Stanford Exploration Project, Report 120, May 3, 2005, pages 237–247

Interpolating with data-space prediction-error filters

William Curry[1]

ABSTRACT

The Madagascar sea elevation dataset presents a problem where data are collected along crossing tracks. These tracks are not straight, and are therefore irregular in the model space. Previous methods assumed that the data were regularly sampled in the model space coordinate system, or did not take into account the regularities in the acquisition of the data. Instead of attempting to find a prediction-error filter in the model space, I estimate two prediction-error filters in a coordinate system based on the data's spatial distribution, and show how to regularize the data with these filters with promising results. I then show how this strategy can be applied to 2D and 3D land surveys when data predicted by reciprocity is included.

INTRODUCTION

The Madagascar seasat sea level dataset is a collection of two passes of the GEOSAT satellite (ascending and descending) over a region of the Southwest Indian Ridge in the Indian Ocean. There is a densely-acquired region of the dataset in the south, which ranges from 40 to 70 degrees (E) longitude and 30 to 40 degrees (S) latitude, while the latitude of the sparsely-acquired data ranges from 20 to 40 degrees (S) latitude, as shown in Figure 1.

The satellite tracks are much like feathered marine geophone cables, sail lines, or shot lines in a 3D seismic survey. Any method that hopes to succeed on 3D seismic data should be able to deal with this toy problem.

Early work on this dataset at SEP (Ecker and Berlioux, 1995; Lomask, 1998, 2002) has mainly dealt with the systematic errors present in the dense dataset (Ecker and Berlioux, 1995), or with ways in which to use information in the dense portion of the data to regularize the missing bins in the northern, sparse portion of the data (Lomask, 1998, 2002). In the latter, it is assumed that the statistics of the data are stationary over both regions. More recent work has started to deal with the interpolation of only the sparse tracks (Curry, 2004; Lomask, 2004).

In this paper, only the lower half of the dataset (30 to 40 degrees (S) latitude) is examined, so that interpolation of the sparse tracks can be compared with the dense tracks. Here, a different approach is presented to dealing with the sparse track problem, where a pair of prediction-error filters (PEFs) are estimated directly on the two sets of tracks. This pair of

[1]**email:** bill@sep.stanford.edu

filters is estimated in the data space, so that the issues of missing data and irregular geometry are no longer present. Once these filters have been estimated, they can be used in tandem to regularize the missing portions of the model space.

Extension of this method to incorporate non-stationary PEFs is quite straightforward. The similarities between the Madagascar data and a Colombian 2D seismic data line are noteworthy enough that this method should be applicable to 2D land data, where the two sets of tracks correspond to positive and negative offsets. The geometry of a 3D cross-swath land seismic survey also has similarities to the Madagascar data, where when data predicted by reciprocity is added irregular crossing tracks are present in cmp_x, offset_x space.

BACKGROUND

The Madagascar regularization problem has been approached using the following fitting goals (Lomask, 2002):

$$\begin{aligned} \mathbf{W}\frac{d}{dt}[\mathbf{L}\mathbf{m}-\mathbf{d}] &\approx \mathbf{0} \\ \epsilon\mathbf{A}\mathbf{m} &\approx \mathbf{0}. \end{aligned} \tag{1}$$

In these fitting goals: $\mathbf{W}$ corresponds to a weight for ends of tracks and spikes in the data, $\frac{d}{dt}$ is a derivative along each track used to eliminate low frequency variations along each track, $\mathbf{L}$ is a linear interpolation operator that moves from values on a regular grid to the data points, $\mathbf{m}$ is the desired gridded model, $\mathbf{d}$ are the data points along the tracks, $\mathbf{A}$ is a regularization operator, and ϵ is a trade-off parameter between the two fitting goals.

The regularization operator ($\mathbf{A}$) typically is a Laplacian, a prediction-error filter (PEF), or a non-stationary PEF (Crawley, 2000). When using a PEF, it first must be estimated on some training data, using a least-squares fitting goal,

$$\mathbf{W}(\mathbf{D}\mathbf{K}\mathbf{a}+\mathbf{d}) \approx \mathbf{0}, \tag{2}$$

in which $\mathbf{W}$ is a weight to exclude equations with missing data, $\mathbf{D}$ is convolution with the data, $\mathbf{K}$ constrains the first filter coefficient to 1, $\mathbf{a}$ is the unknown filter, and $\mathbf{d}$ is a copy of the data.

In order to set a benchmark for how effective any interpolation of the sparse tracks is, a PEF is estimated using fitting goal (2) on the output of fitting goals (1) when using the dense tracks of the Madagascar dataset and a Laplacian regularization operator. The PEF is then used to interpolate the sparse tracks in the same area, using the same fitting goals (1), this time with the regularization being convolution with a PEF. This result provides an upper bound to what an ideal interpolation of the data would be if we already knew the answer. Any new result should be much better than the Laplacian regularization shown in Figure 2b.

PEFS IN THE DATA SPACE

Once we attempt to interpolate this dataset using only the sparse tracks, the above method no longer works, as no region would have enough contiguous data on which to estimate a PEF.

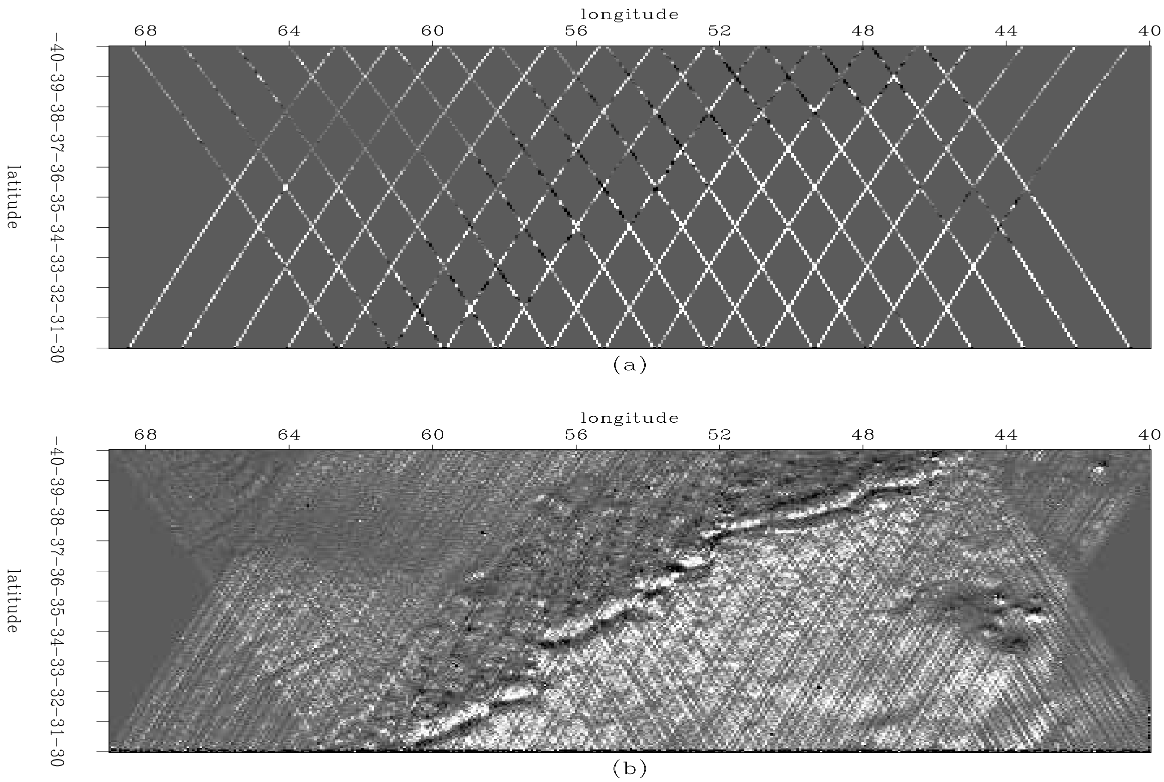

Figure 1: (a) sparse tracks. (b) dense tracks. bill1-data [ER,M]

In this case we will pay more attention to how our data are spatially distributed. The known data points in the model space are distributed along curved crossing tracks, making it very difficult to estimate a PEF in this space, as shown in Figure 1. However, in the data space of the fitting goals (1) of the previous section, the data are sampled in a regular space: a series of regularly-sampled tracks, as shown in Figures 3 and 4.

Since these data are collected in two series of one-dimensional tracks, it would be easiest to estimate a pair of one-dimensional PEFs on these two sets of tracks, as shown in the top halfs of Figures 3 and 4.

We now have two PEFs which have been estimated in a data space, but the model which we wish to regularize with these PEFs is in a different space. This requires the introduction of two additional linear interpolation operators $\mathbf{L_1}$ and $\mathbf{L_2}$, which pull bins from the model space into the ascending and descending track data spaces, respectively. The mappings used for these operators are shown in Figure 5. Now that we have both two prediction-error filters for regularizations operators as well as linear interpolation operators that pull model points into the data space, we can put everthing together in the following fitting goals,

$$
\begin{aligned}
\mathbf{W}\frac{d}{dt}[\mathbf{Lm}-\mathbf{d}] &\approx \mathbf{0} \\
\epsilon \mathbf{A_1 L_1 m} &\approx \mathbf{0} \\
\epsilon \mathbf{A_2 L_2 m} &\approx \mathbf{0},
\end{aligned}
\tag{3}
$$

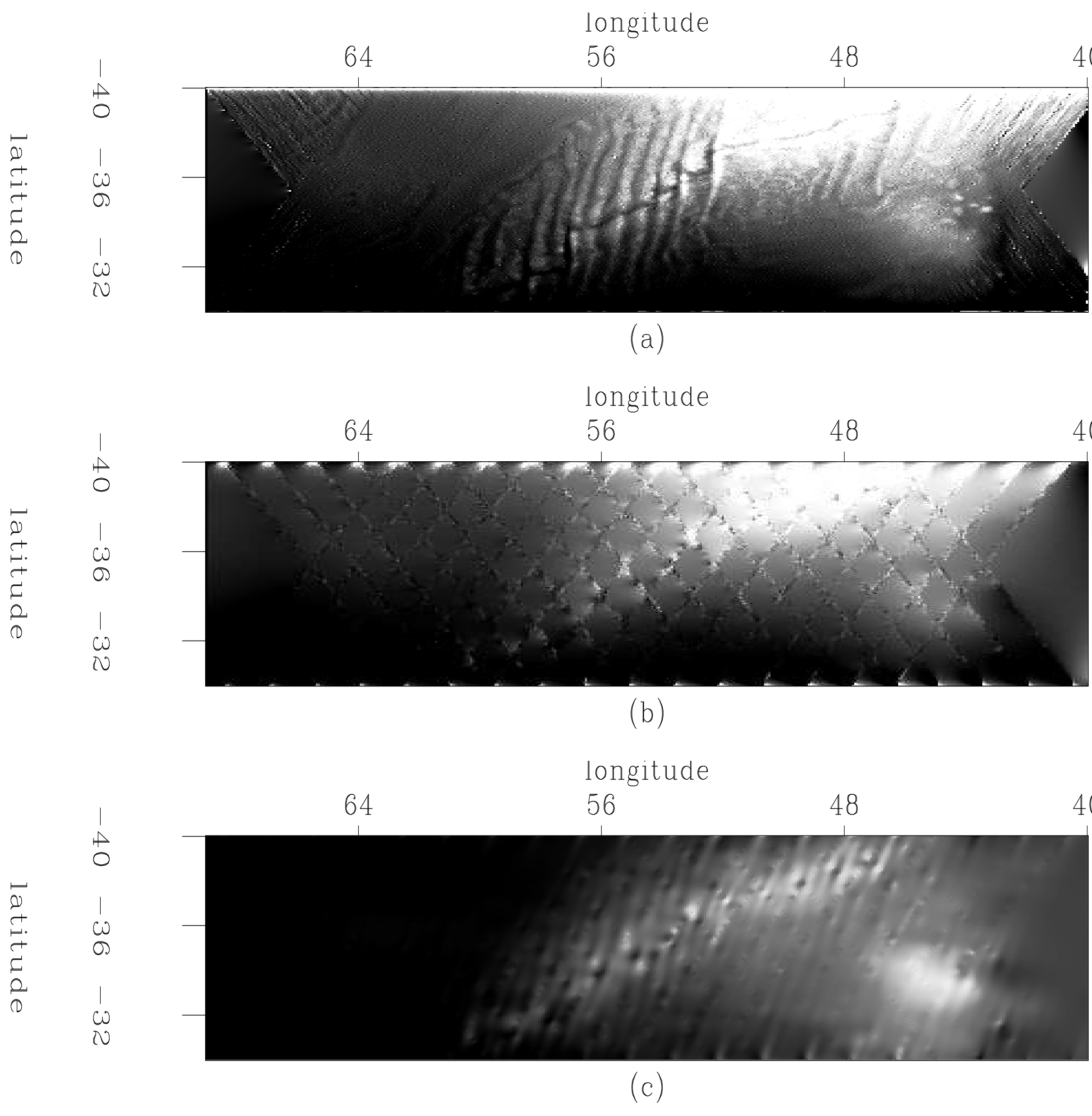

Figure 2: (a) dense data regularized with a Laplacian. (b) sparse data regularized with a Laplacian. (c) sparse data regularized with a PEF trained on the top. bill1-best [ER,M]

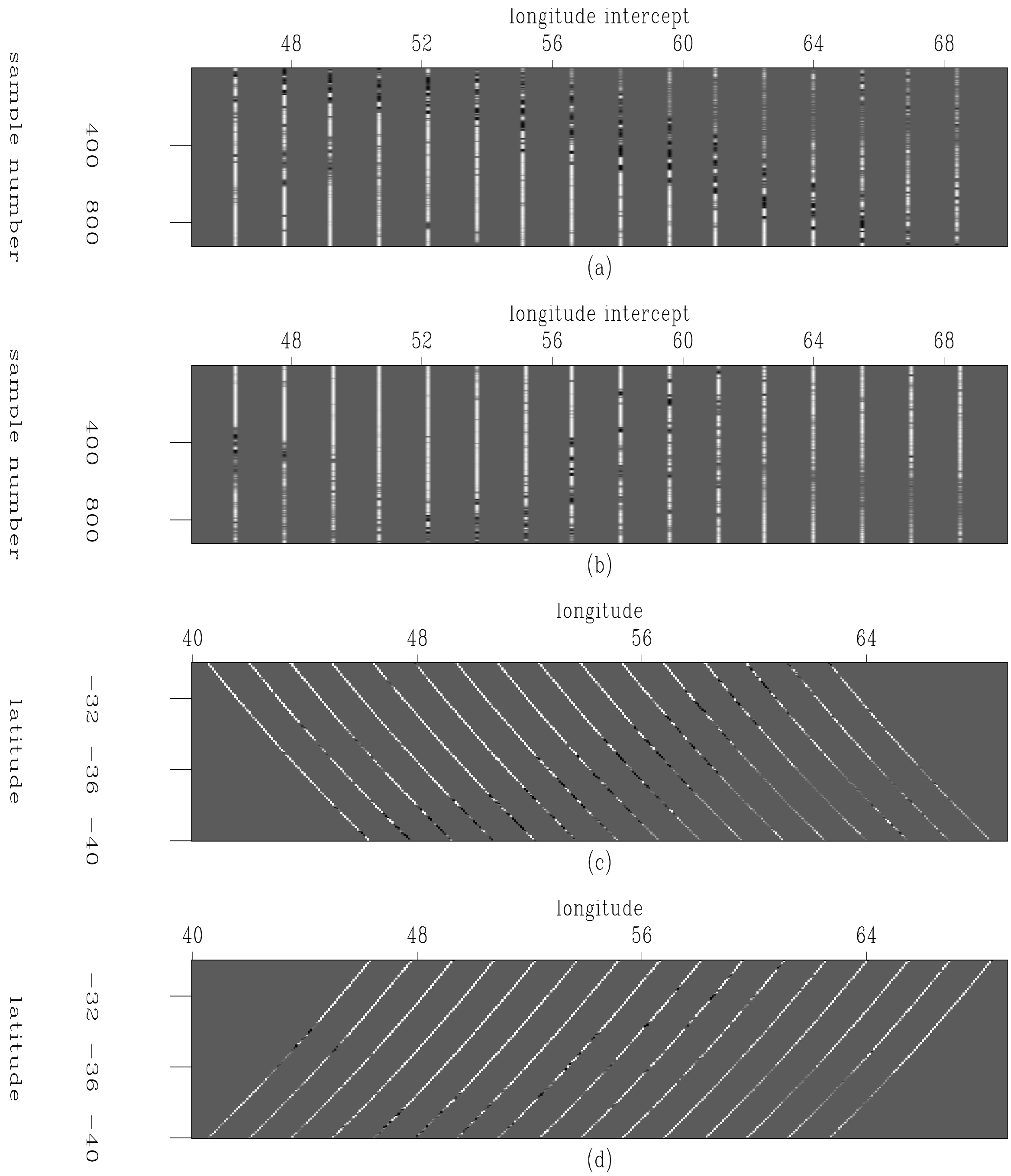

Figure 3: Sparse tracks, (a) ascending and (b) descending in model space, and (c) ascending and (d) descending tracks in data space. bill1-sptracks [ER,M]

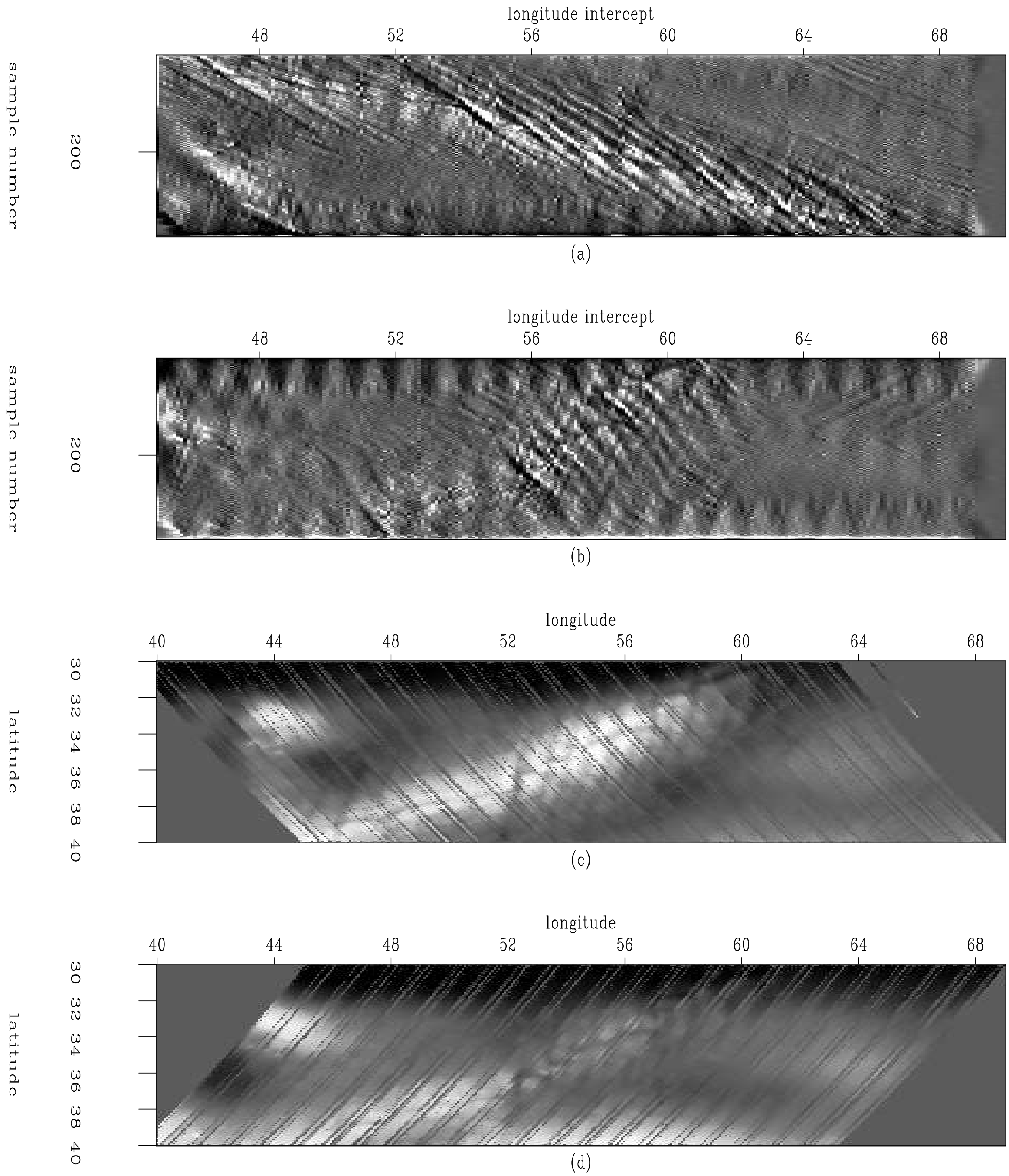

Figure 4: Dense tracks, (a) ascending and (b) descending in model space, and (c) ascending and (d) descending tracks in data space. bill1-detracks [ER,M]

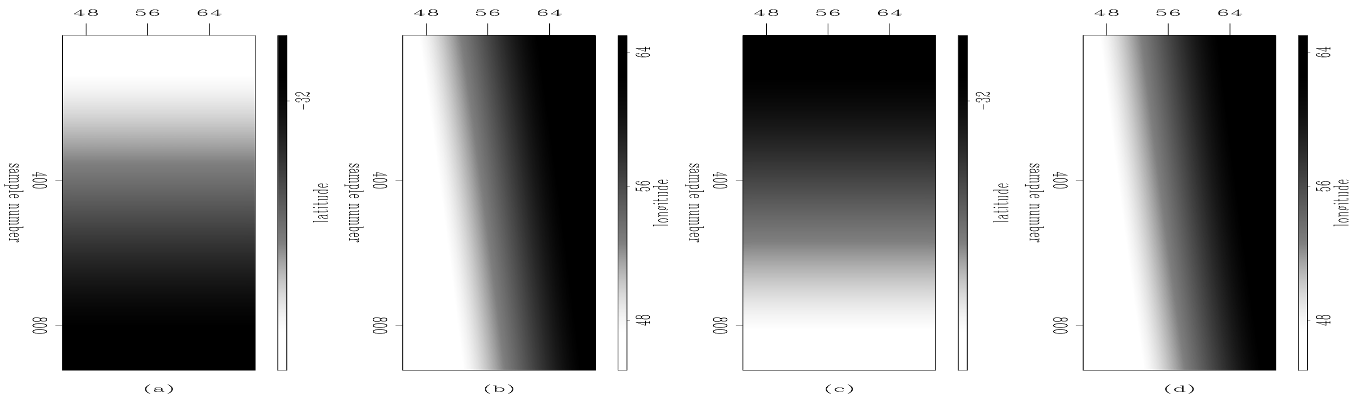

Figure 5: Mappings from data space to model space, all shown in data space. From the top down: ascending tracks latitude, ascending tracks longitude, descending tracks latitude, descending tracks longitude. bill1-maps [ER,M]

where $\mathbf{L}$ pulls model points ($\mathbf{m}$) to where we have data ($\mathbf{d}$), $\mathbf{A_1}$ and $\mathbf{A_2}$ are 1D PEFs that are estimated on the ascending and descending tracks in the data space, respectively, and $\mathbf{L_1}$ and $\mathbf{L_2}$ are linear interpolation operators that pull model points into the ascending and descending track data spaces, respectively. ϵ is a tradeoff parameter between the data fitting and model styling goals.

The 1D PEFs can also be replaced by 2D PEFs that are estimated by scaling the filter so that it covers multiple sparse tracks. If this approach is taken, the interpolation can occur in the data space where the PEFs are estimated (using a single PEF for each of the two track spaces), or in the model space (using both PEFs simultaneously). The more straightforward data-space interpolation is shown in Figure 6.

Figures 6a and b are simply the interpolation of the data space with 2D PEFs estimated on the sparse tracks. Figures 6c and d are those interpolated results mapped back to the model space by using fitting goals (1), where the input data are now the interpolated sets of tracks in the first two panels. Since fitting goals (1) were applied when generating the new tracks, the track derivative is no longer necessary.

The results are mixed, as Figure 6c shows that the trend of the ridge was correctly identified by the PEF estimated on the ascending tracks. The PEF estimated on the descending tracks did not fare so well, as the direction of the ridge in the interpolated tracks of Figure 6d does not match the densely sampled tracks in Figure 4d. This is because the descending tracks are oblique to the structure, so the structure is aliased beyond the point where a spaced PEF can interpolate accurately. In either case, the result is better than that obtained with a Laplacian, and in the case of the ascending tracks is not that far from the PEF estiamted on a fully-sampled model space shown in Figure 2c.

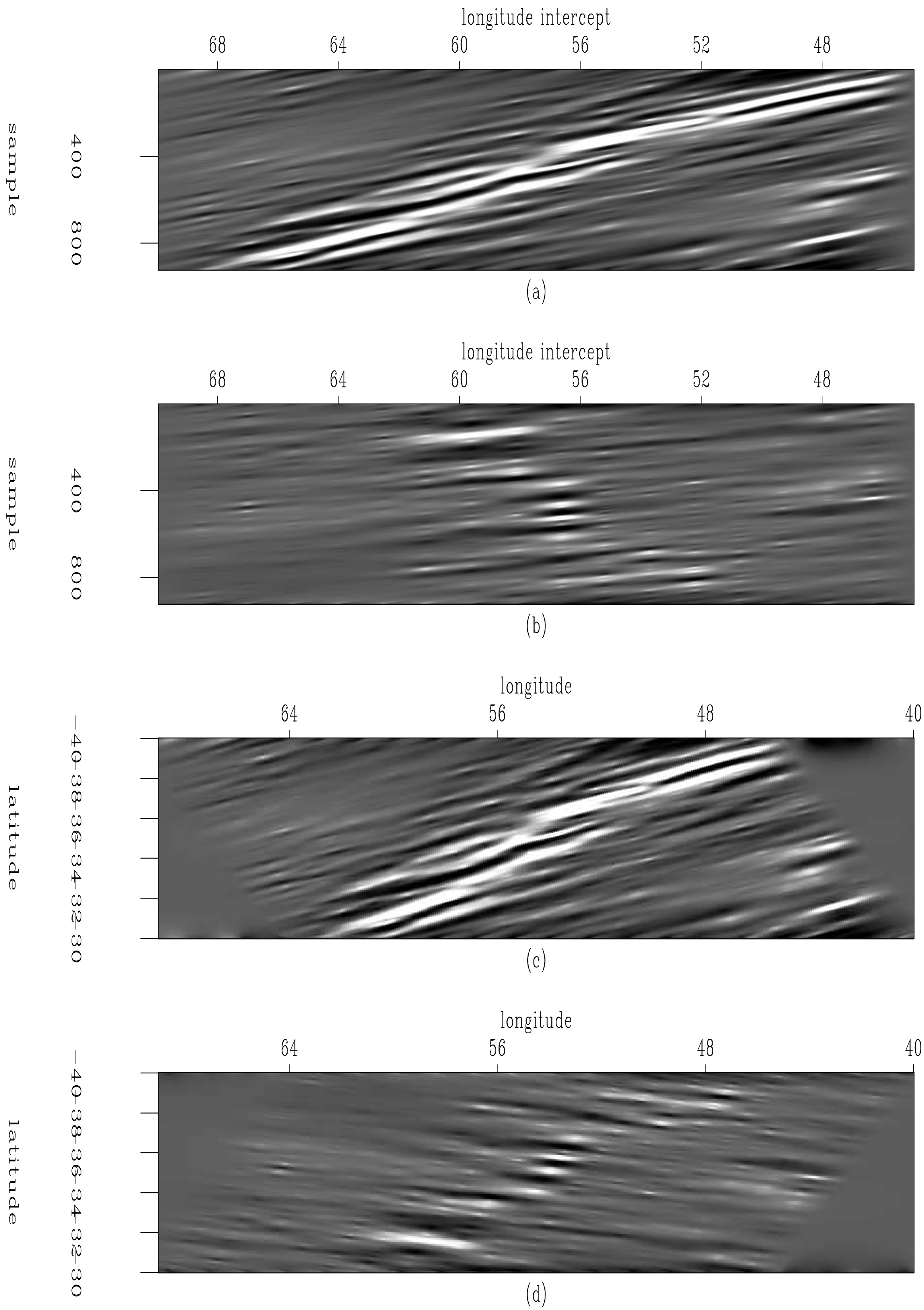

Figure 6: Interpolation of the two different track spaces (a) and (b). (c) and (d) contain the same results mapped into model space. The correct strike of the ridge is identified by the ascending tracks but not the descending tracks. bill1-dataspinterp [ER,M]

CONCLUSIONS AND FUTURE WORK

By estimating a pair of PEFs in the data space, the problem of irregular data acquistion has been avoided. The actual interpolation can be performed in either the data space (track coordinates) or the model space (latitude and longitude). The results turned out to be much better for the ascending tracks than for the descending tracks.

Crossing tracks are also present in land seismic data, where due to reciprocity the negative offsets in the split-spread land experiment cross. In 2D this happens in cmp - absolute offset space as shown in Figure 7, and in 3D this also happens, but in cmp_x - offset_x and cmp_y - offset_y spaces.

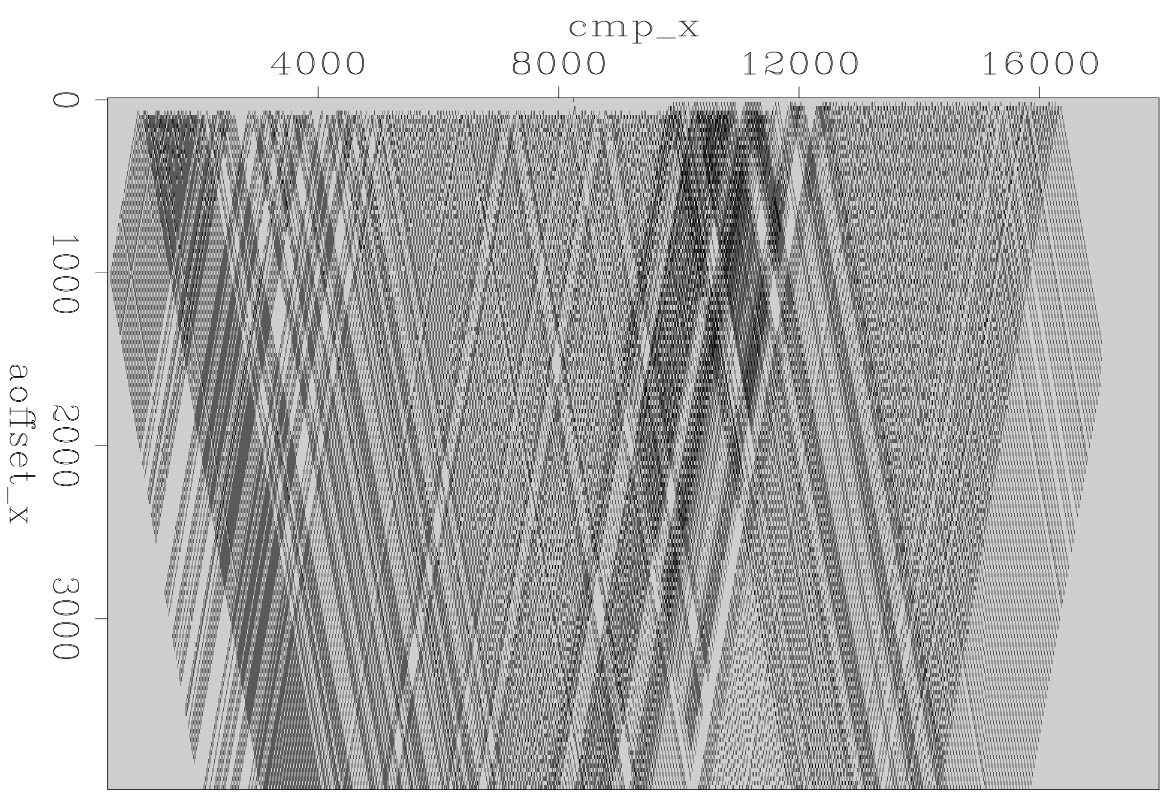

Figure 7: Fold map of an irregularly-sampled 2D land survey in cmp and absolute offset. Similar patterns of crossing tracks are also present in this survey due to reciprocity. bill1-hulia [ER]

REFERENCES

Crawley, S., 2000, Seismic trace interpolation with nonstationary prediction-error filters: Ph.D. thesis, Stanford University.

Curry, W., 2004, Regularizing Madagascar: PEFs from the data space?: SEP–**115**, 347–356.

Ecker, C., and Berlioux, A., 1995, Flying over the ocean southeast of Madagascar: SEP–**84**, 295–306.

Lomask, J., 1998, Madagascar revisited: A missing data problem: SEP–**97**, 207–216.

Lomask, J., 2002, Madagascar satellite data: An inversion test case: SEP–**111**, 337–349.

Lomask, J., 2004, Estimating a 2D stationary PEF on sparse data: SEP–**117**, 129–140.