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Short Note

The effect of model covariance description on global seismology tomography problems

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INTRODUCTION

Global seismology tomography faces a different set of problems than those faced in typical oil exploration tomography projects. Energy travels at significantly wider range of angles in global seismology and the velocity-depth ambiguity of reflection based seismology is not present. On the other hand, global seismologists have orders of magnitude less data. In addition, this data usually has a much lower signal to noise ratio than in the typical oil industry project.

The result is that both types of tomography operators have a large null space. In global seismology, this problem is usually is addressed by limiting the number of model components, either through some type of global harmonic parameterization or simply larger grid cells. Another approach is to introduce an inverse model covariance operator as a regularization operator into the inversion scheme. This regularization operator can be a non-stationary operator that introduce a desired, or hypothesized, structure to the velocity model (Clapp, 2001).

In this paper we apply a series of non-stationary regularization operators, a steering filter (Clapp, 2001), to a global seismology tomography problem. We show how these filters can produce a more aesthetic pleasing image that still fits the data, and we hypothesize they can be used to help evaluate different model hypotheses.

DATA

The data is from an array laid out across the Colorado-Wyoming border. The array is composed of 30 seismometers laid out in line in a semi-straight line with approximate spacing of 2.6 km. The data consists of a series of arrival times from known station locations, at known dips and arrival directions (back azimuths). The left panel of Figure 1 shows an initial velocity model with the stations marked with an '*'. The right panel of Figure 1 shows the distribution, in terms of azimuth and angle, of the earthquakes used in this experiment.

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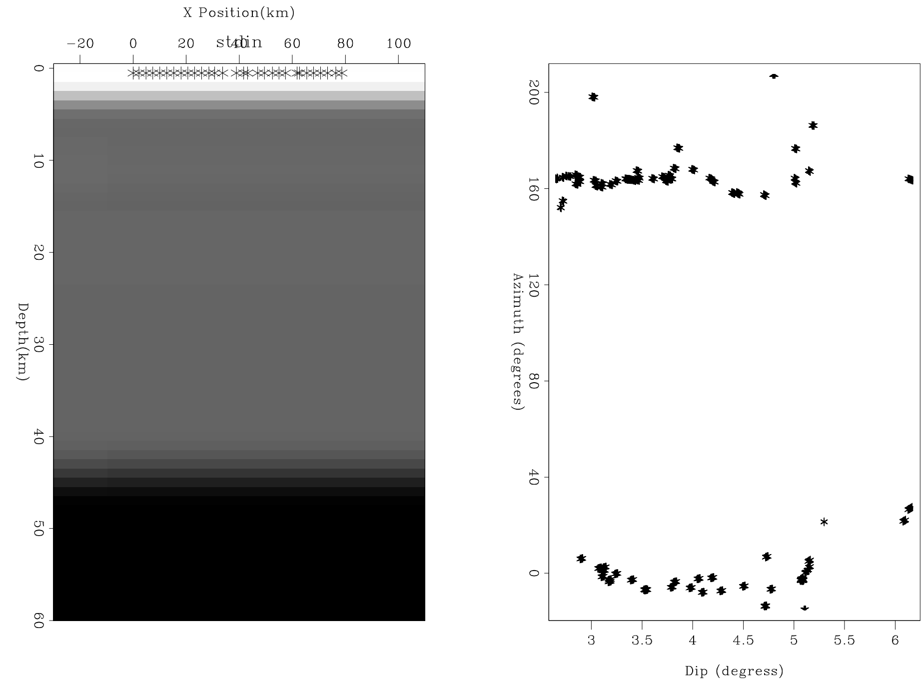

Figure 1: The left panel shows an initial velocity model with the recording stations marked with an '*'. The right panel shows the distribution, in terms of azimuth (compared to receiver line) and dip, of the earthquakes used in this experiment. The large change at 43 km is the Moho. bob4-initial [ER,M]

A linear trend is removed from the arrival times for each event (earthquake). The subtraction of the trend is meant to account for the varying source directions (earthquake position compared to the array). The trend removed arrived times generally form the data $\Delta\mathbf{t}$ for the global tomography problem. Figure 2 shows these arrivals with the trend removed. Note the general shapein the times.

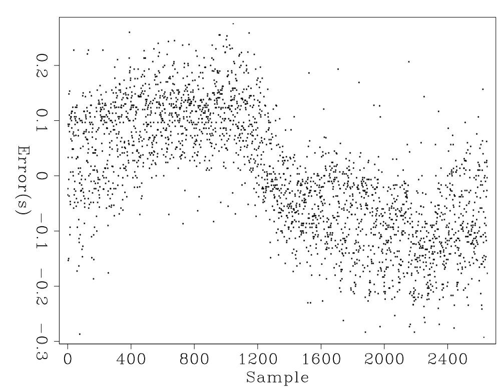

Figure 2: The arrival times with a linear trend removed. Note the pattern in the arrival times. bob4-dt [ER]

TOMOGRAPHY

We linearize the tomography problem around an initial slowness model $\mathbf{s_0}$. Rays are traced through the model based on the recorded arrival direction. The length of the ray segments through each model cell form the basis of the tomography operator $\mathbf{T_0}$. Figure 3 show the rays forming $\mathbf{T_0}$ overlying the initial velocity model. Note that we have decent angular coverage.

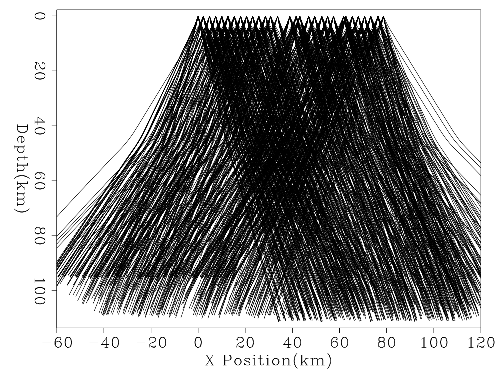

Figure 3: The rays forming $\mathbf{T_0}$ overlying the initial velocity model. bob4-rays [ER]

For this experiment we are limiting ourselves to a 2-D earth model. As shown in Figure 1, our selected set of earthquakes are approximately oriented along the receiver line. For this experiment we assume constant velocity out of plane. We do 2-D ray tracing and then correct all lengths by

$$l_{\text{new}} = \frac{l_{\text{old}}}{\| \cos(\phi) \|}, \tag{1}$$

where l_{old} is the old ray length, l_{new} is the updated ray length and ϕ is the azimuth direction of the earthquake.

Each arrival time also has a variance associated with it. The inverse of these variances form a noise covariance operator $\mathbf{W}_0$ for the inversion. We invert for the change in slowness $\Delta\mathbf{s}$ by minimizing the fitting goal,

$$\mathbf{0} \approx \mathbf{W}_0(\Delta\mathbf{t} - \mathbf{T_0}\Delta\mathbf{s}). \tag{2}$$

Regularization

Our velocity model is sampled in 1km in both depth and x position. Our total number of model points is approximately seven times our number of data points. As a result our model is significantly undeterdetermined. There are several ways to deal with the problem. One solution would be to decrease the number of grid points by sampling differently (coarser regular sampling or some type of irregular sampling). We can improve the situation by back propagating along fatter rays or we can add some type of regularization operator. Potentially the most interesting, and the one chosen for this paper is to add a regularization operator.

The typical choice for a regularization operator is an isotropic roughner. This will tend to fill undetermined portions of the model with isotropic blobs. In many cases this is unrealistic.

Generally velocity follows structure and our structure is laid down as a series of layers that are later deformed by tectonic processes. A better choice for our regularization operator is something that tends to create features that follow structure. Clapp (2001) showed that this can improve the velocity estimate for oil exploration targets. Our regularization operator becomes a *steering filter*, a non-stationary filter which tends to smooth along some predefined dip map. In this case we used three reflectors to build the dip map from: the surface, a basement reflector, and the Moho. We measure the dip along each reflector and then interpolate between them. Figure 4 shows the dip field overlain by the reflectors.

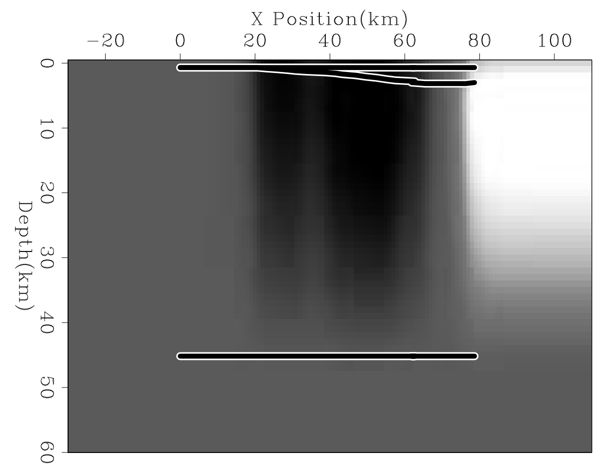

Figure 4: The dip field and reflectors used to construct the steering filter operator. The upward dip on the right edge is due to continuing the dip present at the edge of the reflectors. bob4-dips [ER]

We want to smooth the slowness model, not the change in slowness that we are inverting for. As a result our regularization fitting goal becomes

$$\mathbf{0} \approx \mathbf{A}(\mathbf{s_0} + \Delta\mathbf{s}), \tag{3}$$

where $\mathbf{s_0}$ is the initial slowness and $\mathbf{A}$ is our steering filter. This problem converges quite slowly. As a result we precondition the model using the inverse of our regularization operator (Claerbout, 1999). Our final set of fitting goals become

$$\begin{aligned} \Delta\mathbf{t} &\approx \mathbf{T_0}\mathbf{A}^{-1}\mathbf{p} \\ \mathbf{0} &\approx \epsilon\mathbf{p}, \end{aligned} \tag{4}$$

where $\mathbf{p}$ is the preconditioned variable and ϵ is a twiddle parameter controlling the amount of smoothing. Figure 5 shows the resulting change in velocity and the updated velocity model. Note how the basin structure that has now appears in the model.

RELINEARIZATION

As mentioned above tomography is actually a non-linear problem. We can potentially achieve a better result by relinearization around our updated slowness model. We can write our updated travel times in terms of non-linear iteration i,

$$\Delta\mathbf{t_i} = \Delta\mathbf{t} - \mathbf{T_i}(\mathbf{s_i} - \mathbf{s_0}). \tag{5}$$

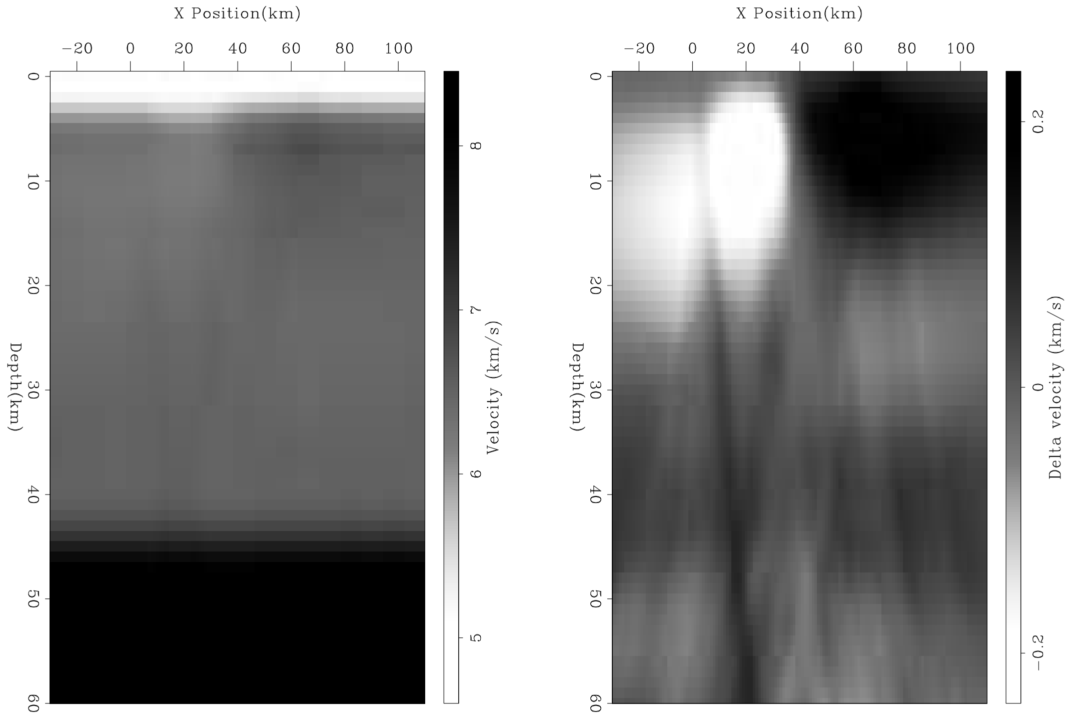

Figure 5: The left panel shows the updated velocity model. The right panel is the change in velocity resulting from 120 iterations of fitting goals (4). bob4-vel1 [ER]

We performed two more non-linear iterations of tomography. The right panel of Figure 6 shows our updated ray field. The ray field shows moderate changes from the initial field (left panel of Figure 6). After two additional non-linear iterations we get a moderately changed velocity model, shown in the left panel of Figure 6. The overall reduction in the RMS travel-time error is about 80%, with the majority coming on the first iteration.

FUTURE WORK

There are several avenues for future research. One obvious step is to solve for a 3-D velocity model. This would require significantly more events and stations than are available in this experiment. Second, the subtraction of the best fit plane wave from the arrival times is less than optimal. In areas of complex velocity this could introduce significant travel-time errors that would be difficult to correct.

The most interesting future research direction is to use the steering filters to test different geologic hypotheses. The difference in the reduction in the travel time errors and the location of large values in the regularization portion of the residual vector would both be good indicators of model suitability.

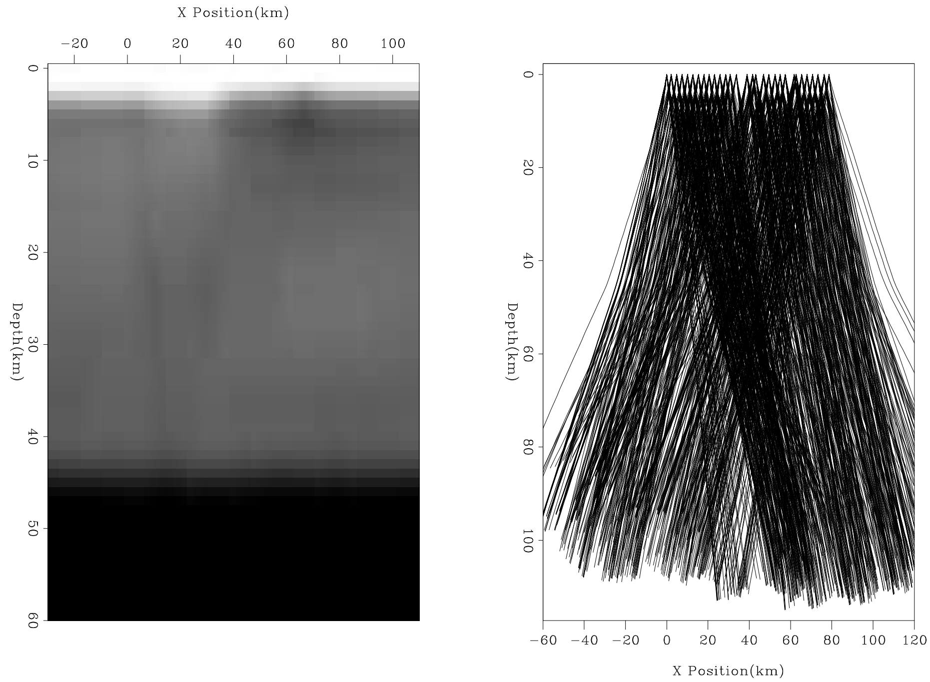

Figure 6: The final set of rays (right) and the final velocity model (left) after three non-linear iterations of tomography. Note how many of the feature seen in Figure 5 have become more defined. bob4-final [ER]

CONCLUSION

We apply a non-stationary filter, a *steering filter*, to a deep crustal tomography problem. We show that the updated velocity model shows more consistency with our geologic model than typically is found. We hypothesize that steering filters could be an effective tool in evaluating different geologic model in global seismologic tomography problems.

REFERENCES

Claerbout, J., 1999, Geophysical estimation by example: Environmental soundings image enhancement: Stanford Exploration Project, `http://sepwww.stanford.edu/sep/prof/`.

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