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Unified seismic-wave imaging - from data space to model space

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ABSTRACT

Under operator, matrix and inverse theory, seismic-wave imaging can be considered a unified process—mapping from data space to model space. The main topics in seismic-wave imaging include (1) seismic-data interpolation, regularization and redatuming, which mainly decrease the imaging noise; (2) seismic-wave illumination analysis, which predicts whether a target reflector can be imaged and evaluates the suitability of an acquisition configuration in the case of rugged topography and severe lateral velocity variations; and (3) seismic-wave migration/inversion imaging algorithms, which give an imaging result with the help of a wave propagator, known a macro-velocity model. The last and most important thing is to build an accurate macro-velocity model. All of the processes can be considered with the conjugate operator/matrix under least-squares theory. In this article, we review the following topics: (1) expression of data space and model space; (2) affiliation between data space and model space; (3) seismic-data preprocessing; (4) seismic-data illumination; (5) migration imaging and inversion imaging as least-squares inverse problems; (6) amplitude-preserving migration imaging with wavefield extrapolation; (7) migration velocity analysis and inversion and (8) some related topics. We express the imaging process with the operator or matrix theory and give some directions for further research.

INTRODUCTION

Seismic-wave imaging can be seen as a mapping from data space to model space. The objective is to position reflectors and to quantitatively estimate the physical parameters of the medium (such as reflectivity, P-wave velocity, S-wave velocity and density).

From the view of the historical development of seismic wave imaging, there are two ways to reach the goal. The first approach is seismic-wave migration technologies commonly used in the oil industry at present, which includes estimating low-wavenumber macro-velocity (with NMO+DMO velocity analysis, migration velocity analysis or traveltime tomography), positioning the reflectors and qualitatively estimating the reflectivity. However, quantitatively estimating reflectivities (which is called amplitude-preserving imaging or true-amplitude imaging) is currently much more actively pursued by the oil industry and academia. Seismic-wave illumination analysis, migration convolution, least-squares migration, and amplitude analysis of angle gathers are being given increased attention by many authors (Chavent and Plessix,

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1999; Gray, 1997; de Bruin et al., 1990; Black et al., 1993; Bleistein et al., 1999; Nemeth et al., 2000; Brandsberg-Dahl et al., 2003). All of these endeavors aim for clearer imaging and more quantitative reflectivity. The second approach is seismic-wave inversion, which directly and quantitatively estimates the physical contrast parameters of rocks (such as P-wave impedance, S-wave impedance and density).

The advantages of the first approach are that reflectors can be positioned step by step, and that reflectivity estimation can take advantage of many signal/noise enhancement technologies, macro-velocity analysis techniques and migration imaging methods. The procedure can be easily controlled and adjusted, and its calculation cost can be afforded by present computer systems. The geological and lithological knowledge about a survey area can be easily used for adjusting the final imaging results. The disadvantage of this conventional approach is its lack of a unified theoretical framework. Therefore, it is difficult to guarantee the reliability of the quantitative estimation of reflectivities.

The second approach has two subcategories: direct inversion methods (Bleistein et al., 1987; Wu and Toksoz, 1987; Miller et al., 1987), and iterative inversion methods (Tarantola, 1984; Mora, 1987, 1988; LeBras and Clayton, 1988; Chavent and Jacewitz, 1995; Pratt, 1999). Both have elegant theoretical expressions. However, the inverse problems are inherently ill-posed and the solutions are unstable and nonunique if the observed data set has certain flaws, including frequency band-limitation, aperture limitation, non-Gaussian noise, and/or an unknown source function. An unsuitable modeling algorithm or non-linearity between the observed data and geophysical parameters will also cause problems. In addition, the calculation costs of these methods often exceed the capacities of present computer systems. Until recently, only 2D inversion algorithms can be used in practice. The direct-inversion approach requires an analytical expression, which can be given analytically only in the case of constant background or slow background variance. Otherwise, the approach cannot produce a satisfactory result. Iterative inversion problems deal with either maximum-probability solutions for a Gaussian probability-density function or least-squares solutions of an l_2-norm problem. These kinds of inverse problems need huge numbers of iterative modeling calculations.

At present, some trends indicate that standard seismic-wave migration imaging and inversion imaging are merging. The problem at the point of intersection is how to quantitatively estimate the reflectivity. How the shot and receiver configurations affect the imaging resolution and the amplitude of the estimated reflectivity is a question relevant to both imaging approaches.

We think that it is worthwhile to investigate how to generate amplitude-preserving common-image gathers with inversion imaging approaches, while also considering how to include seismic-wave-illumination effects and irregular data sets.

In this research proposal, we review the following topics: (1) expression of data space and model space; (2) relationship between data space and model space; (3) seismic-data preprocessing; (4) seismic-data illumination; (5) migration imaging and inversion imaging as a least-squares inverse problem; (6) amplitude-preserving migration imaging with wavefield extrapolation; (7) migration-velocity analysis or macro-velocity inversion and (8) some other related topics. We try to express the imaging process within the context of inverse theory and give some directions for further research.

EXPRESSION OF DATA SPACE AND MODEL SPACE

First, we give a general expression of the data space and model space for seismic-data imaging. In general, seismic data acquired at the surface or in a well is presented as $\mathbf{d}\left(\vec{m},\vec{h},\omega\right)$ or $\mathbf{d}(\vec{s},\vec{g},\omega)$. We can use the following special variables to depict data space: azimuth, offset, and CMP coordinate, or azimuth, offset, shot-point coordinate and receiver-point coordinate respectively. These special variables completely define a data space or a seismic data set.

Model space can be characterized with as many different approaches as there are different applications. We define the model space as one which characterizes the interior of the earth, such as a velocity field, an impedance field, a stacked imaging volume, or common-image gathers, etc.. Basically, the model space is expressed as $\mathbf{m}(i\triangle x, j\triangle y, n\triangle z)$, with evenly discretized intervals. We usually present the velocity model or stacked imaging data volume in this form.

In some models, the subsurface floats in a velocity (or other physical parameter) field. The subsurface is a very important component of a physical parameter model, which shows the geometry of a geological structure. Reflectivities, for example, are defined in the subsurface. Therefore, common-image gathers and AVO/AVA analysis have a close relation to the subsurface. In fact, the subsurface plays a key role in macro-velocity model building (Mora, 1989; Cao et al., 1990; Pratt and Hicks, 1998). Angle gathers are expressed as $\mathbf{m}(i\triangle x, j\triangle y, \gamma(\alpha,\varphi), n\triangle z)$, where γ is an incident angle (between the incident ray and the normal ray of a reflector) or an emerging angle (between the emerging ray and the normal ray); φ is the azimuth angle; and α is the dipping angle of a reflector. In macro-velocity inversion, the model space is commonly expressed as $\mathbf{m}\left(r^k_{x_i}, r^k_{y_j}, v^k_{func}, r^k_{z_n}\right)$, where v^k_{func} is a velocity function attached to a reflector, which is given a concrete formula for each specific application; $r^k_{x_i}$ is the horizontal coordinate of i^{th} point on the k^{th} reflector, and $r^k_{x_i}$ and $r^k_{z_n}$ have a meaning similar to $r^k_{x_i}$.

How to evaluate data space and model space

What is a good observed data space? What is a good estimated model space? It is difficult to answer these two questions, because the answers depend on the practical applications. Up to now, the acquisition systems basically can be divided into three classes: Case $\mathbf{d}_1$: Full-area acquisition system. A 3D survey area is discretized into a regular grid, and each grid point has a receiver point and a shot point. This is an ideal case, in which a data set has an even azimuth interval, offset interval and CMP interval. Such a data set is a complete one. In practice, to minimize acquisition costs, a receiver is put at each grid point, and the shot points are arranged depending on the on-site situations. Case $\mathbf{d}_2$: Wide-azimuth acquisition system with partial sacrifice of cross-line aperture. The land acquisition system belongs to this category. It is difficult to maintain even azimuth intervals and even offset intervals because of complex variations of the surface and near-surface, and the consideration of acquisition efficiency and costs. Commonly, we have to move the shot point to an advantageous location, sacrificing spatial regularization. Case $\mathbf{d}_3$: Narrow-azimuth acquisition system with complete sacrifice of cross-line aperture. The present marine acquisition system belongs in this category. It is easy to achieve an even azimuth and even offset, but the data set is incomplete because it lacks the cross-line aperture. This acquisition system is not suitable in cases of complex structure variance along the cross-line direction. Similarly, we can divide the model space into three categories: Case $\mathbf{m}_1$, with a flat surface and some flat subsurfaces; Case $\mathbf{m}_2$, with a flat surface and some complex subsurfaces; and Case $\mathbf{m}_3$, with a rough topography and some complex subsurfaces. Table 1 gives the relationship between the data space and model space in different cases.

	d_1	**d_2**	**d_3**
m_1	even azimuth interval, even offset interval and even CRP illumination; a complete data set	uneven azimuth interval, uneven offset interval and basically even CRP illumination;	even azimuth interval, even offset interval and even CRP illumination
m_2	even azimuth interval, even offset interval and uneven CRP illumination	uneven azimuth interval, uneven offset interval and uneven CRP illumination	even azimuth interval, even offset interval and uneven CRP illumination
m_3	even azimuth interval, uneven offset interval and uneven CRP illumination	uneven azimuth interval, uneven offset interval and uneven CRP illumination	even azimuth interval, uneven offset interval and uneven CRP illumination

From the table, we know that wide-azimuth acquisition gives wider aperture in the cross-line direction. However, this also causes an uneven azimuth interval and uneven offset interval, which will result in a noisy image. Narrow-azimuth acquisition can give an even azimuth interval and even offset interval but sacrifices the cross-line aperture. Our conclusion can be summarized with following statements: In Case $\mathbf{m}_1$, the regularization of the data space yields an even sampling of the model space. In a geologically symple medium, a good data set is one with an even azimuth interval, an even offset interval and an **even CMP interval**. In Cases $\mathbf{m}_2$ and $\mathbf{m}_3$, a good data set is one with an even azimuth interval, an even offset interval and an **even CRP illumination**. In practice, even CRP illumination commonly means that a data set is irregular. There exists a trade-off between the even illumination and the regular data set. Since an irregular acquisition configuration generally yields a noisy image, a field data set should be preprocessed to be regular. On the other hand, bad illumination causes a vague image or no image, or yields false amplitude; therefore, the illumination deficiency should be compensated with other information from well-logging, rock physics or geology data. From the perspective of prestack imaging, assuming the macro-velocity model is accurate enough, a good model space can be defined as **one which has an amplitude-preserving angle gather on each point of a reflector at each azimuth**, which is the main goal of seismic-wave imaging (Shin and Min, 2001; Plessix and Mulder, 2004).

RELATIONSHIP BETWEEN DATA SPACE AND MODEL SPACE

Data space can be related to model space with the following formula:

$$\mathbf{d}\left(\vec{m},\vec{h},\omega\right) = L\mathbf{m}\left(\vec{m},\gamma\left(\alpha,\varphi\right),n\triangle z\right), \tag{1}$$

where the modeling operator L is can be a Green's function, a one-way or two-way wave equation, or a more complex wave equation suitable for wavefield extrapolation or modeling. Wavefield extrapolation and modeling operators were exhaustively studied by many authors (Hale, 1991c,a; Stoffa et al., 1990; Ristow and Ruhl, 1994; Huang et al., 1999; Biondi, 2002; Li, 1991; Docherty, 1991; Berkhout and Wapenaar, 1989; Wapenaar et al., 1989; Marfurt, 1984). However, prestack depth migration imaging is mostly implemented with one-way acoustic wave equations, which is one of the reasons it cannot achieve true-amplitude images. If a one-way wave equation is used for migration imaging, all wave phenomena except the primary reflection wave are processed as noise. In fact, if quantitative reflectivity is the objective of imaging, a two-way acoustic, or even elastic, wave equation should be used (Mora, 1988; Pratt, 1999; Pratt and Shipp, 1999).

SEISMIC DATA PREPROCESSING

As discussed above, seismic-wave imaging needs a suitable data set. In general cases, real data sets have some drawbacks. For example, the spatial sampling of land data commonly is too coarse and/or irregular; marine data sets commonly show feathering. Therefore, seismic-data preprocessing is necessary. Seismic-data preprocessing deals with the signal-to-noise enhancement, wavelet correction, seismic-data regularization and interpolation, and redatuming. The latter three terms are closely related to seismic-wave imaging. Seismic data regularization, interpolation and redatuming can be seen as a seismic-data mapping under the least-squares theory. An irregular seismic-data set from on-site field acquisition can be expressed as follows:

$$\mathbf{d}^{obs} = L\mathbf{m}, \tag{2}$$

where L is an ideal seismic-wave propagator, and $\mathbf{m}$ is an ideal underground medium model. From the irregular observed seismic-data, an underground medium model can be estimated:

$$\hat{\mathbf{m}}_{inv} = \left[\left(\hat{L}^*\right)^T \hat{L}\right]^{-1} \left(\hat{L}^*\right)^T \mathbf{d}^{obs} \tag{3}$$

where $\hat{L}$ is the practical seismic wave propagator, which can be written as a complex matrix. $\left(\hat{L}^*\right)^T$ is a conjugate transpose matrix of the matrix $\hat{L}$. Substituting the estimated model into equation (3), the estimated and regular data set can be found:

$$\hat{\mathbf{d}}_{reg} = L\left[\left(\hat{L}^*\right)^T \hat{L}\right]^{-1} \left(\hat{L}^*\right)^T \mathbf{d}^{obs}. \tag{4}$$

In equation (4), the ideal wave propagator L is unknown, but it can be replaced with the practical wave propagator $\hat{L}$. Therefore, equation (4) can be rewritten as

$$\hat{\mathbf{d}}_{reg} = \hat{L}\left[\left(\hat{L}^*\right)^T \hat{L}\right]^{-1} \left(\hat{L}^*\right)^T \mathbf{d}^{obs} = \hat{L}H^{-1}\left(\hat{L}^*\right)^T \mathbf{d}^{obs}, \tag{5}$$

where $H = \left(\hat{L}^*\right)^T \hat{L}$ is a Hessian matrix which acts as a filter. If we choose the filter as an ideal full-pass one, that is, $H = I$, equation (5) can be rewritten as

$$\hat{\mathbf{d}}_{reg} = \hat{L}\left(\hat{L}^*\right)^T \mathbf{d}^{obs}, \tag{6}$$

which is a general seismic data mapping frame. Up to now, the seismic data mapping can be implemented with $\mathbf{DMO} + \mathbf{DMO}^{-1}$ or $\mathbf{PSTM} + \mathbf{PSTM}^{-1}$(Biondi et al., 1998; Canning and Gardner, 1996; Ronen, 1987; Bleistein et al., 1999).

Redatuming also can be carried out with equation (5). The formula for redatuming should be modified as follows:

$$\hat{\mathbf{d}}_{datum} = \hat{L}_2\left[\left(\hat{L}_1^*\right)^T \hat{L}_1\right]^{-1}\left(\hat{L}_1^*\right)^T \mathbf{d}^{obs} = \hat{L}_2 H^{-1}\left(\hat{L}_1^*\right)^T \mathbf{d}^{obs}, \tag{7}$$

where $\hat{\mathbf{d}}_{datum}$ is a new and regular data set extrapolated from a topographic surface to another surface which may be a horizontal or non-horizontal datum. $\hat{L}_1^T$ is the propagator corresponding to the topographic surface, and $\hat{L}_2$ is the propagator to a horizontal datum. Now the Hessian matrix H has a relation to the topography and the acquisition configuration. Similarly, if the Hessian matrix is an ideal full-pass filter, equation (7) can be rewritten as

$$\hat{\mathbf{d}}_{datum} = \hat{L}_2\left(\hat{L}_1^*\right)^T \mathbf{d}^{obs}. \tag{8}$$

However, if a suitable Hessian filter is chosen, the quality of data mapping will be improved further. Next, we discuss data regularization with common-offset prestack time migration and the necessity of anti-aliasing for processing land data sets.

Common-offset prestack time migration and data regularization

The time-distance relation for a shot-receiver pair is

$$\sqrt{(x-h_x)^2 + \left(y-h_y\right)^2 + z^2} + \sqrt{(x+h_x)^2 + \left(y+h_y\right)^2 + z^2} = vt_h, \tag{9}$$

where t_h is the two-way traveltime of a non-zero-offset shot-receiver pair, h_x is the in-line component of the half-offset, and h_y is the cross-line component of the half-offset. For simplicity, the connection line of the shot and receiver points is parallel to the x-axis of the Cartesian coordinate system. Therefore, we have the following simple equation which delineates the isochron surface of the prestack migration:

$$\left(\frac{\acute{x}}{a_{\acute{x}}}\right)^2 + \left(\frac{\acute{y}}{a_{\acute{y}}}\right)^2 + \left(\frac{z}{a_z}\right)^2 = 1, \tag{10}$$

where $a_{\acute{x}}$,$a_{\acute{y}}$and a_z are the half-lengths of the axes of the rotary isochron ellipse in the case of constant velocity. If $\acute{y} = z = 0$, then $a_{\acute{x}} = \acute{x} = \frac{vt_h}{2}$; If $\acute{x} = z = 0$, then $a_{\acute{y}} = \acute{y} = \sqrt{\left(\frac{vt_h}{2}\right)^2 - h^2} =$

$\frac{vt_n}{2}$; If $\acute{x} = \acute{y} = 0$, then $a_z = z = \sqrt{\left(\frac{vt_h}{2}\right)^2 - h^2} = \frac{vt_n}{2}$. The variable t_n is the two-way traveltime after NMO. Equation (10) can be rewritten as

$$\left(\frac{\acute{x}}{\frac{vt_h}{2}}\right)^2 + \left(\frac{\acute{y}}{\frac{vt_n}{2}}\right)^2 + \left(\frac{z}{\frac{vt_n}{2}}\right)^2 = 1. \tag{11}$$

Further, equation (11) can be changed into

$$\frac{\acute{x}^2}{1+\left(\frac{2h}{vt_n}\right)^2} + \acute{y}^2 + z^2 = \left(\frac{vt_n}{2}\right)^2. \tag{12}$$

Defining $\acute{X}^2 = \frac{\acute{x}}{\sqrt{1+\left(\frac{2h}{vt_n}\right)^2}}$ yields:

$$\acute{X}^2 + \acute{y}^2 + z^2 = \left(\frac{vt_n}{2}\right)^2. \tag{13}$$

Equation (13) is in the form of a poststack migration. Therefore, prestack migration can be explained as a poststack migration on a post-NMO data set. We know that

$$k_{\acute{X}} = k_{\acute{x}}\sqrt{1+\left(\frac{2h}{vt_n}\right)^2}. \tag{14}$$

Therefore, the dispersion relation of equation (13) is

$$\left(\frac{v}{2}\right)^2 \left[k_{\acute{X}}^2 + k_{\acute{y}}^2 + k_z^2\right] = \omega_n^2. \tag{15}$$

Substituting (14) into the above formula yields

$$\left(\frac{v}{2}\right)^2 \left[k_{\acute{x}}^2 \left(1+\left(\frac{2h}{vt_n}\right)^2\right) + k_{\acute{y}}^2 + k_z^2\right] = \omega_n^2, \tag{16}$$

which can be rewritten as follows:

$$k_z = -sgn(\omega_n)\sqrt{\left(\frac{2\omega_n}{v}\right)^2 - \left(1+\left(\frac{2h}{vt_n}\right)^2\right)k_x^2 - k_y^2}. \tag{17}$$

This is the dispersion relation of the common-offset prestack migration equation. In the time domain, the dispersion relation is

$$k_\tau = -sgn(\omega_n)\sqrt{\omega_n^2 - \left(\frac{v(\tau)}{2}\right)^2 \left[\left(1+\left(\frac{2h}{v(\tau)t_n}\right)^2\right)k_x^2 + k_y^2\right]}. \tag{18}$$

Therefore, common-offset prestack time migration (PSTM) can be implemented with the following relation:

$$\mathbf{m}\left(\omega_\tau, k_x, k_y\right) = \int dt_n\, J \left\{ e^{-i\omega_\tau A t_n} \mathbf{d}\left(t_n, k_x, k_y, h\right) \right\}. \tag{19}$$

The term in the braces represents the wave-field extrapolation, and the integral at $t_n = 0$ serves to extract the imaging values. Then, the common-offset inverse PSTM is

$$\mathbf{d}\left(t_n, k_x, k_y, h\right) = \int d\omega_\tau \frac{1}{A} \left\{ e^{i\omega_\tau A t_n} \mathbf{m}\left(\omega_\tau, k_x, k_y\right) \right\}. \tag{20}$$

Similarly, the term in the braces represents the wave-field extrapolation, which is an inverse migration. The integral is an inverse Fourier transform.

In the presence of moderate lateral velocity variations, prestack time migration can be expressed as follows:

$$\begin{aligned} \mathbf{m}\left(\tau, m_x, m_y\right) &= \int dx_s \int dx_g W_1 e^{-i\omega\left[t-\left(t_s+t_g\right)\right]} \mathbf{d}\left(t, x_s, x_g, h\right), \\ &= \int dx_s \int dx_g W_1 \mathbf{d}\left(t, x_s, x_g, h\right) \delta\left(t = t_s + t_g\right), \end{aligned} \tag{21}$$

where $t = t_s + t_g = \sqrt{\left(\frac{m_x - h_x}{v_{rms}}\right)^2 + \left(\frac{m_y - h_y}{v_{rms}}\right)^2 + \left(\frac{\tau}{2}\right)^2} + \sqrt{\left(\frac{m_x + h_x}{v_{rms}}\right)^2 + \left(\frac{m_y + h_y}{v_{rms}}\right)^2 + \left(\frac{\tau}{2}\right)^2}$. W_1 is the amplitude weight, and τ is the two way traveltime along the imaging ray.

The inverse PSTM is

$$\begin{aligned} \mathbf{d}\left(t, x_s, x_g, h\right) &= \int dm_x \int dm_y W_2 e^{-i\omega\left[t+\left(t_s+t_g\right)\right]} \mathbf{m}\left(\tau, m_x, m_y\right), \\ &= \int dm_x \int dm_y W_2 \mathbf{m}\left(\tau, m_x, m_y\right) \delta\left(t = -\left(t_s + t_g\right)\right). \end{aligned} \tag{22}$$

Bleistein and Stockwell (2000) give a general theory of data mapping. From here, we will develop some practical approaches for data mapping.

Aliasing and anti-aliasing

From the discretized data space, the discretized model space, and their relation formula,

$$\mathbf{d}\left(m_{x_i}, m_{y_j}, h_{x_k}, h_{x_l}, z_n, \omega\right) = L\mathbf{m}\left(m_{x_i}, m_{y_j}, n\triangle z\right), \tag{23}$$

some causes of aliasing can be clearly seen. The sources of aliasing can be divided into the following three types:

(1) Overly coarse sampling intervals. For example, $\triangle\vec{s}$ and/or $\triangle\vec{g}$, $\triangle\vec{m}$ and/or $\triangle\vec{h}$ are too coarse, where $\triangle\vec{s}$ and $\triangle\vec{g}$ are the shot-point and receiver-point intervals respectively, and $\triangle\vec{m}$ and $\triangle\vec{h}$ are the CMP and half-offset intervals, respectively.

(2) Unsuitable modeling or imaging operators, such as the integrated DMO operator (Hale, 1991b), or the Kirchhoff integral operator (Zhang et al., 2003; Biondi, 2001; Abma et al., 1999; Druzhinin, 1999).

(3) Insufficient output resolution, where the output spatial intervals, such as $\triangle\vec{m}$ and $\triangle z$ are too coarse. We can analyze the aliasing in the cases of shot-gather migration, receiver-gather migration, common-offset gather migration, and midpoint half-offset domain migration.

Aliasing in shot gather migration is described by the following equation:

$$I(\vec{m}, z) = \sum_{S_k} \sum_{\omega} S^* (\vec{m}, z|S_k)\, R(\vec{m}, z|S_k), \tag{24}$$

where $S^*(\vec{m}, z|S_k)$ is the conjugate of the extrapolated shot wave-field; $R(\vec{m}, z|S_k)$ is the extrapolated receiver wave-field. S_k stands for the K^{th} common-shot gather. In general, single common-shot-gather imaging presents no aliasing, because the receiver-point interval is easily arranged regularly and small enough. However, imaging an entire 2D line or 3D area will present severe aliasing problems because of the irregular and large shot-point intervals.

Aliasing in receiver-gather migration is described by the following equation:

$$I(\vec{m}, z) = \sum_{R_k} \sum_{\omega} S^*_{sort} \left(\vec{m^k}, z|R_k\right) R_{sort} \left(\vec{m^k}, z|R_k\right), \tag{25}$$

where $S^*_{sort}\left(\vec{m^k}, z|R_k\right)$ is the extrapolated so-called shot wavefield, which corresponds to a specific receiver point; $R_{sort}\left(\vec{m^k}, z|R_k\right)$ is the extrapolated receiver wavefield, which is sorted from shot gathers. R_k stands for the K^{th} common-receiver-gather. In general, single common-receiver-gather imaging profiles are susceptible to aliasing.

Aliasing in midpoint half-offset domain migration follows equation (26):

$$I(\vec{m}, z) = \sum_{h} \sum_{\omega} U\left(\vec{m}, \vec{h}, \omega, z\right), \tag{26}$$

where $U\left(\vec{m}, \vec{h}, \omega, z\right)$ is the extrapolated wavefield with a double-square-root equation. Aliasing will result if $\triangle\vec{m}$ and/or $\triangle\vec{h}$ are too coarse.

Aliasing in common-offset-gather migration is described by the following equation:

$$I(\vec{m}, z) = \sum_{h_k} \sum_{\omega} U(\vec{m}, h_k, \omega, z), \tag{27}$$

where $U(\vec{m}, h_k, \omega, z)$ is the extrapolated wavefield with a double-square-root equation. Aliasing will result if $\triangle\vec{m}$ is too coarse.

In the case of rugged topography, the geometry of the acquisition configuration becomes more and more irregular, especially the shot-point coordinates. Therefore, antialiasing processing is necessary in redatuming, seismic data regularization and migration imaging for land-data imaging.

SEISMIC WAVE ILLUMINATION ANALYSIS

Seismic-wave illumination becomes increasingly problematic in regions with rugged topography and complex geological structure with severe lateral velocity variations. In these hostile cases, we think that seismic-wave illumination is much more important than seismic-data regularization. Without enough illumination for the target reflectors, regular seismic data can not guarantee a quality image.

Seismic-wave illumination is related to the macro-velocity model and the acquisition configuration, both of which are embodied in the Green's function. In fact, seismic-wave illumination analysis inherently is an issue of seismic-wave propagation and observation in the presence of complex velocity structure. Whether a seismic wave reaches a target reflector and whether the reflected wave is received are both important.

Wu and Chen (2002) analyze seismic-wave illumination with Beamlet Propagators. With directional illumination maps, the illumination of a reflector is demonstrated. Berkhout et al. (2001); Volker et al. (2001) discuss how the imaging resolution and amplitude are affected by the acquisition geometries with focal beams: emission-focusing and detection-focusing. However, neither methods deals with the compensation for illumination deficiency from the perspective of inverse imaging.

In least-squares inversion theory, the Hessian matrix—the second-order derivatives of the wavefield about the perturbation of a physical parameter—is given. The Hessian matrix is closely related to the seismic-wave illumination of a target reflector.

The two important issues of seismic-wave illumination analysis are (1) compensating for illumination deficiencies and (2) evaluating acquisition patterns and guiding their design.

Seismic-wave migration imaging can be represented by the following matrix equation:

$$\mathbf{m} = \left(L^*\right)^T \mathbf{d}, \tag{28}$$

where

$$\mathbf{d} = \left(d_{x_1}, d_{x_2}, ..., d_{x_n}\right)^T, \tag{29}$$

and

$$\mathbf{m} = \left(m_{x_1}, m_{x_2}, ..., m_{x_l}\right)^T, \tag{30}$$

$$\left(\mathbf{L}^*\right)^T = \begin{pmatrix} L_{11}^* & L_{12}^* & \cdots & L_{1n}^* \\ L_{21}^* & L_{22}^* & \cdots & L_{2n}^* \\ \cdots & \cdots & \cdots & \cdots \\ L_{l1}^* & L_{l2}^* & \cdots & L_{ln}^* \end{pmatrix}. \tag{31}$$

The index l is the number of imaging points or scattering points along the in-line direction in the Z^{th} layer; n is the number of shot-receiver pairs; L_{ij}^* are the complex amplitudes of the conjugate Green's functions corresponding to the imaging points. In the matrix $(\mathbf{L}^*)^T$, each row is a Green's function for an imaging point in the Z^{th} layer. In fact, equation (28) is the Kirchhoff integral migration formula expressed in matrix form.

However, if seismic-wave illumination is considered, the concept of double focusing (emission focusing and detection focusing) should be introduced into the general migration-imaging formula (28), following Berkhout's notation (Berkhout et al., 2001; Volker et al., 2001):

$$\mathbf{D}\mathbf{L}^U R \mathbf{L}^D \mathbf{S} = \mathbf{d}^{obs}. \tag{32}$$

The matrix formula stands for the emission of a wavefield from the source S and the detection by the receivers D; meanwhile the energy of the wavefield propagates downward to the reflector R with an ideal propagator $\mathbf{L}^D$, and is reflected back to the surface; $\mathbf{L}^U$ is an ideal upward propagator.

Defining $\mathbf{F}^U = (\mathbf{D}^*)^T \left(\mathbf{L}^{*U}\right)^T$ and $\mathbf{F}^D = \left(\mathbf{L}^{*D}\right)^T (\mathbf{S}^*)^T$ gives us the formulae for detection focusing and emission focusing, respectively. Together, they represent the illumination of a point on a reflector.

We will analyze the seismic-wave illumination of a target reflector with the local Hessian matrix and compare this with the double-focusing approaches.

MIGRATION IMAGING AND INVERSION IMAGING AS A LEAST-SQUARES PROBLEM

Seismic-wave imaging can be expressed as a least-squares inversion problem,

$$min|\mathbf{d}^{cal} - \mathbf{d}^{obs}|^2, \tag{33}$$

where, given an underground geological model characterized with some parameters such as P-wave velocity, S-wave velocity, and/or density, or a reflectivity image, we then create a synthetic data set which minimizes the "distance" between the calculated data set and the observed data set.

The solution of the inverse problem is expressed as follows:

$$\begin{aligned} \hat{\mathbf{m}_{inv}} &= \left[\left(\hat{L}^*\right)^T \hat{L}\right]^{-1} \left(\hat{L}^*\right)^T \hat{\mathbf{d}}^{obs} = \mathbf{H}^{-1} \left(\hat{L}^*\right)^T \hat{\mathbf{d}}^{obs} \\ &= \mathbf{H}^{-1} \hat{\mathbf{m}_{mig}}, \end{aligned} \tag{34}$$

or

$$\begin{aligned} \hat{\mathbf{m}_{inv}} &= \left[\left(\hat{L}^*\right)^T \hat{L}\right]^{-1} \left(\hat{L}^*\right)^T \hat{\mathbf{d}}^{reg} = \mathbf{H}^{-1} \left(\hat{L}^*\right)^T \hat{\mathbf{d}}^{reg} \\ &= \mathbf{H}^{-1} \hat{\mathbf{m}_{mig}}, \end{aligned} \tag{35}$$

or

$$\begin{aligned} \hat{\mathbf{m}_{inv}} &= \left[\left(\hat{L}^*\right)^T \hat{L}\right]^{-1} \left(\hat{L}^*\right)^T \hat{\mathbf{d}}^{datum} = \mathbf{H}^{-1} \left(\hat{L}^*\right)^T \hat{\mathbf{d}}^{datum} \\ &= \mathbf{H}^{-1} \hat{\mathbf{m}_{mig}}. \end{aligned} \tag{36}$$

Similarly, if we assume Hessian matrix is a unitary matrix, equation (35), (36), and (37) all degenerate to

$$\hat{\mathbf{m}_{mig}} = \left(\hat{L}^*\right)^T \hat{\mathbf{d}}^{obs}. \tag{37}$$

Migration imaging avoids the matrix inversion by replacing the general inverse with a conjugate-transpose operator $\left(\hat{L}^*\right)^T$. The advantage of the processing is to change an ill-posed inverse problem into a well-posed wavefield backpropagation problem, which is quite stable and robust (Ronen and Liner, 2000; Duquet et al., 2000; Chavent and Plessix, 1999; Nemeth et al., 1999; Chavent and Plessix, 1999). In fact, migration imaging mainly locates the reflector and gives only a qualitative estimate of the reflectivity. $\left(\hat{L}^*\right)^T$ is the two-way or one-way propagator, which commonly is expressed in the form of the conjugate Green's function.

In fact, the quantitative estimation of the reflectivity should take advantage of inverse imaging. If we consider the reflectivity imaging as a weighting summation, equation (37) gives an unsuitable weight function. Bleistein and Stockwell (2000) discuss in detail about how to choose a suitable weight function.

The inverse of the Hessian matrix is just a deconvolution operator, which modifies the unsuitable weight function of the migration imaging. Therefore, equation (35) can give more accurate estimate of the reflectivity than can the migration imaging (equation (37).

If equation (35) is rewritten as

$$\hat{\mathbf{m}}_{inv} = \mathbf{H}^{-1}\hat{\mathbf{m}}_{mig}, \tag{38}$$

it is clear that Hessian matrix is a deconvolution operator, which improves the resolution of migration results (Hu et al., 2001). We will consider how to quantitatively estimate the reflectivity with inverse imaging and determine the conditions under which direct inverse imaging and iterative inverse imaging are equivalent.

RELATIONSHIP BETWEEN WAVEFIELD-EXTRAPOLATION IMAGING AND INVERSE IMAGING

At the scattering point, we can define a "distance" or norm as

$$E(R) = \sum_{\omega_{min}}^{\omega_{max}} (U_S - U_I R)^2 \, d\omega, \tag{39}$$

where R is the reflectivity, U_S is the wavefield downward extrapolated to a reflector, and U_I is the wavefield downward propagated to the reflector. The scattering wavefield U_S should be equal to or close to the convolution result between the wavefield U_I and the reflectivity. Letting

$$\frac{\partial E}{\partial R} = 0, \tag{40}$$

we have

$$-2\sum_{\omega_{min}}^{\omega_{max}}\left(U_S(\omega)-U_I(\omega)R\right)U_I(\omega)\,d\omega=0, \tag{41}$$

If the incident wavefield equals zero, equation (41) is satisfied. However this case has no physical meaning. If the incident wavefield does not equal zero, then,

$$R=\frac{\sum\limits_{\omega_{min}}^{\omega_{max}} U_S}{\sum\limits_{\omega_{min}}^{\omega_{max}} U_I}. \tag{42}$$

In the complex domain, we can rewrite equation (42) as

$$R=\frac{\sum\limits_{\omega_{min}}^{\omega_{max}} U_S U_I^*}{\sum\limits_{\omega_{min}}^{\omega_{max}} U_I U_I^*}. \tag{43}$$

If the incident wave is quite weak, the following regularization should be introduced:

$$R=\frac{\sum\limits_{\omega_{min}}^{\omega_{max}} U_S U_I^*}{\sum\limits_{\omega_{min}}^{\omega_{max}} \left(U_I U_I^*+\varepsilon\right)}, \tag{44}$$

where ε is the regularization coefficient. In fact, the reflectivity is related to the incident angle to a reflector of the plane-wave component of a seismic wave. Therefore, we should modify equation (44) into the following form to reach the angle gathers:

$$R(p)=\frac{\sum\limits_{\omega_{min}}^{\omega_{max}} U_S(\omega,p)\,U_I^*(\omega,p)}{\sum\limits_{\omega_{min}}^{\omega_{max}} \left(U_I(\omega,p)\,U_I^*(\omega,p)+\varepsilon\right)}, \tag{45}$$

where $U_S(\omega,p)$ and $U_I^*(\omega,p)$ are a scattering plane wavefield and an incident plane wavefield, respectively. In fact, the extrapolated wavefield can be defined as

$$U_S=\left(\hat{L}^*\right)^T\hat{\mathbf{d}}^{obs}. \tag{46}$$

Therefore, in the frequency domain, equation (44) can be rewritten as

$$R=\frac{\sum\limits_{\omega_{min}}^{\omega_{max}} \left(\hat{L}^{*U}\right)^T\hat{\mathbf{d}}^{obs}\left(\hat{L}^{*D}\right)^T}{\sum\limits_{\omega_{min}}^{\omega_{max}} \left(\hat{L}^D\left(\hat{L}^{*D}\right)^T+\varepsilon\right)}. \tag{47}$$

We will further discuss this topic later to clarify the relationship.

RELATED TOPICS

Angle gathers

Common-image gathers are closely related to the angle reflectivity, which can be used for AVA analysis or AVA inversion. Unfortunately, macro-velocity errors will cause amplitude aberrations in common-image gathers. Therefore, some traps in AVA analysis or inversion should be carefully avoided. However, residual depth or time differences are present in the common-image gathers if macro-velocity field has errors; these differences can be used for migration velocity analysis.

On the other hand, the amplitude-preserving common-image gathers may be generated from inverse imaging.

Wavefield propagator

In seismic-wave migration imaging, the conjugate-transpose matrix $\left(\hat{L}^*\right)^T$ stands for the back-propagation of the observed wavefield. Therefore, the wavefield propagators are the basis for seismic-wave imaging.

For constructing a wavefield propagator, we introduce the following methods: (1) a hybrid wavefield propagator, that is, the split-step-Fourier propagator plus optimal interpolation with a self-adaptive reference velocity choice; and (2) a local and directional wavefield propagator, which can be designed with the local Fourier transform and local plane wave/Gaussian beam, for target-oriented imaging (Hill, 2001, 1990; Soubaras, 2003).

MIGRATION VELOCITY ANALYSIS/INVERSION

The macro-velocity field has a decisive influence on seismic-wave imaging. Unfortunately, it is not easy to accurately estimate the velocity field from the seismic data. Up to now, the residual depth/time difference in the common-image gathers has been used for migration-velocity analysis (MVA) or inverting the macro-velocity distribution. However, in the case of complex topography and gelogical structures, MVA is not a successful approach. Therefore, seismic-wave imaging in complex survey areas has a long way to go. We propose the following approach to inverting the macro-velocity field. The norm is defined as

$$E = W_1\left(U_S^{k+1} - U_S^k\right)^2 + W_2\left(\triangle S_m^{k+1} - \triangle S_m^k\right)^2 + W_3\left(R^{k+1} - R^k\right)^2, \tag{48}$$

where k stands for the iterative number; U_S is the calculated scattering wavefield. R is the position of the main reflectors, which can be identified from the migrated profile. $\triangle S_m$ is the slowness disturbance field. W_1,W_2 and W_3 are the different weights. According to Bleistein (2000,p.39), the calculated scattering wavefield can be given by

$$U_S\left(\vec{x}_g, \vec{x}_s, \omega\right) = \omega^2 \int_0^\infty \frac{\alpha\left(\vec{x}\right)}{c^2\left(\vec{x}\right)} U_I\left(\vec{x}, \vec{x}_s, \omega\right) g\left(\vec{x}, \vec{x}_g, \omega\right) d\vec{x}, \tag{49}$$

where $\alpha(\vec{x}) = \frac{c^2(\vec{x})}{c^2(\vec{x})} - 1$. Alternatively, the calculated scattering wavefield (Huang et al., 1999) also can be given by

$$\frac{\partial U_S\left(\omega, k_x, k_y, z\right)}{\partial z} = \frac{i}{k_z} FT_{x,y}\left[\omega \triangle S(x,y,z) U_I(\omega, x, y, z)\right], \tag{50}$$

where $\triangle S(x,y,z) = S(x,y,z) - S_{ref}(x,y,z)$ is the slowness disturbance, U_I is the incident wave field, and k_z is the vertical wavenumber. The incident wave field U_I can be calculated with the following equation:

$$\frac{\partial U_I\left(\omega, k_x, k_y, z\right)}{\partial z} = i k_0 k_z U_I\left(\omega, k_x, k_y, z\right). \tag{51}$$

where $k_0 = \frac{\omega}{v_r}$, $k_z = \sqrt{1 - \left(\frac{k_T}{k_0}\right)^2}$ and $k_T = \sqrt{k_x^2 + k_y^2}$.

DISCUSSION AND CONCLUSION

Migration algorithms extract the depth locations and relative amplitude behavior of reflectors in the earth from measured seismic data. However, these classic approaches cannot give a quantitative estimate of the reflectivity. In fact, seismic-wave imaging can be performed with the operator and matrix operations, based on least-squares inverse theory. The inverse imaging approaches have the potential to generate quantitive estimates of the reflectivity and to cope with seismic-data regularization and seismic-wave illumination. The ideas we proposed using these theories will open avenues for further research.

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REFERENCES

Abma, R., Sun, J., and Bernitsas, N., 1999, Antialiasing methods in Kirchhoff migration: Geophysics, **64**, 1783–1792.

Berkhout, A. J., Ongkiehong, O., Volker, A., and Blacquiere, G., 2001, Comprehensive assessment of seismic acquisition geometries by focal beamsŮPart I: Theoretical considerations: Geophysics, **66**, 911–917.

Berkhout, A. J., and Wapenaar, C. P. A., 1989, One-way versions of the Kirchhoff integral: Geophysics, **54**, 460–467.

Biondi, B., 2001, Kirchhoff imaging beyond aliasing: Geophysics, **66**, 654–666.

—, 2002, Stable wide-angle Fourier finite-difference downward extrapolation of 3-D wavefields: Geophysics, **67**, 872–882.

Biondi, B., Fomel, S., and Chemingui, N., 1998, Azimuth moveout for 3-D prestack imaging: Geophysics, **63**, 574–588.

Black, J. L., Schleicher, K. L., and Zhang, L., 1993, True-amplitude imaging and dip moveout: Geophysics, **58**, 47–66.

Bleistein, N., C. J. K., and Stockwell, J. W. J., 2000, Mathematics of multidimensional seismic imaging, migration, and inversion Springer., Chapter 7.

Bleistein, N., Cohen, J., and Jaramillo, H., 1999, True-amplitude transformation to zero offset of data from curved reflectors: Geophysics, **64**, 112–129.

Bleistein, N., Cohen, J. K., and Hagin, F. G., 1987, Two and one-half dimensional Born inversion with an arbitrary reference: Geophysics, **52**, 26–36.

Brandsberg-Dahl, S., de Hoop, M. V., and Ursin, B., 2003, Focusing in dip and AVA compensation on scattering-angle/azimuth common image gathers: Geophysics, **68**, 232–254.

de Bruin, C. G. M., Wapenaar, C. P. A., and Berkhout, A. J., 1990, Angle-dependent reflectivity by means of prestack migration: Geophysics, **55**, 1223–1234.

Canning, A., and Gardner, G. H. F., 1996, Regularizing 3-D data sets with DMO: Geophysics, **61**, 1103–1114.

Cao, D., Beydoun, W. B., Singh, S. C., and Tarantola, A., 1990, A simultaneous inversion for background velocity and impedance maps: Geophysics, **55**, 458–469.

Chavent, G., and Jacewitz, C. A., 1995, Determination of background velocities by multiple migration fitting: Geophysics, **60**, 476–490.

Chavent, G., and Plessix, R. E., 1999, An optimal true-amplitude least-squares prestack depth-migration operator: Geophysics, **64**, 508–515.

Docherty, P., 1991, A brief comparison of some Kirchhoff integral formulas for migration and inversion: Geophysics, **56**, 1164–1169.

Druzhinin, A., 1999, Anti-aliased kirchhoff-helmholtz transformations: Geophysical Prospecting, **47**, 757–783.

Duquet, B., Marfurt, K. J., and Dellinger, J., 2000, Kirchhoff modeling, inversion for reflectivity, and subsurface illumination: Geophysics, **65**, 1195–1209.

Gray, S. H., 1997, True-amplitude seismic migration: A comparison of three approaches: Geophysics, **62**, 929–936.

Hale, D., 1991a, 3-D depth migration via McClellan transformations: Geophysics, **56**, 1778–1785.

—, 1991b, A nonaliased integral method for dip moveout: Geophysics, **56**, 795–805.

—, 1991c, Stable explicit depth extrapolation of seismic wavefields: Geophysics, **56**, 1770–1777.

Hill, N. R., 1990, Gaussian beam migration: Geophysics, **55**, 1416–1428.

—, 2001, Prestack Gaussian-beam depth migration: Geophysics, **66**, 1240–1250.

Hu, J., Schuster, G. T., and Valasek, P., 2001, Poststack migration deconvolution: Geophysics, **66**, 939–952.

Huang, L. Y., Fehler, M. C., and Wu, R. S., 1999, Extended local Born Fourier migration method: Geophysics, **64**, 1524–1534.

LeBras, R., and Clayton, R. W., 1988, An iterative inversion of back-scattered acoustic waves: Geophysics, **53**, 501–508.

Li, Z., 1991, Compensating finite-difference errors in 3-D migration and modeling: Geophysics, **56**, 1650–1660.

Marfurt, K. J., 1984, Accuracy of finite-difference and finite-element modeling of the scalar and elastic wave-equations: Geophysics, **49**, 533–549.

Miller, D., Oristaglio, M., and Beylkin, G., 1987, A new slant on seismic imaging - Migration and integral geometry: Geophysics, **52**, 943–964.

Mora, P., 1988, Elastic wave-field inversion of reflection and transmission data: Geophysics, **53**, 750–759.

—, 1989, Inversion = migration + tomography: Geophysics, **54**, 1575–1586.

Mora, P. R., 1987, Nonlinear two-dimensional elastic inversion of multioffset seismic data: Geophysics, **52**, 1211–1228.

Nemeth, T., Sun, H., and Schuster, G. T., 2000, Separation of signal and coherent noise by migration filtering: Geophysics, **65**, 574–583.

Nemeth, T., Wu, C., and Schuster, G. T., 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, 208–221.

Plessix, R.-E., and Mulder, W. A., 2004, Frequency-domain finite-difference amplitude-preserving migration: Geophys. J. Int., **157**, 975–987.

Pratt, R. G., 1999, Seismic waveform inversion in the frequency domain, Part 1: Theory and verification in a physical scale model: Geophysics, **64**, 888–901.

Pratt, R. G., and Shipp, R. M., 1999, Seismic waveform inversion in the frequency domain, Part 2: Fault delineation in sediments using crosshole data: Geophysics, **64**, 902–914.

Pratt, R. G., S. C., and Hicks, G. J., 1998, Gauss-newton and full newton methods in frequency-space seismic waveform inversion: Geophys. J. Int., **133**, 341–362.

Ristow, D., and Ruhl, T., 1994, Fourier finite-difference migration: Geophysics, **59**, 1882–1893.

Ronen, J., 1987, Wave equation trace interpolation: Geophysics, **52**, 973–984.

Ronen, S., and Liner, C. L., 2000, Least-squares DMO and migration: Geophysics, **65**, 1364–1371.

Shin, C., J. S., and Min, D., 2001, Improved amplitude preservation for prestack depth migration by inverse scattering theory: Geophyscal Prospecting, **49**, 592–606.

Soubaras, R., 2003, Angle gathers for shot-record migration by local harmonic decomposition: 73th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded abstracts, 889–892.

Stoffa, P. L., Fokkema, J. T., de Luna Freire, R. M., and Kessinger, W. P., 1990, Split-step Fourier migration: Geophysics, **55**, 410–421.

Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, 1259–1266.

Volker, A., Blacquiere, G., Berkhout, A. J., and Ongkiehong, O., 2001, Comprehensive assessment of seismic acquisition geometries by focal beams – Part II: Practical aspects and examples: Geophysics, **66**, 918–931.

Wapenaar, C. P. A., Peels, G. L., Budejicky, V., and Berkhout, A. J., 1989, Inverse extrapolation of primary seismic waves: Geophysics, **54**, 853–863.

Wu, R., and Chen, L., 2002, Mapping directional illumination and acquisition-aperture efficacy by beamlet propagators: 72th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded abstracts, 1352–1355.

Wu, R. S., and Toksoz, M. N., 1987, Diffraction tomography and multisource holography applied to seismic imaging: Geophysics, **52**, 11–25.

Zhang, Y., Sun, J., and Gray, S., 2003, Aliasing in wavefield extrapolation prestack migration: Geophysics, **68**, 629–633.