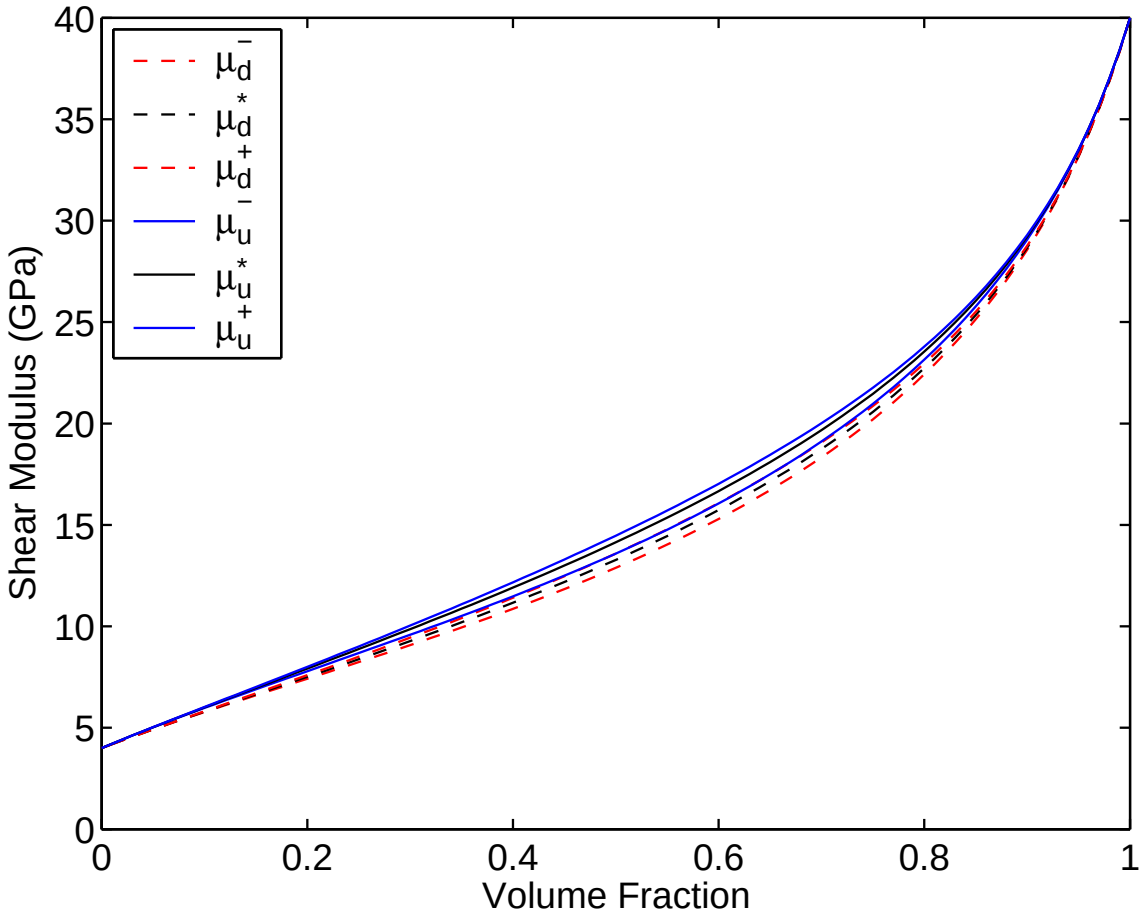

Figure 1: Illustrating the shear modulus results for the random polycrystals of porous laminates model for homogeneous saturation when each grain is composed of two constituents: (1) $K_d^{(1)} = 35.0$ GPa, $\mu_d^{(1)} = 4.0$ GPa and (2) $K_d^{(2)} = 35.0$ GPa, $\mu_d^{(2)} = 40.0$ GPa. Skempton's coefficient is taken to be $B = 0.0$ when the system is gas saturated, and $B = 1.0$ when the system is fully liquid saturated. The effective stress coefficients for the layers are both $\alpha = 0.75$, and $\alpha^* = 0.75$ also. The computed undrained bulk modulus is $K_u = 140$ GPa. Volume fraction of the layers varies from 0 to 100% of constituent number 2. jim2-Fig1 [NR]

Constant Drained Bulk Modulus, Patchy Fluid Saturation

To add one level of complication, consider next the same porous framework as before, but now suppose that the saturation is patchy (White, 1975; Berryman *et al.*, 1998; Norris, 1993;

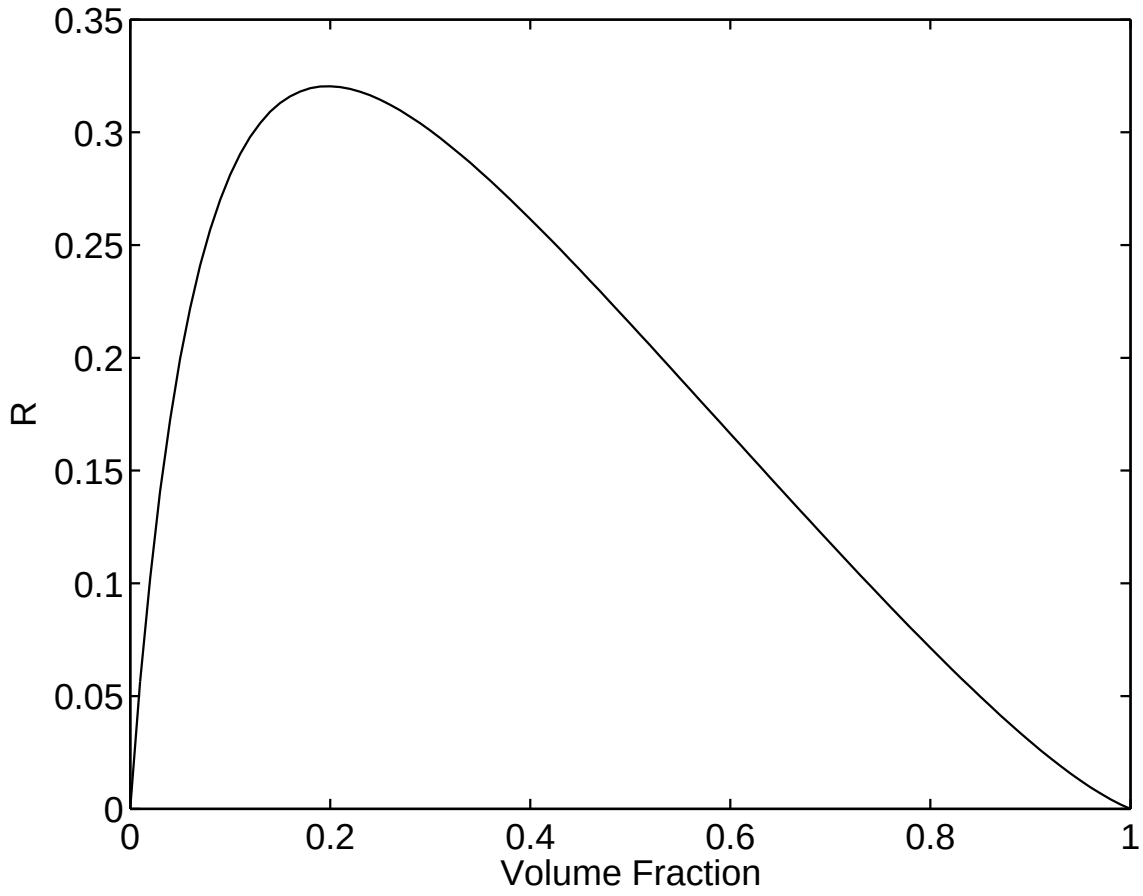

Figure 2: Plot of the ratio R from equation (24), being the ratio of compliance differences due to fluid saturation. These results are for the same model described in Figure 1 for homogeneous saturation. The values of R should be compared to those predicted by Mavko and Jizba (1991) for very low porosity and flat cracks, when $R = 4/15 \simeq 0.267$. We find in contrast that the random polycrystals of porous laminates model for the case considered always has $R \leq 0.32$. jim2-Fig2 [NR]

Dvorkin *et al.*, 1999; Johnson, 2001; Berryman *et al.*, 2002a), rather than homogeneous. The idea is that some of the layers in the grains will have a liquid saturant having $K_f = K_l$, while others will have a gas saturant having $K_f = K_g$. In general I assume that $K_g \ll K_l$ so that for most purposes the gas saturated parts of the system satisfy $K_u \simeq K_d$, *i.e.*, undrained moduli are to a very good approximation the same as the drained moduli for these layers. If this were not so, then I could treat the second saturant in exactly the same way as I will treat the liquid saturant; but there would be no new ideas required to do this, so I will not stress this approach here.

For this system, the drained constants are all the same as in the preceding example. In particular, the overall volume effective stress coefficient α^* is the same.

The undrained constants differ for this system however because the undrained bulk modulus is not constant in the layers. Gassmann's formula does not provide an answer for this overall bulk modulus because the system is not homogeneous. But Backus averaging determines all the elastic constants in a straightforward way for this system [see Berryman (2004a)]. The correct results are obtained for all the constants related to Voigt and Reuss averages [Eqs. (4)-(9) for both bulk and shear moduli] as long as the K's shown explicitly in (2) are properly interpreted as the undrained constants K_u from (17) for the fluids having bulk moduli K_l or K_g in the appropriate layers.

One explicit result found useful to quote from some earlier work (Berryman, 2004a) is

$$G^v_{\rm eff} = c_{66} - \frac{4c^u_{33}}{3}\left[\left\langle\frac{\Delta\mu^2}{K_u+4\mu/3}\right\rangle\left\langle\frac{1}{K_u+4\mu/3}\right\rangle - \left\langle\frac{\Delta\mu}{K_u+4\mu/3}\right\rangle^2\right], \tag{25}$$

where $c^u_{33} = \langle 1/(K_u+4\mu/3)\rangle^{-1}$ and the bracket notation has the same meaning as in the Backus formulas (2). The difference $\Delta\mu \equiv \mu - c_{66}$ is the deviation of the layer shear modulus locally from the overall average across all the layers. The term in square brackets in (25) is always non-negative. As K_u in the layers ranges (parametrically) from zero to infinity, the corrections from the square bracket term times the factor $\frac{4c^u_{33}}{3}$ can be shown to decrease from $c_{66} - c_{44}$ to zero. Thus, $G^v_{\rm eff}$ in the layered model ranges for all possible layered poroelastic systems from c_{44} to c_{66}.

Figure 3 shows that the drained bulk modulus does not change with volume fraction, since all the layers have the same drained bulk modulus. The undrained bulk modulus can have some small variations, however, due to variations in the shear modulus, as is shown by the small spread in the bulk modulus bounds. Uncorrelated Hashin-Shtrikman bounds [computed by evaluating (3) at μ's having the lowest and highest shear modulus values among all those in the layers] are also shown here for comparison purposes. Clearly, the Peselnick-Meister-Watt correlated bounds based on the polycrystals of laminates microstructure are much tighter. Figure 4 shows that the overall shear modulus has only relatively weak dependence (though stronger than that in Figure 3) on patchy saturation when the bulk modulus itself is uniform. Figure 5 shows that shear modulus changes with saturation, while small, are present and not very tightly coupled to the bulk modulus changes (drained to undrained). This observation is seen to be especially significant at the lowest volume fractions of liquid, as the changes in

shear compliance are greater here (by a factor of about 3) than the corresponding changes in the bulk compliance.

Two Distinct Gassmann Materials, Uniform Fluid Saturation

This example and the next one will remove the restriction that the porous frame material is uniform. To have as much control as possible, we limit the heterogeneity to just two types of drained bulk moduli, $K_d^{(1)}$ and $K_d^{(2)}$. These occur with a frequency measured by the volume fractions f_1 and f_2, respectively. These porous materials fill the space, so $f_1 + f_2 = 1$. The effective stress coefficient is known exactly for this model and is given by (20). This result is true both for homogeneously saturated two-component media (Berryman and Milton, 1991) as treated in this example, or for the type of patchy saturation treated in the next example. Proof of this statement is provided in Appendix A. For Gassmann's equations in each material, we also need either the fluid bulk modulus together with the layer porosities $\phi^{(1)}$ and $\phi^{(2)}$, or we just need the Skempton coefficient, B. For simplicity, we take $B = 0.0$ for uniform gas saturation, and $B = 1.0$ for uniform liquid saturation. (Although $B = 1$ may not be exactly correct for real liquid-saturated reservoirs, only the product αB is important for the modeling examples that follow. So desired differences in B can be introduced through differences in α. In this way we hope to capture the essence of this problem using the minimum number of free parameters.) This summarizes the part of the modeling that is the same in this example and the next.

We will now assume that the fluid saturation is uniform throughout the stated model material: $(1l, 2l)$. [Notation indicates first layer is liquid filled (l) and second layer is also liquid filled. The alternative is that some layers are gas filled (g).]

In Figure 6 there appear to be only two curves for bulk modulus, but in fact six curves are plotted here. All three of the drained curves are so close to each other that they cannot be distinguished on the scale of this plot. Similarly, all three of the undrained curves are equally indistinguishable.

Figure 7 appears to be both qualitatively and quantitatively very similar to Figure 1. But this time we find the inequality $\mu_u^- > \mu_d^+$ is never violated. So there is no doubt that shear modulus is affected by pore fluids in this system.

Figure 8 shows that the maximum value of $R \simeq 0.2$ occurs around $f_2 \simeq 0.3$. For this case, 4/15 is an upper bound on R, but I know this is not a general result.

Two Distinct Gassmann Materials, Patchy Fluid Saturation

This final set of examples will use the same model framework as the preceding example. However, two fluids will be present simultaneously in this case. If the two fluids (g, l) are assumed to saturate only one or the other types of Gassmann materials, then we have a relatively simple two component model: $(1g, 2l)$. On the other hand, the setup is now general enough to permit a variety of other possibilities. For example, porous material 1 might be saturated with either

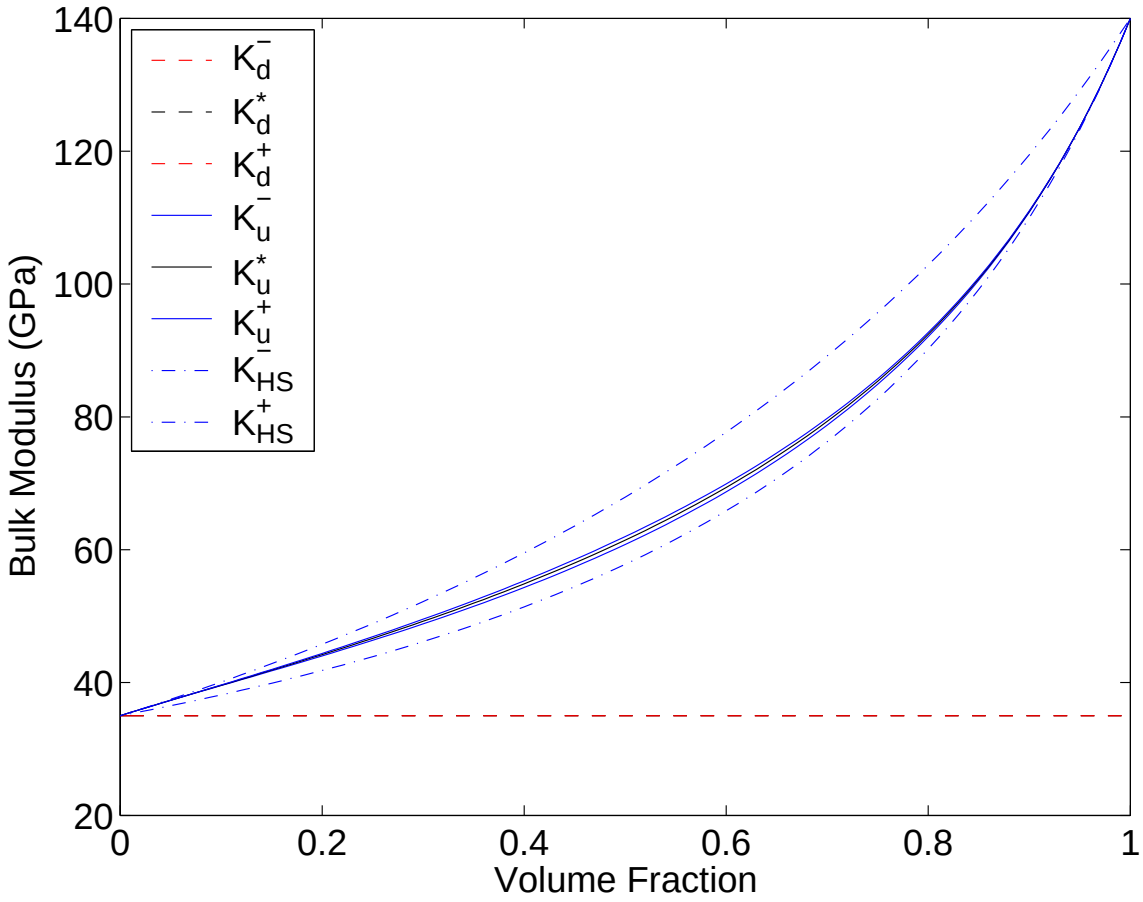

Figure 3: Illustrating the bulk modulus results for the random polycrystals of porous laminates model for patchy saturation when each grain is composed of two constituents: (1) $K_d^{(1)} = 35.0$ GPa, $\mu_d^{(1)} = 4.0$ GPa and (2) $K_d^{(2)} = 35.0$ GPa, $\mu_d^{(2)} = 40.0$ GPa. Skempton's coefficient is taken to be $B = 0.0$ for constituent 1 (gas saturated), and $B = 1.0$ for constituent 2 (liquid saturated). The effective stress coefficients for the layers are both given by $\alpha = 0.75$, so $\alpha^* = 0.75$ also. Porosity does not play a direct role in the calculation when we are using B as the fluid substitution parameter. Volume fraction of the layers varies from 0 to 100% of constituent number 2. To emphasize the tightness of the polycrystal (correlated) bounds, uncorrelated Hashin-Shtrikman bounds $K_{HS}^{\pm}$ on the undrained bulk modulus are also shown. jim2-Fig3 [NR]

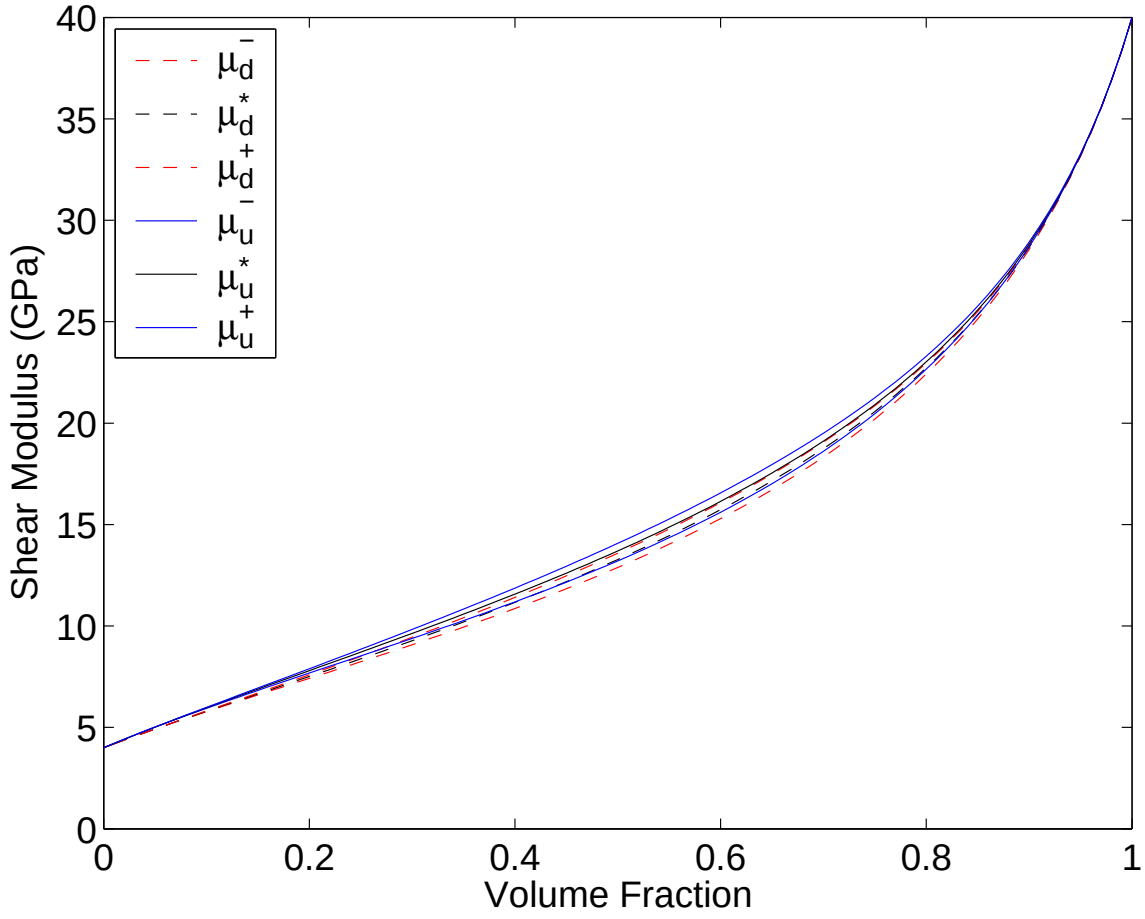

Figure 4: Illustrating the shear modulus results for the random polycrystals of porous laminates model. Model parameters are the same as in Figure 3 for patchy saturation. jim2-Fig4 [NR]

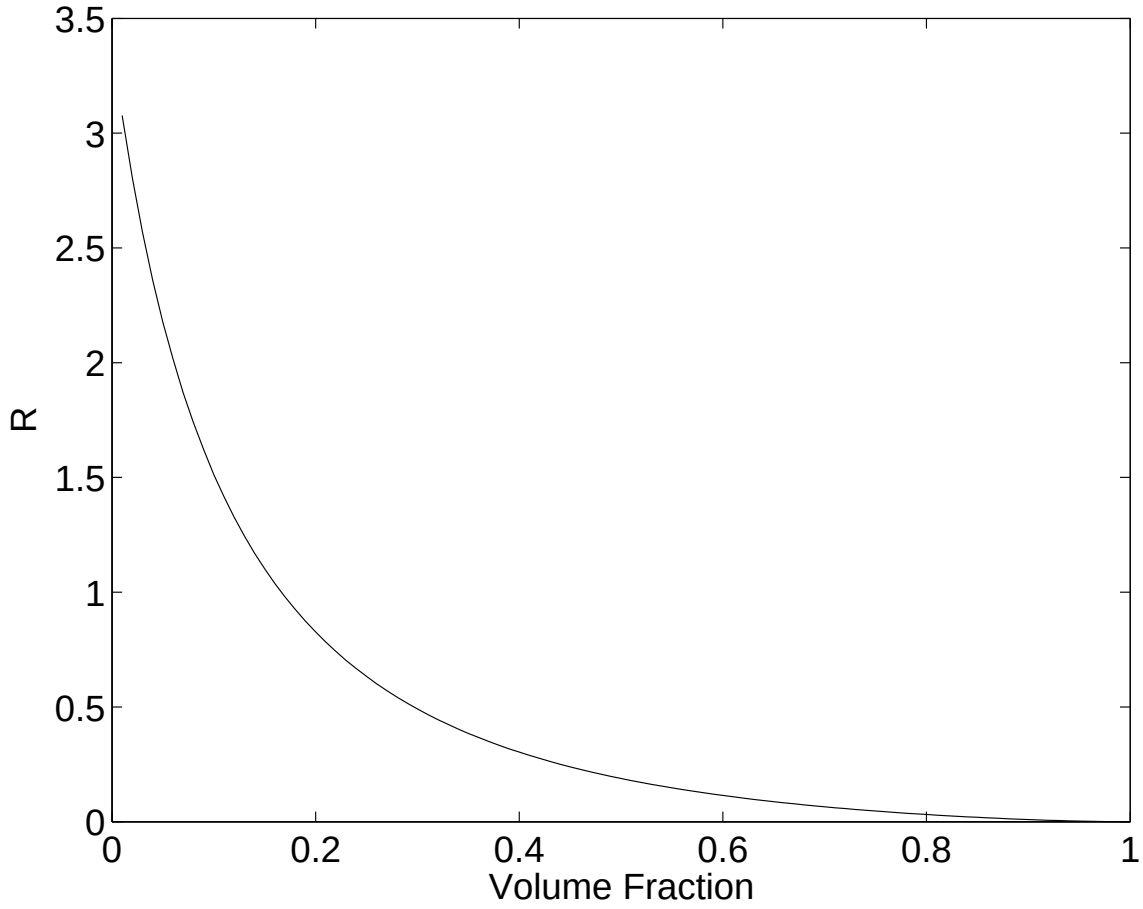

Figure 5: Plot of the ratio R from equation (24), being the ratio of compliance differences due to fluid saturation. These results are for the same model described in Figure 3 for patchy saturation. The values of R should be compared to those predicted by Mavko and Jizba (1991) for very low porosity and flat cracks, when $R = 4/15 \simeq 0.267$. We find in contrast that, for partial and patchy saturation, R can take any positive value, or zero. jim2-Fig5 [NR]

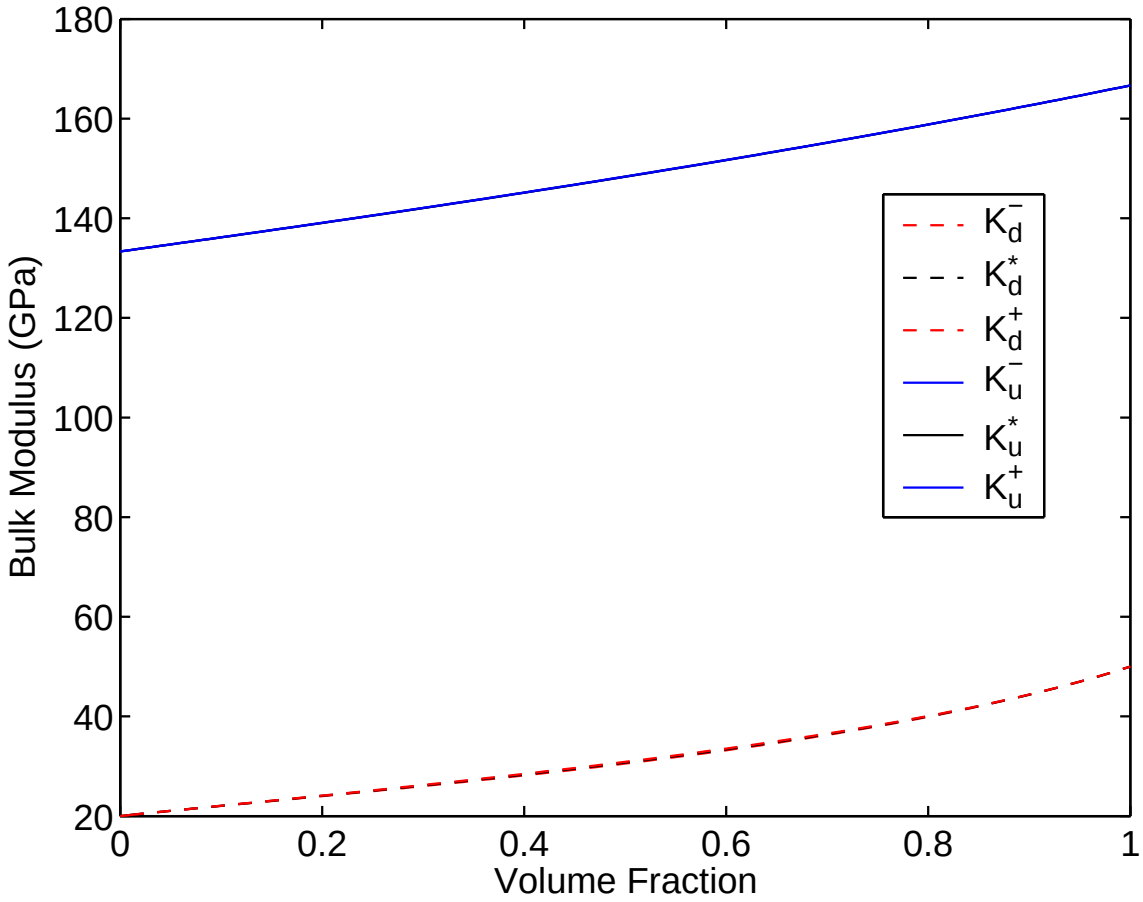

Figure 6: Illustrating the bulk modulus results for the random polycrystals of porous laminates model for homogeneous saturation when each grain is composed of two constituents: (1) $K_d^{(1)} = 20.0$ GPa, $\mu_d^{(1)} = 4.0$ GPa and (2) $K_d^{(2)} = 50.0$ GPa, $\mu_d^{(2)} = 40.0$ GPa. Skempton's coefficient is taken to be $B = 0.0$ when the system is gas saturated, and $B = 1.0$ when the system is fully liquid saturated. The effective stress coefficients for the layers are, respectively, $\alpha^{(1)} = 0.85$ and $\alpha^{(2)} = 0.70$. Porosity does not play a direct role in the calculation when we are using B as the fluid substitution parameter. Volume fraction of the layers varies from 0 to 100% of constituent number 2. jim2-Fig6 [NR]

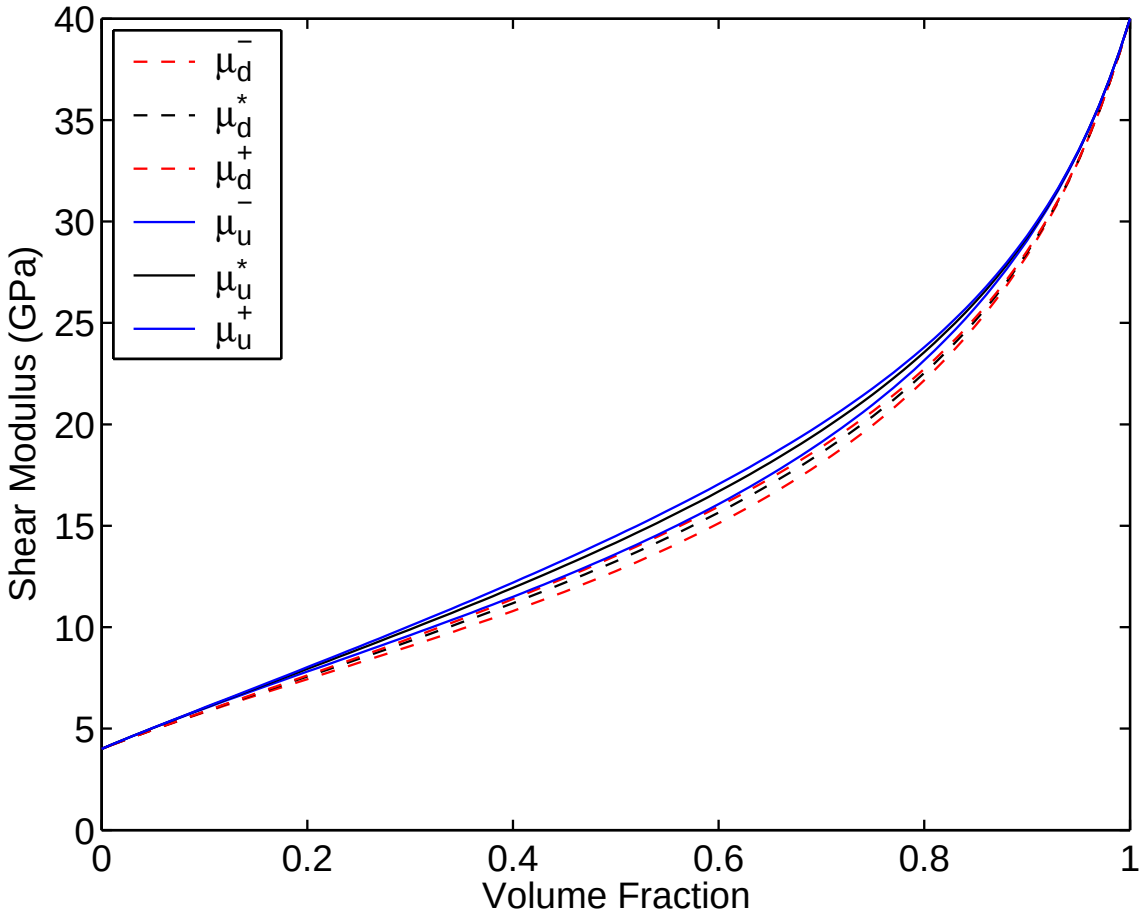

Figure 7: Illustrating the shear modulus results for the random polycrystals of porous laminates model. Model parameters are the same as in Figure 6 for homogeneous saturation. jim2-Fig7 [NR]

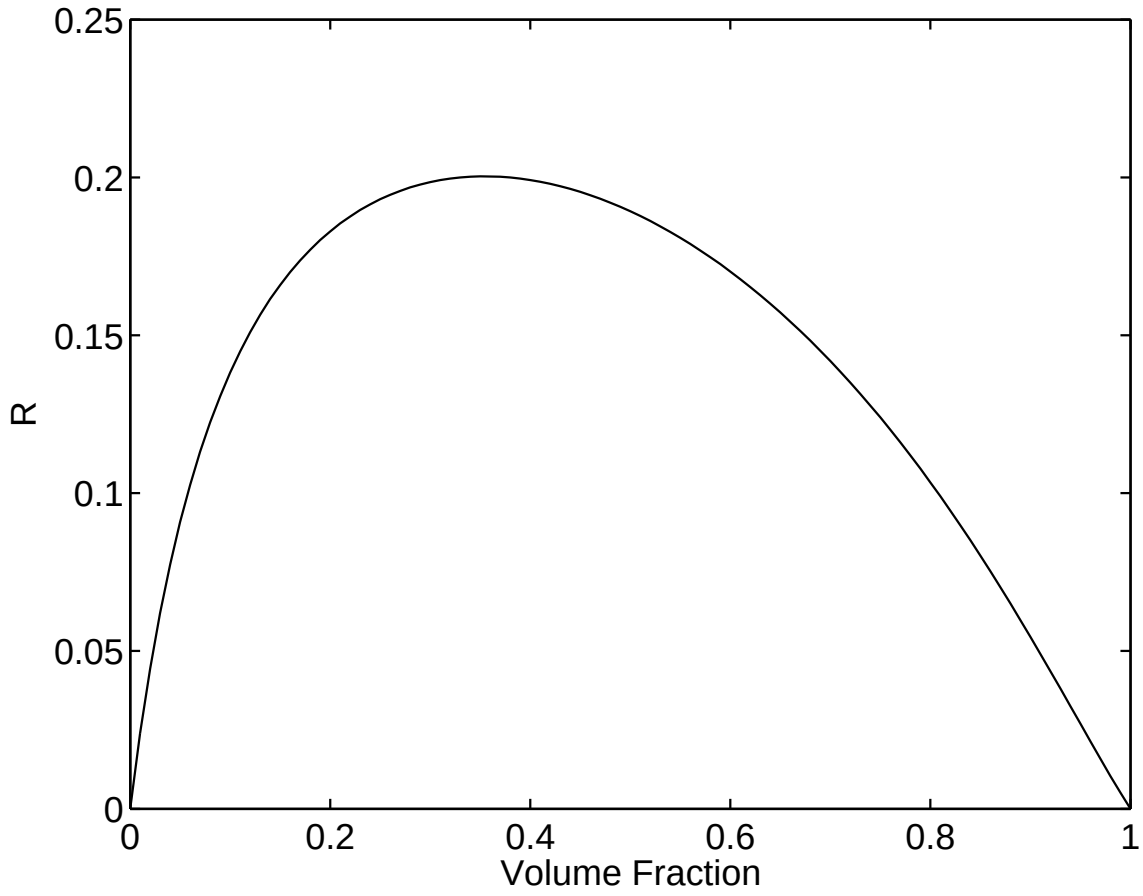

Figure 8: Plot of the ratio R from equation (24), being the ratio of compliance differences due to fluid saturation. These results are for the same model described in Figure 6 for homogeneous saturation. The values of R should be compared to those predicted by Mavko and Jizba (1991) for very low porosity and flat cracks, when $R = 4/15 \simeq 0.267$. We find in contrast that the random polycrystals of porous laminates model for the case considered always has $R \leq 0.20$. jim2-Fig8 [NR]

gas or liquid, and the same for porous material 2: $(1g, 1l, 2g, 2l)$. We could also suppose that some of the layers have homogeneously (h) mixed saturation of both liquid and gas, *i.e.*, a partially rather than patchy saturated layer: $(1g, 1h, 1l, 2g, 2h, 2l)$. Although this additional complication is not a problem for the numerical modeling, the large increase in the number of possible cases needing enumeration becomes a bit too burdensome for the short presentation envisioned here. (There is an infinite number of ways these types of materials having homogeneously mixed regions could be incorporated.) So we will instead limit discussion to the two cases mentioned before: (a) just two types of patchy saturated layers $(1g, 2l)$, or (b) four types of patchy saturated layers $(1g, 1l, 2g, 2l)$. Since the case (b) is expected to be more complex but not expected to contain any new ideas, we will limit the discussion further to case (a).

In Figure 9 as in Figure 6, the three drained curves for bulk modulus are so close together that they cannot be distinguished on the scale of this plot (although they can be distinguished if the plot is magnified). In contrast to Figure 6, the three undrained bulk modulus curves can now be distinguished, but they are still quite close together. The undrained curves start out at the same values as the drained curves because at zero volume fraction of constituent 2 the only fluid in the system is air. Then, as the volume fraction of constituent 2 increases, we add liquid up to the point where the final values at full liquid saturation are the same as in Figure 6. Again uncorrelated Hashin-Shtrikman bounds are shown for purposes of comparison, as in Figure 3. The Peselnick-Meister-Watt bounds on undrained bulk modulus — making use of the laminated grain/crystal substructure and the polycrystalline nature of the overall reservoir model — clearly are much tighter. Together Figures 3 and 9 also show that the polycrystalline-based bounding method produces a great improvement over the uncorrelated Hashin-Shtrikman bounds, whose microstructural information is limited to volume fraction data. This result is accomplished without having very detailed knowledge of the spatial correlations, just by using the fact that the local microstructure is layered. Knowledge of local layering is therefore a very important piece of microstructural information that has not been used to greatest advantage in prior applications of bounding methods for up-scaling purposes.

For Figure 10, the results are not so simple, as the six curves are all very close to each other. Undrained curves are always above the corresponding drained curves, but in general there is little separation to be seen here. Figure 11, like Figure 5, shows that the shear modulus changes with saturation are not really tightly coupled to the bulk modulus changes, and especially so at the lowest volume fractions of liquid, as the changes in shear compliance are again greater in magnitude there than the changes in the bulk compliance.

CONCLUSIONS

The “random polycrystals of porous laminates” model introduced and studied here has been shown to be a useful tool for studying some very difficult technical issues concerning how geomechanical constants of reservoirs behave as a function of changes of pore fluid and varying degrees of heterogeneity. This model has the advantage that rigorous bounds [the Hashin-Shtrikman bounds of Peselnick and Meister (1965) and Watt and Peselnick (1980)] on the geomechanical constants (bulk and shear moduli) are available. Furthermore, due to the re-

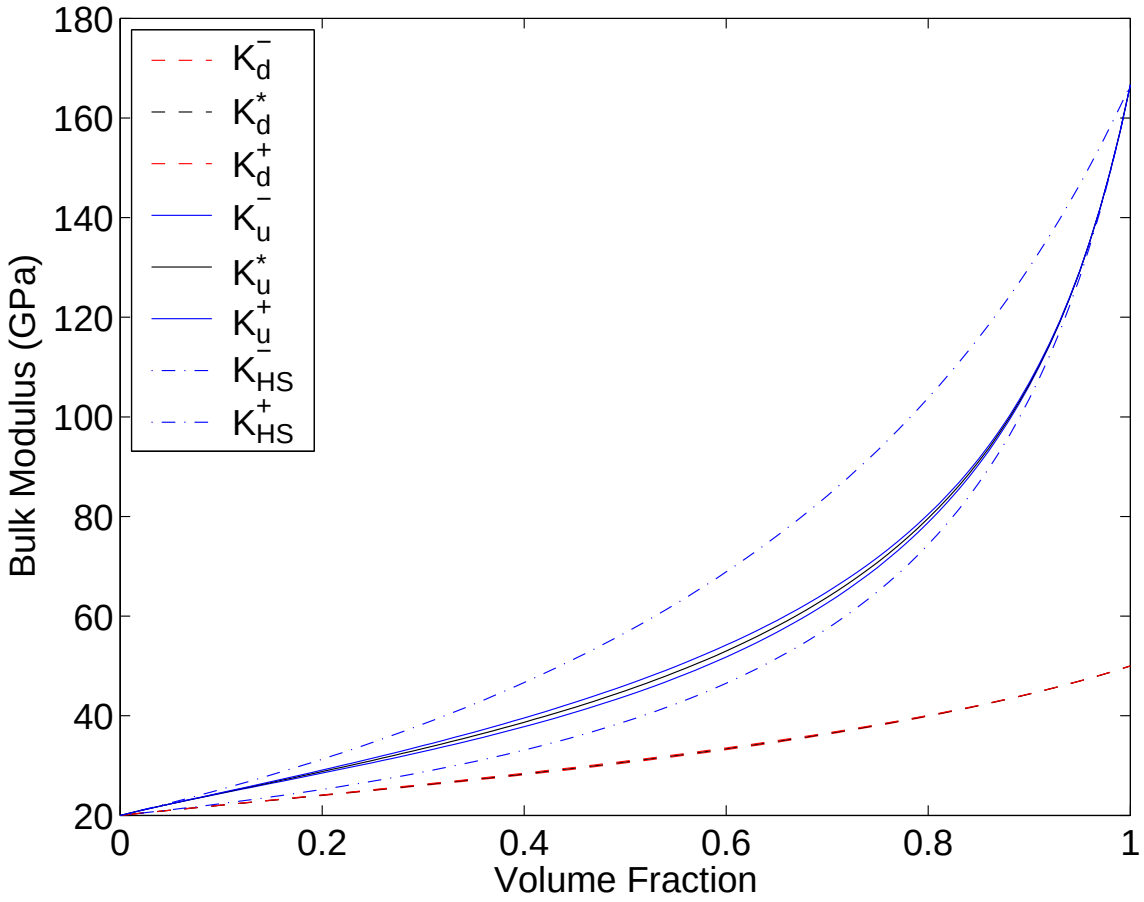

Figure 9: Illustrating the bulk modulus results for the random polycrystals of porous laminates model for patchy saturation when each grain is composed of two constituents: (1) $K_d^{(1)} = 20.0$ GPa, $\mu_d^{(1)} = 4.0$ GPa and (2) $K_d^{(2)} = 50.0$ GPa, $\mu_d^{(2)} = 40.0$ GPa. Skempton's coefficient is taken to be $B = 0.0$ for constituent 1 (gas saturated), and $B = 1.0$ for constituent 2 (liquid saturated). The effective stress coefficients for the layers are, respectively, $\alpha^{(1)} = 0.85$ and $\alpha^{(2)} = 0.70$. Porosity does not play a direct role in the calculation when we are using B as the fluid substitution parameter. Volume fraction of the layers varies from 0 to 100% of constituent number 2. To emphasize the accuracy of the polycrystal bounds and self-consistent estimates, uncorrelated Hashin-Shtrikman bounds $K_{HS}^{\pm}$ on undrained bulk modulus are also shown. jim2-Fig9 [NR]

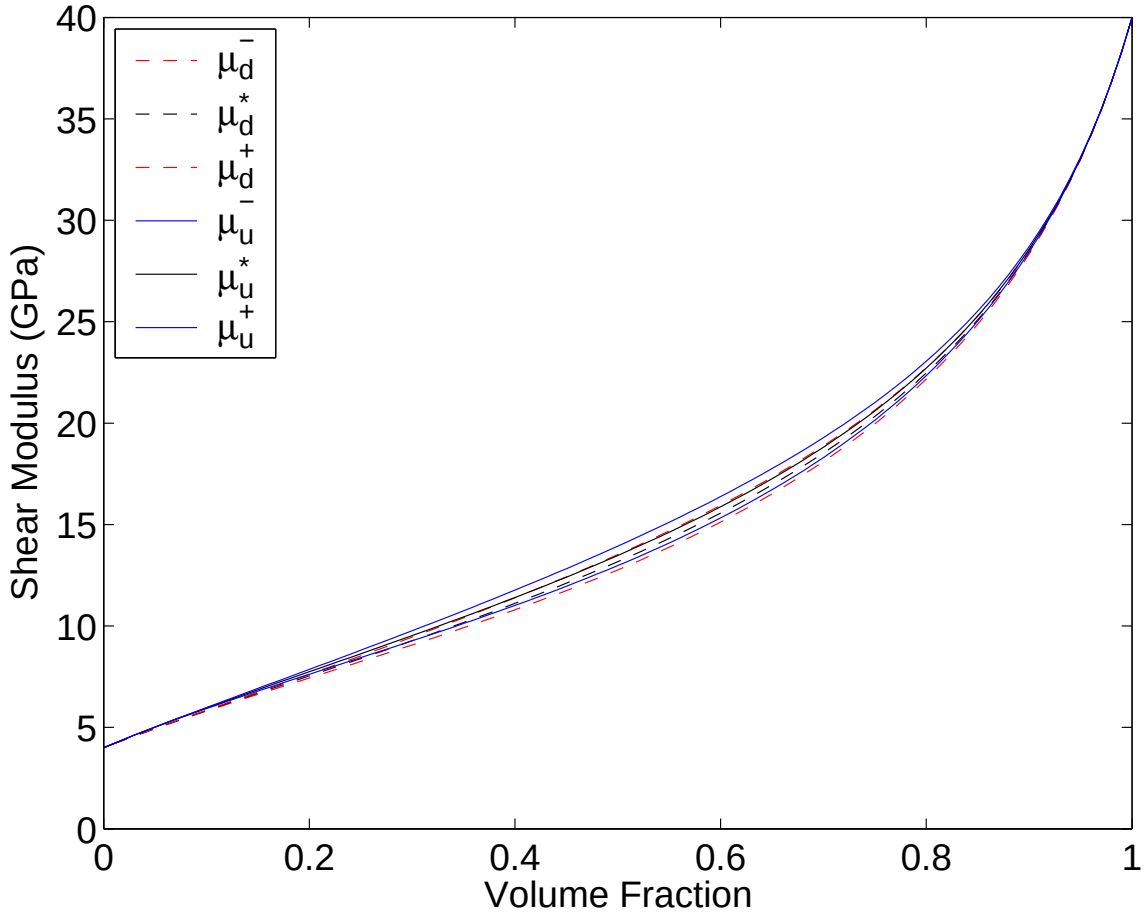

Figure 10: Illustrating the shear modulus results for the random polycrystals of porous laminates model. Model parameters are the same as in Figure 9 for patchy saturation. jim2-Fig10 [NR]

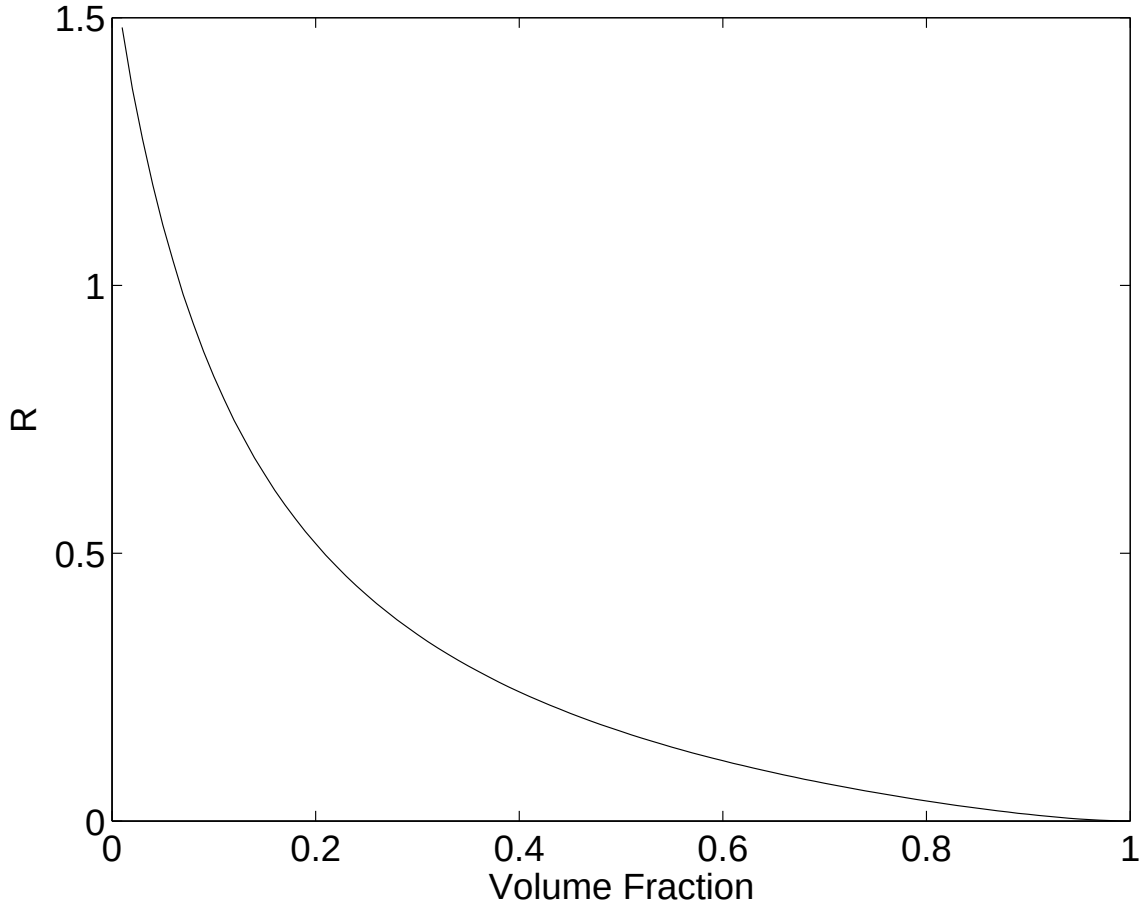

Figure 11: Plot of the ratio R from equation (24), being the ratio of compliance differences due to fluid saturation. These results are for the same model described in Figure 9 for patchy saturation. The values of R should be compared to those predicted by Mavko and Jizba (1991) for very low porosity and flat cracks, when $R = 4/15 \simeq 0.267$. We find in contrast that, for partial and patchy saturation, R can take any positive value, or zero. Note that the magnitude of this effect is smaller than in Figure 5, even though the degree of heterogeneity for bulk modulus is greater here. jim2-Fig11 [NR]

fined formulation of these bounds presented here, it is also possible to obtain self-consistent estimates directly from these bounds (Berryman, 2004b; 2005). This situation is particularly beneficial as the rigorous bounds then provide immediate theoretical error bars for the self-consistent estimates – a situation that is sometimes but not always true for other effective medium theories (Berryman, 1995). The model should therefore prove useful for a range of applications in geomechanics.

The results obtained for the specific application considered here, *i.e.*, pore fluid effects on shear modulus, show that the pore fluid interaction with overall shear behavior is complicated. The changes from drained to undrained behavior for shear modulus can range from being a negligible effect (as it is according to Gassmann's results for microhomogeneous and isotropic media) to being a bigger effect than the changes in bulk modulus under some circumstances (see Figures 5 and 11 showing that the ratio of compliance differences $R > 1$ in some cases). Influences of pore geometry can also be studied in this model if desired, but this complication was avoided here by parameterizing the fluid effects through the use of Skempton's coefficient B. All the pore microgeometry effects were thereby hidden in the present analysis, but these could be brought out in future studies of the same and/or many other related systems.

Another related result of some importance to analysis of partially and patchy saturated systems was obtained in Appendix B. The results are illustrated in Figure 12 and show that deviations from a system satisfying Hill's equation (3) need not be small if the shear modulus heterogeneity is large. The analysis does show, however, that if shear modulus variation is small, then the observed deviations from predictions of Hill's equation should also be correspondingly small.

An implicit assumption made throughout the present paper is that the porosity and — most importantly — the fluid permeability of the geomechanical system under consideration is relatively uniform. Then, the pore fluid pressures equilibrate on essentially the same timescale throughout the whole system. If this is not true, as it would not be in a double-porosity dual-permeability system (Berryman and Wang, 1995), then the present analysis needs to be modified to account for the presence of more than one pertinent timescale. One direction for future work along these lines will therefore be focused on this more complex, but nevertheless important, problem commonly encountered in real earth reservoirs. The random polycrystal of porous laminates model is flexible enough to allow this set of problems to be studied within a very similar framework.

APPENDIX A: EFFECTIVE STRESS COEFFICIENT AND PARTIAL SATURATION

Although Eq. (20) for the overall volume effective stress coefficient α^* is known to be true for homogeneous pore saturation, we also need to have a corresponding result here for patchy saturation. It turns out that the same formula applies for arbitrarily patchy saturated media, as long as there are only two types of solid components. To show this, consider

$$\begin{pmatrix} \delta e^{(n)} \\ -\delta\zeta^{(n)} \end{pmatrix} = \frac{1}{K_d^{(n)}} \begin{pmatrix} 1 & -\alpha^{(n)} \\ -\alpha^{(n)} & \alpha^{(n)}/B^{(n)} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(n)} \end{pmatrix}, \tag{26}$$

where $\delta e^{(n)}$ and $\delta\zeta^{(n)}$ are the change in overall strain and the increment of fluid content in component n, where $n = 1, 2$. [There are also similar formulas for all the overall properties with (n) replaced by $*$ for the corresponding effective properties. See Berryman and Wang (1995) for more discussion.] Similarly, the change in overall confining (external) pressure is δp_c, and the pore pressure change of component n is $\delta p_f^{(n)}$. The porous material coefficients are defined as in the main text, $K_d^{(n)}$ is the drained bulk modulus, $\alpha^{(n)}$ is the volume effective stress coefficient, and $B^{(n)}$ is Skempton's coefficient for the n-th constituent.

Now the rest of the argument follows that given in Berryman and Milton (1991) exactly, since it is not important what fluid is in the pores when trying to determine the overall effective stress coefficient at long times (when fluid pressure in the system has equilibrated). We simply postulate the existence of any fixed ratio $r = \delta p_f^{(1)}/\delta p_c = \delta p_f^{(2)}/\delta p_c$ such that $\delta e^{(1)} = \delta e^{(2)}$. If there is such a ratio (valid at appropriately long times), then $\delta e^* = \delta e^{(1)} = \delta e^{(2)}$ also follows immediately and we have the condition that must be satisfied:

$$\frac{\delta p_c}{K_d^{(1)}}\left[1-\alpha^{(1)}r\right] = \frac{\delta p_c}{K_d^{(2)}}\left[1-\alpha^{(2)}r\right], \tag{27}$$

which is just a linear relation for ratio r. The result shows that the postulated value of r does exist unless the denominator of the following expression vanishes:

$$r = \frac{1/K_d^{(1)} - 1/K_d^{(2)}}{\alpha^{(1)}/K_d^{(1)} - \alpha^{(2)}/K_d^{(2)}}. \tag{28}$$

If the numerator of (28) vanishes, the results are trivial because Gassmann's microhomogeneity condition is then satisfied. Once the value of r is known, it is easy to see that $\delta e^* = \delta e^{(1)} = \delta e^{(2)}$ implies

$$\frac{\delta p_c}{K_d^*}\left[1-\alpha^* r\right] = \frac{\delta p_c}{K_d^{(1)}}\left[1-\alpha^{(1)}r\right]. \tag{29}$$

This equation can be rearranged into the form (20), as has been shown previously by Berryman and Milton (1991).

Arguments similar to the one just given have also been used, for example, in the context of thermal expansion by Benveniste and Dvorak (1990) and Dvorak and Benveniste (1997), who call this approach "the theory of uniform fields." It turns out this method is not restricted to isotropic constituents as one might infer from the arguments presented here and also in (Berryman and Milton, 1991).

A somewhat more difficult task than the one just accomplished involves deducing the overall effective pore bulk modulus K_ϕ^* as was also done previously by Berryman and Milton (1991). However, this coefficient does not play any direct role in our present analysis, so we will leave this exercise to the interested reader.

APPENDIX B: HILL'S EQUATION AND HETEROGENEOUS POROUS MEDIA

One very common approximation made in studies of partially and patchy saturated porous media (Norris, 1993; Mavko *et al.*, 1998; Johnson, 2001) is based on an assumption that the estimates are being made over a small enough region that it is reasonable to take the shear modulus of the porous frame as constant, even though the bulk modulus over the same small region may vary. Then, when Gassmann's results apply locally, the shear modulus satisfies $\mu_d = \mu_u$, and so remains constant throughout this same region regardless of the distribution of fluids in the pores. When these assumptions are valid, Hill's equation (3) may be used to compute the effective bulk modulus K^*, regardless of anisotropy or of how many constituents might be present. Furthermore, Hill's equation will apply equally to the drained K_d^* and undrained K_u^* bulk moduli of such a poroelastic system; K_n for the layers must be substituted accordingly for the drained and undrained cases.

This approximation based on Hill's equation is very appealing for applications because of its analytical beauty and overall simplicity, but its use in heterogeneous media has never been given a rigorous justification. In particular, the assumption of variable bulk modulus in a heterogeneous system having constant shear modulus is surely one worthy of careful consideration. It seems more likely (at least to me) that the variations in the bulk modulus in an earth system will be mimicked by the shear modulus and, therefore, that the proposed method is in truth an oversimplification of this complex problem.

The model system presented here (*i.e.*, the random polycrystal of porous laminates) offers one means of checking whether this use of Hill's equation might be justified or not.

It turns out that, when $N = 2$, Hill's equation (3) can be inverted to give μ as a functional of K^* (Milton, 1997). The result is given by

$$\mu = \frac{3K_1K_2}{4K_r}\left(\frac{K^* - K_r}{K_v - K^*}\right), \tag{30}$$

where

$$K_v = \sum_{n=1}^{2} f_n K_n \qquad \text{and} \qquad K_r = \left[\sum_{n=1}^{2} \frac{f_n}{K_n}\right]^{-1}. \tag{31}$$

So I can do two calculations based on the results presented here for heterogeneous systems. We can compute effective shear moduli $\mu_d^{\rm eff}$ and $\mu_u^{\rm eff}$ by taking the self-consistent values to be the true values of the drained and undrained K^*, and layer values of $K_d^{(n)}$ and $K_u^{(n)}$ as the values for K_1 and K_2. The volume fractions are those already used in these calculations. So everything is known and the computations are straightforward. We want to check whether the resulting values of effective shear moduli $\mu_d^{\rm eff}$ and $\mu_u^{\rm eff}$ computed this way are approximately constant and/or approximately equal to each other. If they are, then Hill's equation, although not rigorously appropriate in these systems, nevertheless could be capturing some of the observed behavior. If this is not true, then the results would be showing that great care should be exercised in using these formulas for analyzing real data.

My results are illustrated in Figure 12. I find that $\mu_d^{\text{eff}} \simeq \mu_u^{\text{eff}}$. However, except for the volume fractions near 50%, the values of both μ^{eff}'s are very different from the actual shear moduli of the random polycrystals of porous laminates model. The μ^{eff}'s are high when the μ^*'s are low, and vice versa. This observation is a very strong negative result, showing that large errors in analysis can be introduced for systems such as these that are very heterogeneous in shear.

On the positive side, it is also clear from Figure 12 that if the spread of layer μ's is nonzero but small, then the use of Hill's equation can be well justified. The error in shear estimates will never be greater than the spread in the layer shear modulus values, so if this is a small (though nonzero) number, then the errors will be finite but correspondingly small.

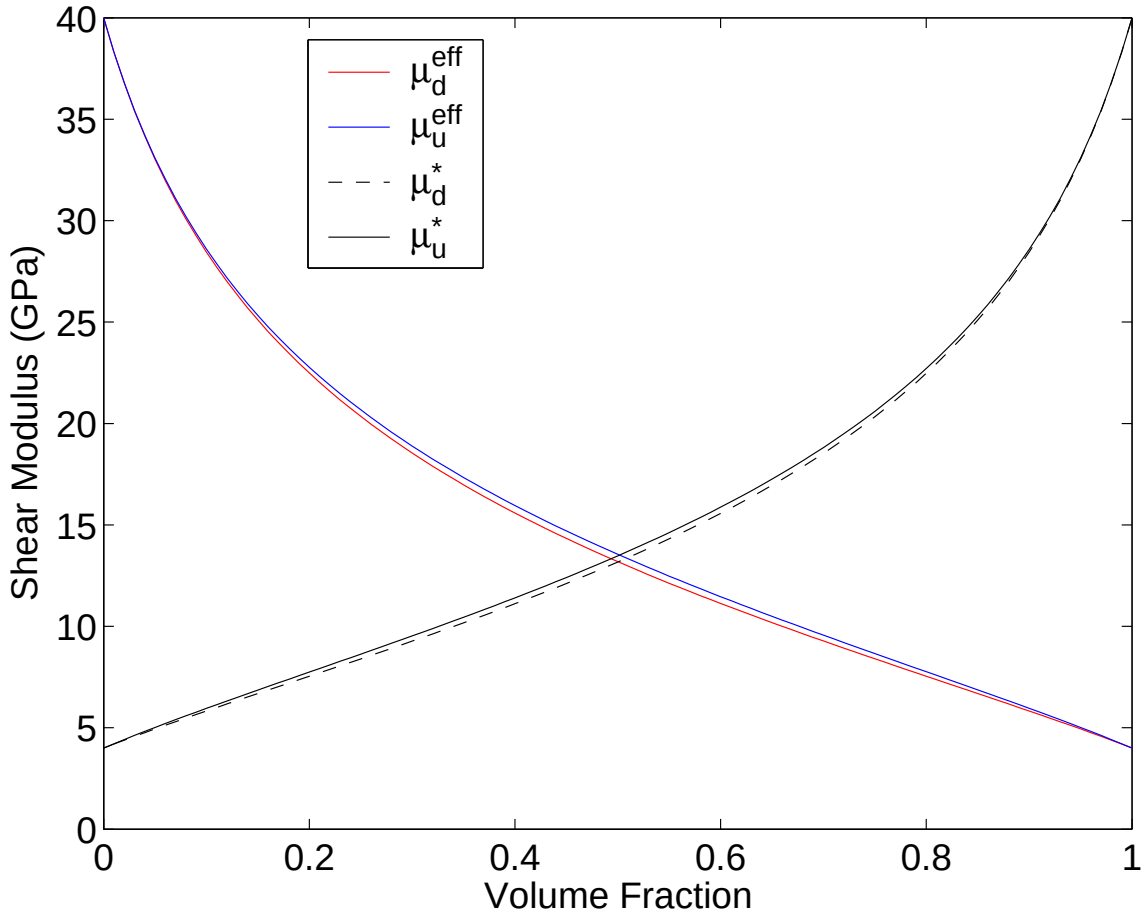

Figure 12: Illustrating computations of an effective shear modulus obtained by inverting Hill's equation for drained (μ_d^{eff}) and for undrained patchy saturation (μ_u^{eff}) conditions. Model parameters are the same as in Figure 9 for patchy saturation. For comparison the curves for self-consistent shear moduli μ_d^* and μ_u^* from Figure 10 are replotted here. jim2-Fig12 [NR]

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