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Short Note

An educated guess on the Vp/Vs ratio

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INTRODUCTION

Data processing of converted waves generally yields estimated values for both P velocity and S velocity in the area of study. These values are usually seen in the form of two parameters: 1) the multiplication of both velocity fields, and 2) the ratio of both velocity fields. Traditionally the ratio of the P and S velocities, which is known as the γ value, is the result of an extensive combined analysis on the PS data and the single P-mode data. Knowledge of γ is important not only for seismic processing but also for rock property estimation. Traditionally, γ is estimated through a combined processing of the PS data and the PP data, as described by Thomsen (1998) and Audebert et al. (1999).

In this note, I present an analytical procedure to estimate an initial value of γ that depends only on the most basic processing scheme, the NMO stacking process. Several authors have discussed the stacking process for converted waves (Tessmer and Behle, 1988; Castle, 1988; Iverson et al., 1989; Huub Den Rooijen, 1991). Tessmer and Behle (1988) apply conventional NMO to converted waves where the RMS stacking velocity is designated as the *converted-wave* velocity. This NMO procedure uses a hyperbolic approximation of the moveout equation; so, there is not a satisfactory correction of the moveout.

I introduce a non-hyperbolic moveout equation that characterizes converted waves. This moveout equation consists of three main terms. The third term depends on the γ function giving us an equation to estimate an approximately constant value of γ, directly from the PS data alone.

THEORY: NON-HYPERBOLIC MOVEOUT

The main characteristic of converted-wave data is their non-hyperbolic moveout. However, for certain offset/depth ratios, it is possible to approximate the non-hyperbolic moveout as a hyperbola (Tessmer and Behle, 1988).

Tessmer and Behle (1988) extend the work of Taner and Koehler (1969) for converted waves. They apply a second-order approximation to the moveout equation to converted-wave

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data. In such cases the stacking velocity corresponds to the product of both P and S velocities known as converted-wave velocity.

Castle (1988) presents the third-order-approximation coefficient terms for the converted-wave moveout equation. In this note, I simplify this term and present it as a function of γ alone.

Equation (1) is the expanded traveltime function of reflected PP or SS data presented by Taner and Koehler (1969):

$$t^2 = c_1 + c_2 x^2 + c_3 x^4, \tag{1}$$

where x represents full offset, $c_1 = b_1^2$, $c_2 = \frac{b_1}{b_2}$, and $c_3 = \frac{b_2^2 - b_1 b_3}{4 b_2^4}$, with

$$b_m = \sum_{k=1}^{n} z_k (\alpha_k^{2m-3} + \beta_k^{2m-3}), \tag{2}$$

where k indicates the stratigraphic layers present in the model. Here and hereafter, α_k and β_k respectively denote the P velocity and the S velocity for the k^{th}-layer. Tessmer and Behle (1988) show that

$$c_1 = \left(\sum_{k=1}^{n} z_k \left(\frac{1}{\alpha_k} + \frac{1}{\beta_k} \right) \right)^2 = t_0^2, \tag{3}$$

and

$$c_2 = \frac{\sum_{k=1}^{n} z_k \left(\frac{1}{\alpha_k} + \frac{1}{\beta_k} \right)}{\sum_{k=1}^{n} z_k (\alpha_k + \beta_k)} = \frac{1}{v_{rms}^2}, \tag{4}$$

where $v_{rms}^2 = \alpha_{rms} \cdot \beta_{rms}$, this is only true when γ is constant. The formal definition for c_3 is as follows (Castle, 1988):

$$c_3 = \frac{\left(\sum_{k=1}^{n} z_k (\alpha_k + \beta_k) \right)^2 - \sum_{k=1}^{n} z_k \left(\frac{1}{\alpha_k} + \frac{1}{\beta_k} \right) \cdot \sum_{k=1}^{n} z_k (\alpha_k^3 + \beta_k^3)}{4 \left(\sum_{k=1}^{n} z_k (\alpha_k + \beta_k) \right)^4}. \tag{5}$$

For one layer, equation (5) simplifies to

$$c_3 = \frac{z^2 \left[(\alpha + \beta)^2 - (\frac{1}{\alpha} + \frac{1}{\beta})(\alpha^3 + \beta^3) \right]}{4 z^4 (\alpha + \beta)^4}, \tag{6}$$

which reduces to

$$c_3 = \frac{2\alpha\beta - \frac{\alpha^3}{\beta} - \frac{\beta^3}{\alpha}}{4 z^2 (\alpha + \beta)^4}. \tag{7}$$

Now, the simple trick I use to make an educated guess for γ with PS data alone is to consider [from the results of c_1 and c_2, equations (3) and (4)] that $\alpha_{rms}^2 = v_{rms}^2 \gamma$ and $\beta_{rms}^2 = v_{rms}^2 \gamma^{-1}$, remember that γ is approximately constant in all layers. With these assumptions, equation (5) simplifies to

$$c_3 = \frac{2-\left(\gamma^2+\gamma^{-2}\right)}{4t_0^2 v_{rms}^4 \left(\gamma^{1/2}+\gamma^{-1/2}\right)^4}. \tag{8}$$

Introducing the final results for c_1, c_2 and c_3 into equation (1), I obtain an equation to perform non-hyperbolic moveout for PS data that is dependent on only two parameters: 1) the multiplication of the P and S velocities, or the converted wave rms velocity (v_{rms}), and 2) the Vp/Vs ratio (γ). It is also important to note that this equation is valid for a constant Poisson's ratio in all layers. With these simplifications and equations, it is possible to obtain an approximate value of γ using PS data alone.

$$t^2 = t_0^2 + \frac{x^2}{v_{rms}^2} + \frac{x^4}{t_0^2 v_{rms}^4}\left[\frac{2-\left(\gamma^2+\gamma^{-2}\right)}{4\left(\gamma^{1/2}+\gamma^{-1/2}\right)^4}\right] \tag{9}$$

Equation (9) is the central result of this paper. It is possible to note that the moveout equation is more than a hyperbolic relation, since it involves a third term. Another important characteristic of equation (9) is that it depends only on two parameters: 1) the converted-waves *rms* velocity, and 2) the Vp/Vs ratio. This important characteristic will allow us to invert for a value of γ. It is also important to note that the sensitivity of equation (9) to γ probably is not too high, since the third term of the equation also depends on the offset-depth ratio.

It is important to note that for the specific case of $\alpha = \beta$ (this never happens in practice), i.e., no converted waves, the value of γ equals 1, and equation (9) reduces to the conventional normal moveout equation. This is also a result of the one layer assumption.

NUMERICAL EXAMPLES

Figure 1 presents a simple flat-layer model. Figures 2 and 3 show the result of modeling the two-way travel time with the conventional normal moveout equation, and with the non-hyperbolic moveout equation, using an initial velocity of 1100 m/s, a velocity gradient of 125 s^{-1} and a constant value of $\gamma = 2$.

Both results (Figures 2 and 3) present a hyperbolic moveout at near offset or small offset-to-depth ratio. This result resembles the well known theoretical presentation of Tessmer and Behle (1988). However, the non-hyperbolic moveout is dominant for large offsets and shallow depths, as can be observed in Figure 2.

I also apply the non-hyperbolic moveout equation to a PS CMP gather from the Alba dataset acquired on the Alba oil field in the North sea. The data set is a multicomponent 3-D Ocean Bottom Seismic experiment. Figure 4 shows the original CMP gather before (left), after (center) non-hyperbolic moveout, and after traditional hyperbolic moveout (right). I use

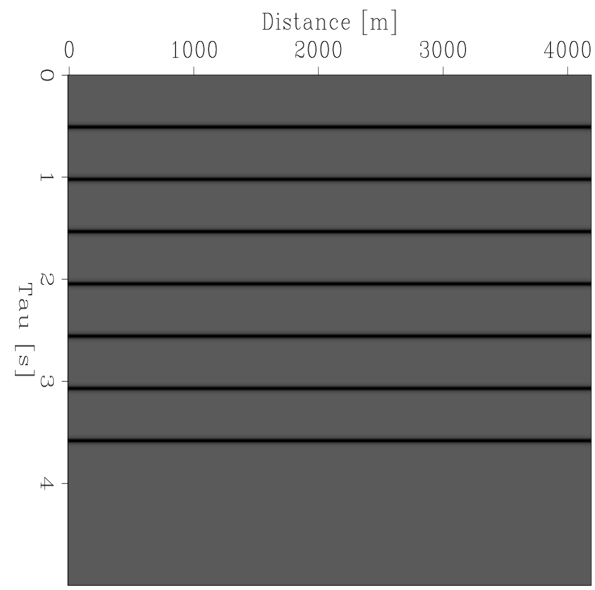

Figure 1: Reflectivity model. daniel2-mod5 [ER]

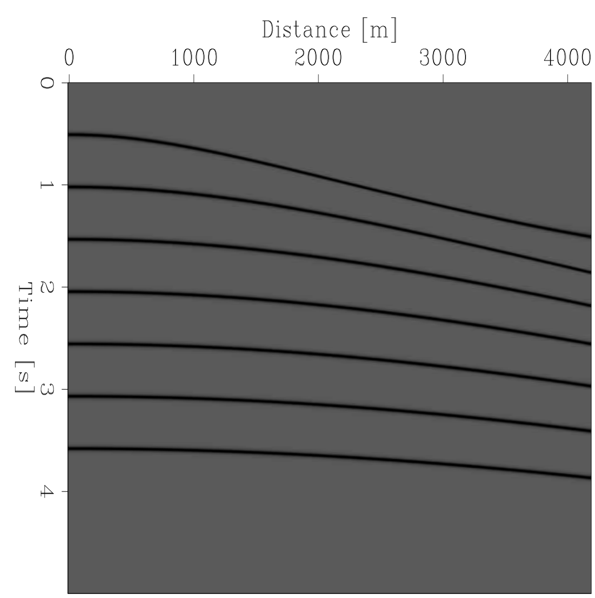

Figure 2: Modeled data, using the non-hyperbolic equation with a constant value of $\gamma = 2$. daniel2-mod5NMO [ER,M]

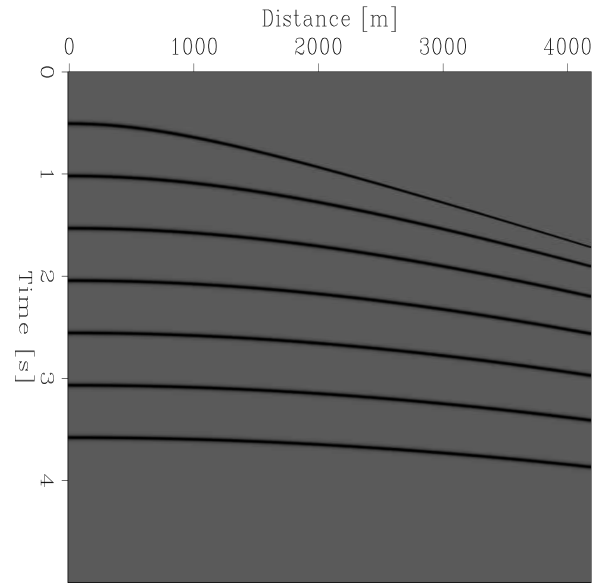

Figure 3: Modeled data, using the the traditional hyperbolic equation. daniel2-mod5NMO1 [ER,M]

an initial velocity value of 1100 m/s, and a velocity gradient of 125 s^{-1} and a constant $\gamma = 2.0$. Note that even though the events are not totally corrected, the non-hyperbolic correction gives a better result for shallow events at large offsets. These events are flatter after the non-hyperbolic moveout correction than with the hyperbolic moveout correction.

CONCLUSIONS AND FUTURE WORK

The non-hyperbolic equation introduced here is an approximation; therefore, I suggest it should be used only as a way to obtain an initial constant value for the Vp/Vs ratio. This ratio should be optimized later, using for example an iterative velocity-analysis technique with wave-equation migration velocity analysis.

A future goal is to produce a γ-scan technique, similar to a velocity scan, to obtain the best γ value. This final value will probably be a key element for more advanced velocity analysis techniques.

Also, with the definition of a direct and more appropriate formulation for the PS travel time with both P and S velocities, one tentative future step is to generate an inversion scheme to estimate both P and S velocities from a single PS section.

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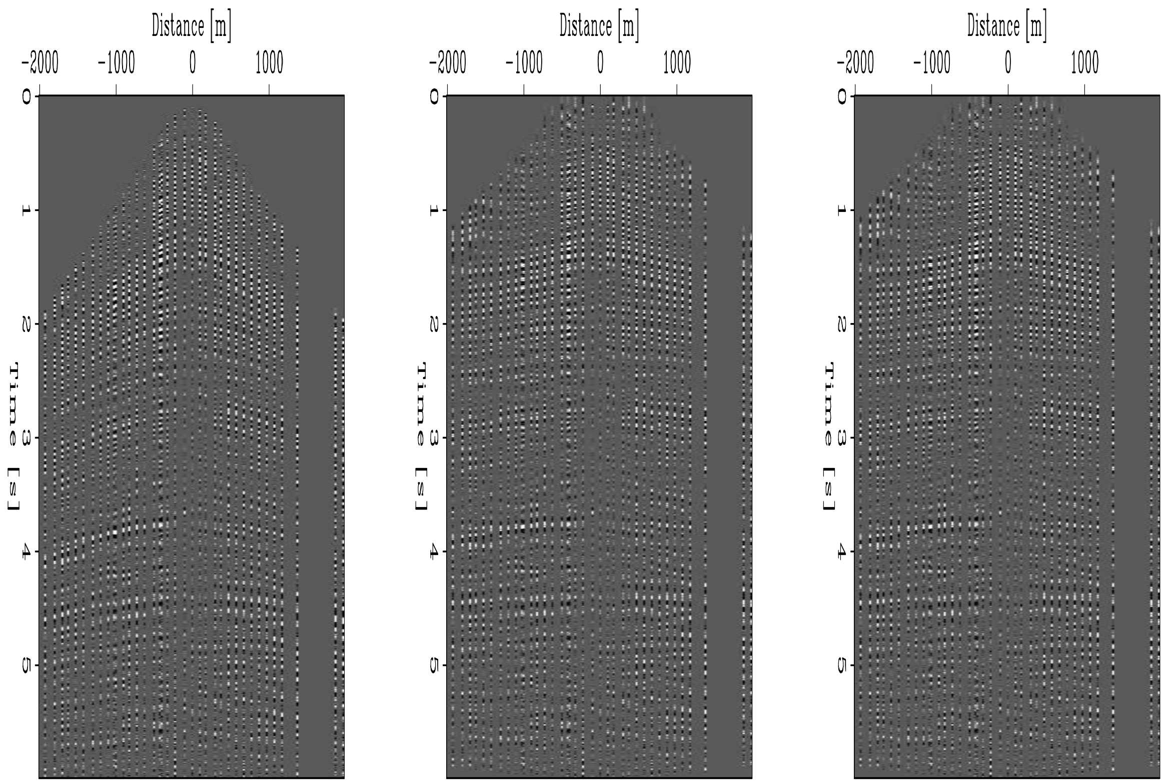

Figure 4: PS CMP gather from the Alba dataset, original gather (left), after non-hyperbolic moveout (center), and after traditional hyperbolic moveout (right). I performed non-hyperbolic moveout and traditional moveout both with an initial velocity of 1100 m/s, a velocity gradient of 125 s^{-1} with $\gamma = 2.0$ for the non-hyperbolic case. daniel2-cmp_comp [ER,M]

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