Stanford Exploration Project, Report 117, October 23, 2004, pages 43–50

Regularized inversion for imaging: Effect on the data space

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ABSTRACT

Imaging in areas of complex subsurfaces is difficult due to poor illumination. This poor illumination is partially caused by seismic energy being directed outside of the survey bounds. Imaging in areas with poor illumination can be improved by using Regularized Inversion with model Preconditioning (RIP). RIP helps compensate for poor illumination by regularizing amplitudes in the image. By compensating for the lost energy, RIP in essence expands the data space.

INTRODUCTION

Subsurface imaging in complex areas, particularly around salt, is plagued by poor illumination. This poor illumination is caused by seismic energy being lost due to such processes as evanescence, mode conversion, or being directed outside of the recording geometry. In this paper, I am concerned with the seismic energy that is directed outside the bounds of the seismic survey by the complex structures. This energy can be thought of as lost data.

One method for compensating for poor illumination is Regularized Inversion with model Preconditioning (RIP). RIP uses a migration operator and a regularization operator in a least-squares inversion. RIP regularizes the image of the subsurface in a way that is consistent with the recorded data. Since it is trying to compensate for the illumination problems, it is filling in parts of the image that correspond to the energy that left the surface bounds. It is as if RIP is recovering the lost data.

In this paper, I will first explain the theory for regularized inversion with model preconditioning. I will discuss how energy that leaves the survey area affects the inversion process. I will show that by expanding the data space and introducing a weighting operator that accounts for the actual recording geometry, we can account for much of the energy that escapes the survey bounds.

BASIC THEORY

My inversion scheme is based on the downward continuation migration explained by Prucha et al. (1999a). To summarize, this migration is carried out by downward continuing the wavefield

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in frequency space, slant stacking at each depth, and extracting the image at zero time. The result is an image in depth (z), common reflection point (CRP), and offset ray parameter (p_h) space. Offset ray parameter is related to the reflection angle (θ) and the dip angle of the reflector (ϕ) in 2-D as:

$$\frac{\partial t}{\partial h} = p_h = \frac{2 \sin\theta \cos\phi}{V(z, \mathrm{CRP})}. \tag{1}$$

In complex areas, the image produced by downward continuation migration will suffer from poor illumination. To compensate for this, I use the migration as an operator in a least-squares inversion. The inversion procedure used in this paper can be expressed as fitting goals as follows:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{Lm} - \mathbf{d} \\ \mathbf{0} &\approx \epsilon \mathbf{Am}. \end{aligned} \tag{2}$$

The first equation is the "data fitting goal," meaning that it is responsible for making a model that is consistent with the data. The second equation is the "model styling goal," meaning that it allows us to impose some idea of what the model should look like using the regularization operator **A**. The model styling goal also helps to prevent a divergent result.

In the data fitting goal, **d** is the input data and **m** is the image obtained through inversion. **L** is a linear operator, in this case it is the adjoint of the angle-domain wave-equation migration scheme summarized above and explained thoroughly by Prucha et al. (1999b). In the model styling goal, **A** is a regularization operator and ϵ controls the strength of the regularization.

Unfortunately, the inversion process described by fitting goals (2) can take many iterations to produce a satisfactory result. I can reduce the necessary number of iterations by making the problem a preconditioned one. I use the preconditioning transformation $\mathbf{m} = \mathbf{A}^{-1}\mathbf{p}$ (Fomel et al., 1997; Fomel and Claerbout, 2003) to give us these fitting goals:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{LA}^{-1}\mathbf{p} - \mathbf{d} \\ \mathbf{0} &\approx \epsilon \mathbf{p}. \end{aligned} \tag{3}$$

$\mathbf{A}^{-1}$ is obtained by mapping the multi-dimensional regularization operator **A** to helical space and applying polynomial division (Claerbout, 1998). I call this minimization scheme Regularized Inversion with model Preconditioning (RIP).

The question now is what the regularization operator **A** is. In this paper, I will use two different regularization schemes. The first, which I call geophysical regularization, acts horizontally along the offset ray parameter axis. Rather than using the derivative operator used by Kuehl and Sacchi (2001) or the steering filter used by Prucha et al. (2000), I have created a symmetrical filter by cascading two steering filters that are mirror images of each other. The other regularization scheme that will be demonstrated in this paper is called geological regularization. This operator acts along user-specified dips in the CRP-depth plane. It is a steering filter constructed from the dips of picked reflectors.

LOST SEISMIC ENERGY AND THE FITTING GOALS

In a perfect world, we would have perfect data and the data fitting goal would allow us to construct a perfect model of the subsurface without needing any regularization. However, we never have perfect data. It is always corrupted by noise and the acquisition geometry is always a compromise between cost and what would provide sufficient data to image the subsurface. The latter is a particularly bad problem in areas of complex subsurface. Subsurface volumes with high velocity contrasts, such as salt bodies or even the simple low velocity lens model seen in Figure 1, are difficult to image at least partially due to seismic energy being directed outside of the limited survey area. The rays in Figure 1 show how seismic energy that is reflected at the flat reflector is affected by the low velocity lens. The maximum offset used in this paper is 4000 m, so the energy that is redirected by the lens will not be recorded. If we look at the synthetic data generated for this model (left panel of Figure 2), we can see the bounds of the survey cutting off the recorded events. The energy that leaves the survey bounds causes the migration result (right panel of Figure 2) to have holes in the common image gathers (CIGs).

Figure 2 indicates that we need to reconsider our fitting goals. Our model is incomplete because we are missing data. If we include the data that leaves the survey ($\mathbf{d_L}$), our fitting goals become:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{L}\mathbf{A}^{-1}\mathbf{p} - \begin{bmatrix} \mathbf{d} \\ \mathbf{d_L} \end{bmatrix} \\ \mathbf{0} &\approx \epsilon\mathbf{p}. \end{aligned} \tag{4}$$

Initially, $\mathbf{d_L}$ is set to zero, since by definition we haven't recorded it. However, as we perform conjugate-gradient iterations of the fitting goals, regularizing the model with each iteration, we are filling in the parts of the model that correspond to $\mathbf{d_L}$ and thereby reconstructing some version of $\mathbf{d_L}$.

In practice, RIP includes $\mathbf{d_L}$ by padding the original data space with zero traces. However, since we are doing conjugate-gradient iterations, it is important to mask out the padded traces during each iteration. Otherwise the "recovered" $\mathbf{d_L}$ would exist in the data residual space and confuse the minimization. Defining

$$\mathbf{d_{pad}} = \begin{bmatrix} \mathbf{d} \\ \mathbf{d_L} \end{bmatrix} \tag{5}$$

and including the weighting operator $\mathbf{W}$, the fitting goals become:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{W}\left(\mathbf{L}\mathbf{A}^{-1}\mathbf{p} - \mathbf{d_{pad}}\right) \\ \mathbf{0} &\approx \epsilon\mathbf{p}. \end{aligned} \tag{6}$$

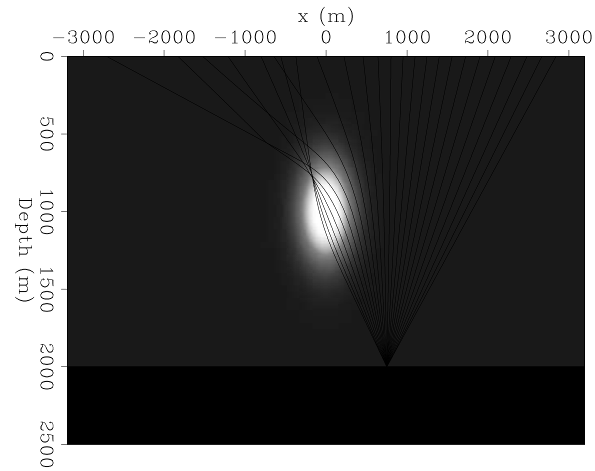

Figure 1: Velocity model with a flat reflector under a low velocity lens. The maximum offset used for this experiment is 4000 m, so some of the rays representing seismic energy are directed outside of the survey bounds. Synthetic provided by Bill Symes and The Rice Inversion Project. marie1-symesray [ER]

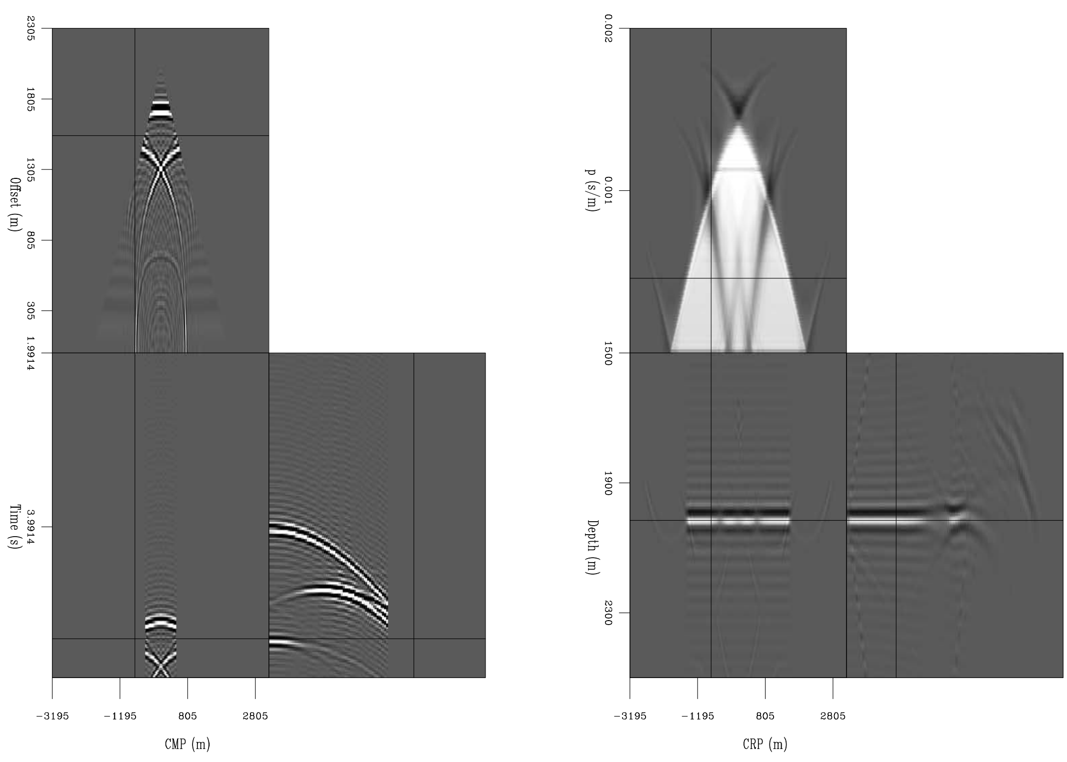

Figure 2: Left: The recorded data displayed as a flattened cube. The top panel is a time slice, the lower left is a common offset section and the lower right is a CMP gather. The triangular shape seen in the time slice is caused by the limited survey geometry. Right: The migration result. The top panel is a depth slice, lower left is a common offset ray parameter section, and lower right is a CIG. Note the holes caused by illumination problems in the reflector. marie1-dat.mig [CR]

EFFECT ON THE DATA SPACE

To see how RIP affects the data space, I first ran 3 iterations of RIP with geophysical regularization. The resulting image (right panel of Figure 3) shows that the shadow zones are beginning to fill in. The data space corresponding to this model (left panel of Figure 3) shows that we now have energy extending outside of the original data (left panel of Figure 2). This "recovered" data has lower amplitude than the recorded data, but theory suggests that with more iterations it would become similar in amplitude.

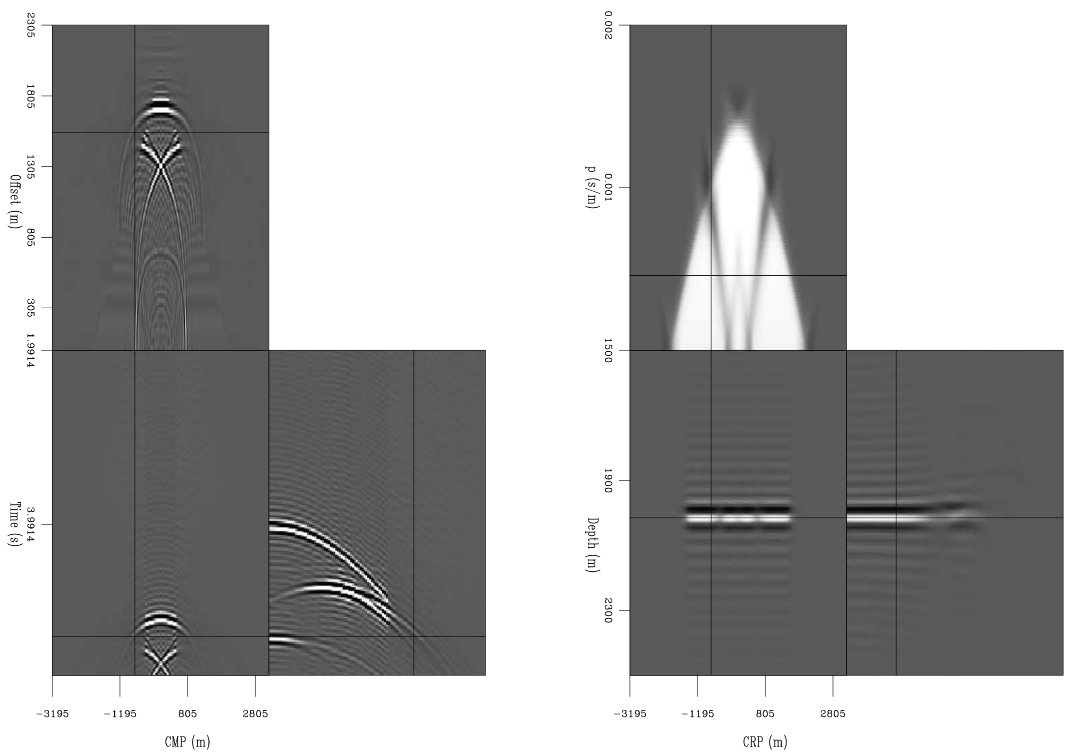

Figure 3: Left: Data space after 3 iterations of geophysical RIP. Right: RIP result after 3 iterations. marie1-datex.3it1eps [CR]

I also ran 3 iterations of RIP with geological regularization. This result (right panel of Figure 4) shows that the shadow zones are almost completely gone. The corresponding data space (left panel of Figure 4) has extended events with almost the same amplitude as the original data. The similarity in amplitude after only 3 iterations is due to the use of regularization along p_h and in the CRP-depth plane. In cases where the reflectors used to create the steering filter are easily interpreted, as for this simple example, geological regularization will provide a better image faster than geophysical regularization.

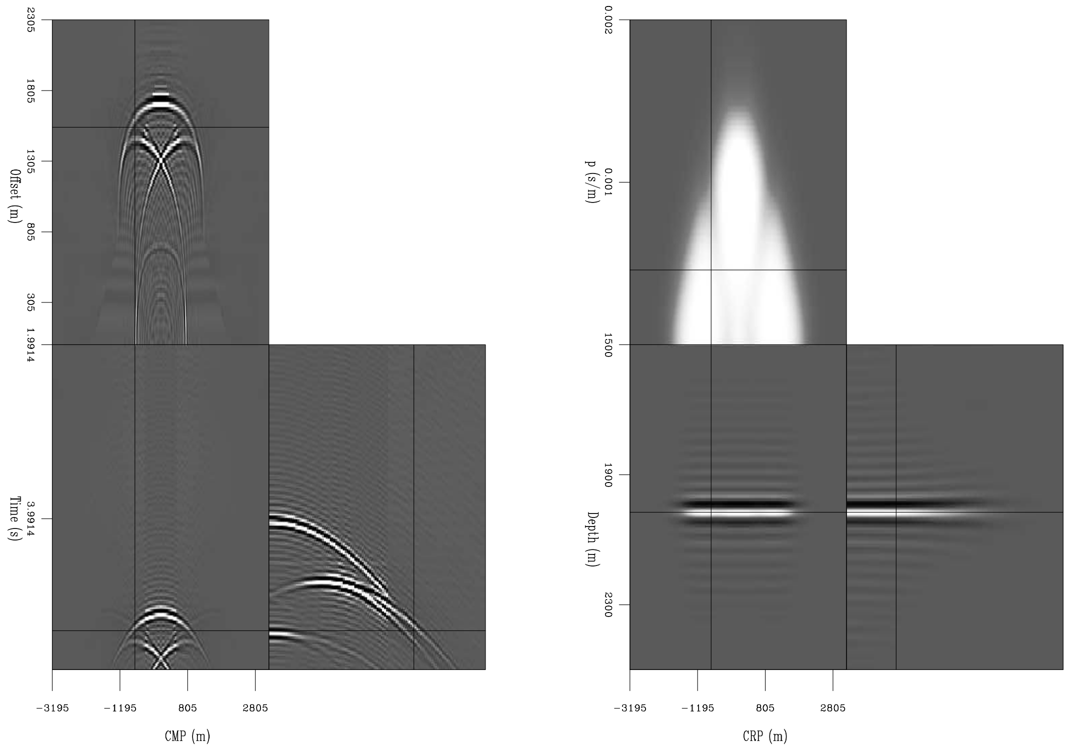

Figure 4: Left: Data space after 3 iterations of geological RIP. Right: RIP result after 3 iterations. marie1-datex.2d.3it1eps [CR]

CONCLUSIONS

Regularized Inversion with model Preconditioning (RIP) helps to compensate for poor illumination. RIP produces images with more consistent amplitudes in shadow zones. This partially corresponds to energy that has left the survey area. I have demonstrated that RIP, both with geophysical and geological regularization, essentially expands the data space to recover that lost energy.

ACKNOWLEDGMENTS

I would like to thank Bill Symes and The Rice Inversion Project for the synthetic dataset used in my experiments.

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