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Short Note

Converted-wave common-azimuth migration

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INTRODUCTION

Multicomponent seismic data may hold a wealth of information for oil exploration and reservoir characterization. Multicomponent seismic contains energy from converted waves that is not seen in conventional seismic; therefore, the development of new techniques to process converted-wave data is important. Much progress has been made in many areas of converted-wave seismic processing, such as stacking, DMO, migration and velocity analysis (Tessmer and Behle, 1988; Iverson et al., 1989; Huub Den Rooijen, 1991; Alfaraj, 1992; Harrison and Stewart, 1993). However, more advanced techniques for single-mode PP seismic still have few converted-wave counterparts.

Common-azimuth migration is an efficient and robust technique for obtaining accurate single-mode PP 3-D seismic images. This technique takes advantage of the reduced dimensionality of the computational domain. It assumes that the data have only the zero cross-line offset; that is, all the traces in the data share the same azimuth (Biondi and Palacharla, 1996). Due to the growing number of 3-D multicomponent seismic data sets in areas where an accurate processing is required to obtain better subsurface images and/or estimate rock properties, wavefield-based continuation methods, such as common-azimuth migration, for converted-mode data are of great importance and are very much needed in the oil industry today.

Converted-wave common-azimuth migration is very similar to conventional common-azimuth migration. However, it uses different propagation velocities for different wavefields. We compare the differences between single-mode and converted-mode common-azimuth migration.

CONVERTED-WAVE COMMON-AZIMUTH MIGRATION

Point-scatterer geometry is a good starting point to discuss converted-waves prestack common-azimuth migration. The equation for the travel time is the sum of a downgoing travel path with

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P-velocity (v_p) and an upcoming travel path with S-velocity (v_s),

$$t = \frac{\sqrt{z^2 + \|\mathbf{s}-\mathbf{x}\|^2}}{v_p} + \frac{\sqrt{z^2 + \|\mathbf{g}-\mathbf{x}\|^2}}{v_s}, \tag{1}$$

where **s** and **g** represent the source and receiver vector locations and **x** is the point-scatterer subsurface position. Common-azimuth migration is a wavefield-based, downward-continuation algorithm. The algorithm is based on a recursive solution of the one-way wave equation (Claerbout, 1985). The basic continuation step used to compute the wavefield at depth $z+\Delta z$ from the wavefield at depth z can be expressed in the frequency-wavenumber domain as follows:

$$P_{z+\Delta z}(\omega, \mathbf{k_m}, \mathbf{k_h}) = P_z(\omega, \mathbf{k_m}, \mathbf{k_h}) e^{ik_z \Delta z}. \tag{2}$$

After each depth-propagation step, the propagated wavefield is equivalent to the data that would have been recorded if all sources and receivers were placed at the new depth level (Schultz and Sherwood, 1980). The wavefields are propagated with two different velocities, a P-velocity for the downgoing wavefield and an S-velocity for the upcoming wavefield. The basic downward continuation for converted waves is performed by applying the Double-Square-Root (DSR) equation:

$$k_z\left(\omega, \mathbf{k_s}, \mathbf{k_g}\right) = \mathrm{DSR}\left(\omega, \mathbf{k_s}, \mathbf{k_g}\right) = -\sqrt{\frac{\omega^2}{v_p^2(\mathbf{s},z)} - \mathbf{k_s}^2} - \sqrt{\frac{\omega^2}{v_s^2(\mathbf{g},z)} - \mathbf{k_g}^2}, \tag{3}$$

or in midpoint-offset coordinates,

$$\mathrm{DSR}(\omega, \mathbf{k_m}, \mathbf{k_h}) = -\sqrt{\frac{\omega^2}{v_p^2(\mathbf{s},z)} - \frac{1}{4}(\mathbf{k_m}-\mathbf{k_h})\cdot(\mathbf{k_m}-\mathbf{k_h})} - \sqrt{\frac{\omega^2}{v_s^2(\mathbf{g},z)} - \frac{1}{4}(\mathbf{k_m}+\mathbf{k_h})\cdot(\mathbf{k_m}+\mathbf{k_h})}. \tag{4}$$

The common-azimuth downward-continuation operator takes advantage of the reduced dimensionality of the data space, which results from using a common-azimuth resorting of the data. Rosales and Biondi (2004) discuss how to do this resorting for converted-wave data. The general continuation operator can then be expressed as follows (Biondi and Palacharla, 1996):

$$\begin{aligned} P_{z+\Delta z}\left(\omega, \mathbf{k_m}, k_{x_h}, y_h = 0\right) &= \int_{-\infty}^{+\infty} dk_{y_h} P_z\left(\omega, \mathbf{k_m}, k_{x_h}, y_h = 0\right) e^{-ik_z\Delta z} \\ &= P_z\left(\omega, \mathbf{k_m}, k_{x_h}, y_h = 0\right) \left\{ \int_{-\infty}^{+\infty} dk_{y_h} e^{-ik_z\Delta z} \right\} \\ &\approx P_z\left(\omega, \mathbf{k_m}, k_{x_h}, y_h = 0\right) A\left(\omega, \mathbf{k_m}, k_{x_h}\right) e^{-i\widehat{k_z}\Delta z}. \end{aligned} \tag{5}$$

Since common-azimuth data is independent of k_{y_h}, the integral can be pulled inside and analytically approximated by the stationary-phase method (Bleinstein, 1984). The application of the stationary-phase method is based on a high-frequency approximation. By geometrical means we derive the stationary-path approximation for converted waves.

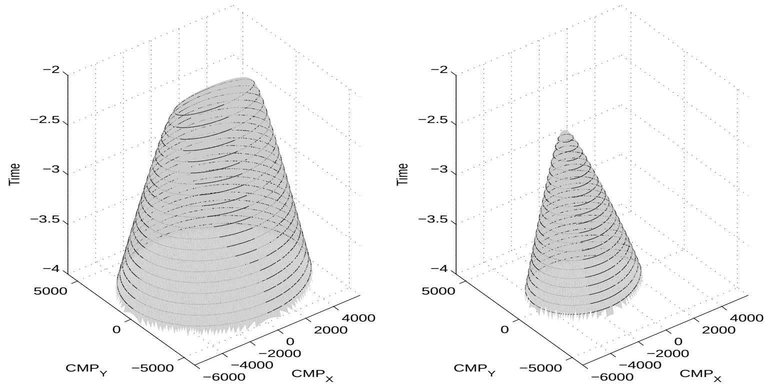

Figure 1: Summation surfaces for a constant medium with a of P-velocity of 3000 m/s, an S velocity of 1500 m/s, and an offset of 2 km. Left: PP summation surface. Right: PS summation surface. daniel1-impcheops [CR]

The expression for $\widehat{k_z}$ comes from substituting the stationary-path approximation into the expression for the full DSR of equation (4):

$$\widehat{k_z} = \mathrm{DSR}\left[\omega, \mathbf{k_m}, k_{h_x}, \widehat{k}_{h_y}(z), z\right] \tag{6}$$

where

$$\widehat{k}_{h_y}(z) = k_{ym} \frac{\sqrt{\frac{\omega^2}{v_s^2(\mathbf{g},z)} - \frac{1}{4}(k_{xm}+k_{xh})^2} - \sqrt{\frac{\omega^2}{v_p^2(\mathbf{s},z)} - \frac{1}{4}(k_{xm}-k_{xh})^2}}{\sqrt{\frac{\omega^2}{v_s^2(\mathbf{g},z)} - \frac{1}{4}(k_{xm}+k_{xh})^2} + \sqrt{\frac{\omega^2}{v_p^2(\mathbf{s},z)} - \frac{1}{4}(k_{xm}-k_{xh})^2}}. \tag{7}$$

IMPULSE RESPONSE

Figure 1 presents the summation surfaces [equation (1)] for an impulse response at a depth of 500 m, a P-velocity of 3000 m/s, an S-velocity of 1500 m/s, and an in-line offset of 3000 m. The left panel shows the single-mode PP summation surface, and the right panel shows the converted-mode PS summation surface. Similarly, Figure 2 shows the spreading surfaces; that is, the theoretical solution for depth of equation (1), Appendix A shows the calculations. The left panel presents the single-mode PP spreading surface, the center panel shows the converted-mode PS spreading surface, and the right panel compares the contour lines for both spreading surfaces.

Figure 3 shows the common-azimuth impulse response for a constant P-velocity of 2500 m/s, and a constant S-velocity of 1250 m/s, and an in-line offset of 200 m. The left panel exhibits the response for the single-mode PP common-azimuth migration operator, and the right panel exhibits the response for the converted-mode PS migration operator.

CONCLUSIONS

We presented the converted-mode PS 3-D common-azimuth migration operator. The difference between this operator and the single-mode PP operator is the use of two different veloc-

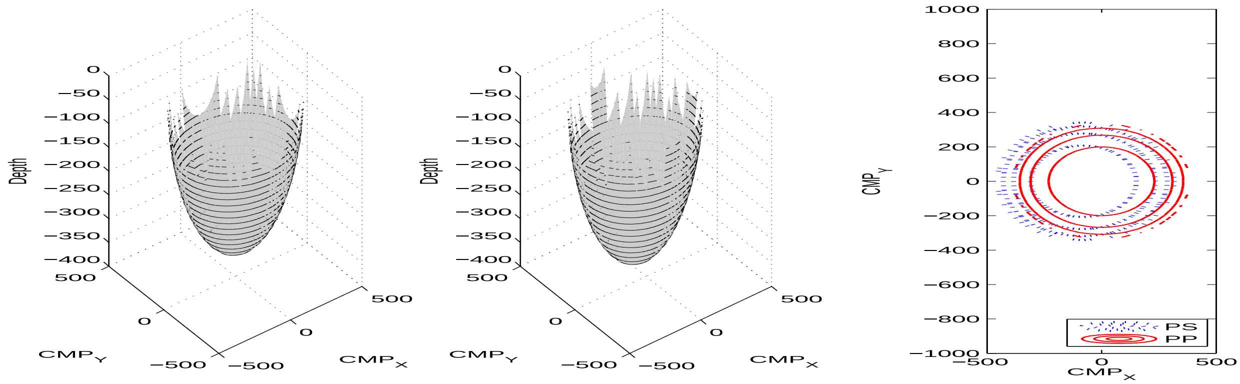

Figure 2: Spreading surfaces for an impulse at 0.320 s PP traveltime and 0.480 s PS traveltime, an offset of 200 m, and assuming constant P-velocity of 2500 m and S-velocity of 1250 m. Left: PP spreading surface. Center: PS spreading surface. Right: contour lines comparison from both spreading surfaces. daniel1-imptheory [CR]

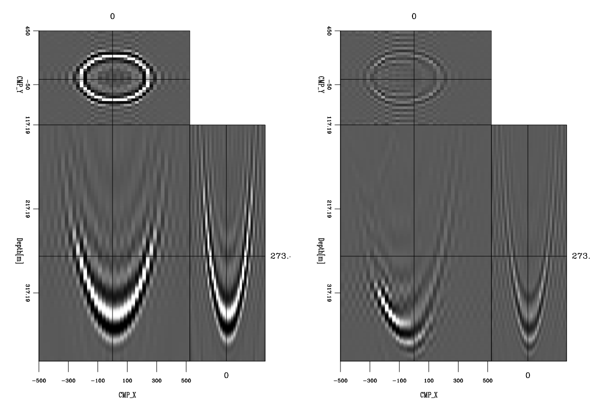

Figure 3: Impulse response for a point diffractor at 0.320 s PP traveltime and 0.480 s PS traveltime, and in a constant-velocity medium with a P-velocity of 2500 m/s and S-velocity of 1250 m/s. Left: presents the single-mode PP. Right: panel presents the converted-mode PS. daniel1-imp.resp [CR]

ity fields. Therefore, a more careful implementation is needed to ensure the correct velocity model. We demonstrate that the subsurface area covered by the PS common-azimuth migration operator is different than that covered by the PP common-azimuth migration operator; therefore, only the area that the two surfaces share can be used for rock-properties analysis based on the two complementary images. This might have important impacts on the reservoir-characterization process.

APPENDIX A

This section derives the exact solution for the common azimuth prestack migration for a reflecting point within an homogeneous Earth. The total travel time is

$$t_D = \frac{\sqrt{z_\xi^2 + \|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2}}{v_s} + \frac{\sqrt{z_\xi^2 + \|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2}}{v_p}. \qquad \text{(A-1)}$$

The following procedure shows how to go from $t_D(z_\xi, \mathbf{m}, \mathbf{h})$ to $z_\xi(t_D, \mathbf{m}, \mathbf{h})$:

$$t_D v_p = \phi \sqrt{z_\xi^2 + \|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2} + \sqrt{z_\xi^2 + \|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2}, \tag{A-2}$$

where ϕ represents the P-to-S velocities ratio. If we make the following definitions,

$$\begin{aligned} 2A &= t_D v_p, \\ \alpha &= z_\xi + \|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2, \\ \beta &= z_\xi + \|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2, \end{aligned} \tag{A-3}$$

(A-2) becomes

$$2A = \phi\sqrt{\alpha} + \sqrt{\beta}. \tag{A-4}$$

We square both sides to get a new equation with only one square root:

$$4A^2 - (\phi^2\alpha + \beta) = 2\phi\sqrt{\alpha\beta}. \tag{A-5}$$

Squaring again to eliminate the square root, and combining elements, we obtain

$$16A^4 - 8A^2(\phi^2\alpha + \beta) + (\phi^2\alpha - \beta)^2 = 0. \tag{A-6}$$

This expression is a 4^{th} degree polynomial in z_ξ; which is:

$$\begin{aligned} 0 &= 16A^4 - 8A^2((\phi^2+1)z_\xi^2 + \phi^2(\|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2)^2 + (\|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2)^2) \\ &+ ((\phi^2-1)z_\xi^2 + \phi^2(\|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2)^2 - (\|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2)^2)^2. \end{aligned} \tag{A-7}$$

This can also be writen as follows

$$\begin{aligned} 0 &= (\phi^2-1)^2 z_\xi^4 \\ &+ (2\phi^2(\|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2)^2 - (\|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2)^2(\phi^2-1) - 8(t_D v_p)^2(\phi^2+1))z_\xi^2 \\ &+ 16(t_D v_p)^4 - 8(t_D v_p)^2\phi^2(\|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2)^2 + (\|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2)^2 \\ &+ (\phi^2(\|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2)^2 - (\|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2)^2)^2 \end{aligned} \tag{A-8}$$

This polynomial equation has 4 solutions, which take the following well known form:

$$z_\xi = \pm\sqrt{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}, \tag{A-9}$$

where

$$\begin{aligned} a &= \left(\phi^2 - 1\right)^2, \\ b &= 2\phi^2(\|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2)^2 - (\|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2)^2(\phi^2-1) - 8(t_D v_p)^2(\phi^2+1), \\ c &= 16(t_D v_p)^4 - 8(t_D v_p)^2\phi^2(\|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2)^2 + (\|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2)^2 \\ &+ (\phi^2(\|\xi_{\mathbf{xy}} - \mathbf{m} + h_D\|^2)^2 - (\|\xi_{\mathbf{xy}} - \mathbf{m} - h_D\|^2)^2)^2. \end{aligned} \tag{A-10}$$

REFERENCES

Alfaraj, M., 1992, Transformation to zero offset for mode converted waves by Fourier transform: 62nd Annual Internat. Mtg., Society Of Exploration Geophysicists, Expanded Abstracts, 974–977.

Biondi, B., and Palacharla, G., 1996, 3-D prestack migration of common-azimuth data: Geophysics, **61**, 1822–1832.

Bleinstein, N., 1984, Mathematical methods for wave phenomena: Academic Press.

Claerbout, J. F., 1985, Imaging the earth's interior: Blackwell Scientific Publications.

Harrison, M. P., and Stewart, R. R., 1993, Poststack migration of P-SV seismic data: Geophysics, **58**, no. 8, 1127–1135.

Huub Den Rooijen, P. G. M., 1991, Stacking of P-SV seismic reflection data using dip moveout: Geophysical Prospecting, **39**, no. 4, 585–598.

Iverson, W. P., Fahmy, B. A., and Smithson, S. B., 1989, VpVs from mode-converted P-SV reflections: Geophysics, **54**, no. 7, 843–852.

Rosales, D. A., and Biondi, B., 2004, Acquisition geometry regularization for multicomponent data: 74rd Annual Internat. Mtg., Society Of Exploration Geophysicists, Expanded Abstracts, 1162–1165.

Schultz, P. S., and Sherwood, J. W. C., 1980, Depth migration before stack: Geophysics, **45**, 376–393.

Tessmer, G., and Behle, A., 1988, Common reflection point data-stacking technique for converted waves: Geophysical Prospecting, **36**, no. 5, 671–688.