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Short Note

Velocity uncertainty: Non-linearity and the starting guess

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INTRODUCTION

The last few years have seen a significant increase in research assessing risk. Several papers deal with assessing risk from a geostatistical framework (Shanor et al., 2002; Gambus-Ordaz et al., 2002). The general methodology is to create equi-probable models based on simplified covariance descriptions and probability functions. Each point is visited in turn and a value selected based on *a priori* probability distribution, surrounding point value, and the covariance description.

In previous work I showed how we accomplish something similar in a global inversion problem. As long as a decorrelator exists (such as a regularization operator that is an inverse noise covariance operator) adding random noise into the residual space will create equi-probable models (Clapp, 2001a). This methodology can be applied to tomography in two distinct ways. If random noise is added to components in the residual vector corresponding to the regularization operator, we produce models that have not only correct covariance but also a reasonable variance. These models add fine layered features that standard tomography can not resolve. From realization to realization these features change in shape and amplitude. They do not effect the kinematics of the final image, but do have an effect on the amplitudes (Clapp, 2003a).

The second choice, adding noise to the portion of the residual vector corresponding to the data fitting goal, does have a more significant effect on the velocity and the final image (Clapp, 2004). Adding noise in this space corresponds to selecting an alternate set of data points. These new data points aren't simply random perturbations from the original model. In the case of tomography, they are similar to not selecting the maximum amplitude of a move-out measure, but a trend of lower or higher move-out. The added complexity is that the migration velocity analysis problem is not linear. We routinely will do several non-linear iterations to come up with the final answer. How to best deal with this non-linearity is unclear.

In this paper I take a slightly different tact from the one taken Clapp (2004). I perform four iterations of non-linear tomography. In the first two iterations I create five equi-probable realizations for the move-out functions. For the last two iterations I choose the minimum-energy

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move-out function. The resulting twenty-five models provide an interesting and instructive measure of the uncertainty involved in standard migration velocity analysis and its effect on final image.

REVIEW

In inversion we try to estimate some model $\mathbf{m}$ given some data $\mathbf{d}$ and an operator $\mathbf{L}$ that maps between the quantities. If our problem is poorly constrained, we can employ Tikhonov regularization (Tikhonov and Arsenin, 1977), adding a roughening operator $\mathbf{A}$ to our objective function Q. To balance the two components of the objective function we introduce a twiddle parameter ϵ and end up with

$$Q(\mathbf{m}) = ||\mathbf{d} - \mathbf{L}\mathbf{m}||^2 + \epsilon^2 ||\mathbf{A}\mathbf{m}||^2. \tag{1}$$

The two terms in our objective function serve different purposes. The first deals with *data fitting* and the second *model styling*. We can write the minimization in a slightly different form in terms of two *fitting goals*,

$$\begin{aligned} \mathbf{0} &\approx \mathbf{r_{data}} = \mathbf{d} - \mathbf{L}\mathbf{m} \\ \mathbf{0} &\approx \mathbf{r_{model}} = \epsilon \mathbf{A}\mathbf{m}, \end{aligned} \tag{2}$$

where $\mathbf{0}$ is a vector of zeros, $\mathbf{r_{data}}$ is the data residual vector, and $\mathbf{r_{model}}$ is the model. Our regularization operator, at best, usually only accounts for second order statistics, producing a model that is often unrealistic. In previous papers (Clapp, 2000, 2001a) I showed how by adding Gaussian random noise to the $\mathbf{r_{model}}$ we can add variance to our models and give the a more realistic texture.

If we decorrelate our data residual vector by adding an inverse noise covariance operator $\mathbf{N}$,

$$\begin{aligned} \mathbf{0} &\approx \mathbf{r_{data}} = \mathbf{N}(\mathbf{d} - \mathbf{L}\mathbf{m}) \\ \mathbf{0} &\approx \mathbf{r_{model}} = \epsilon \mathbf{A}\mathbf{m}, \end{aligned} \tag{3}$$

we can account for uncertainty in our data (Clapp, 2001b). This is similar, but not the same as, using stochastic simulation (Isaaks and Srivastava, 1989a,b) to create several different datasets. The two most notable differences are that we can handle much more spatially variant and complex covariance descriptions and we have the effect of a model styling goal in our inversion.

Tomography

In order to use this methodology our tomography problem has to be set up in a similar fashion to that of fitting goals (3). This isn't necessarily straight forward. Our first problem is that tomography is a non-linear process. The standard approach in ray-based tomography is to linearize around an initial slowness model $\mathbf{s_0}$. Our linearized tomography operator $\mathbf{T_0}$ is formed by rays traced through the background slowness. We then write a linear relation between the change in slowness $\Delta \mathbf{s}$ and the change in travel-time $\Delta \mathbf{t}$.

When doing migration velocity analysis in the depth domain, we are not dealing with travel-times but instead move-out as a function of some parameter (offset or azimuth) (Stork, 1992). Biondi and Symes (2003) showed how for angle domain migration there is a link between travel-time error dt, local dip ϕ, the local slowness s depth of the reflection z, the reflection angle θ, and scaling γ of the background slowness model. This relation can be written in terms of an operator $\mathbf{D}$ which maps from $1. - \gamma$ to $\Delta \mathbf{t}$ and whose elements are

$$D(\theta, \phi, z, s) = \frac{zs \sin(\theta)^2}{\cos(\phi) * (\cos(\theta)^2 - \sin(\theta)^2)}. \tag{4}$$

For our regularization operator we can use a steering filter (Clapp et al., 1997; Clapp, 2001a) oriented along reflector dips. Our basic linearized fitting goals become

$$\begin{aligned} \mathbf{0} &\approx \mathbf{r_{data}} = \mathbf{D}\gamma - \mathbf{T_0}\Delta \mathbf{s} \\ \mathbf{0} &\approx \mathbf{r_{model}} = \epsilon \mathbf{A}(\mathbf{s_0} + \Delta \mathbf{s}). \end{aligned} \tag{5}$$

The added term in our regularization fitting goal $\mathbf{As_0}$ is due to the fact that we want to smooth *slowness* not *change in slowness*. Clapp (2003a) and Chen and Clapp (2002) showed that adding noise to $\mathbf{r_{model}}$ produced velocity models with what looked like thin layers that had little effect on image kinematics but noticeable effects on amplitudes.

We run into problems when we want to explore the effect of adding noise to $\mathbf{r_{data}}$. Our γ values, and therefore our data fitting error exist in some irregular space (potentially consistent angle sampling, but irregular in space). This makes making an effective noise covariance operator difficult.

Multiple realization methodology

Clapp (2004) suggested breaking up the tomography problem into two portions: creating several realizations of γ maps and using them as input to the tomography problem. Estimating the **gamma** field is in itself difficult. The standard approach is to calculate semblance over a range of move-out values. The move-out at given point is then the maximum semblance at the location. To reduce noise, the semblance field is often smoothed. This is still far from in ideal solution. We are constantly fighting a battle between selecting local minima (not enough smoothing) and missing important move-out features (too much smoothing).

In Clapp (2004) the various γ maps were created by selecting a smooth set of random number and converting them into γ values based on a normal score transform (Isaaks and Srivastava, 1989a). This approach was somewhat successful, but suffered from the fact that we don't, and effectively can't scan over an infinite set of move-outs. Therefore our distribution function is misleading. Methods to correct for the limited range proved *adhoc*.

Instead I am going to start from the approach outlined in Clapp (2003b). My goal is to estimate a smooth set of semblance values $\mathbf{g}_{\text{smooth}}$. I begin by selecting the maximum semblance at each point $\mathbf{g}_{\text{max}}$. I solve the simple minimization problem

$$\begin{aligned} \mathbf{0} &\approx \mathbf{r_{data}} = \mathbf{W_g}(\mathbf{g}_{\text{max}} - \mathbf{g}_{\text{smooth}}) \\ \mathbf{0} &\approx \mathbf{r_{model}} = \epsilon \mathbf{A} \mathbf{g}_{\text{smooth}}, \end{aligned} \tag{6}$$

where $\mathbf{A}$ is again a steering filter, and $\mathbf{W_g}$ is a function of the semblance value at each location. After estimating $\mathbf{g}_{\text{smooth}}$ I select the maximum within a range around $\mathbf{g}_{\text{smooth}}$ to form a new $\mathbf{g}_{\text{max}}$, and repeat the estimation. At each iteration, the window I search around and the amount of smoothing (ϵ) decreases. To create a series of models I introduce random noise into $\mathbf{r_{data}}$ scaled by the variance in the semblance at each location. With different sets of random noise I get different realistic models.

EXAMPLE

Clapp (2004) took nine different realizations of a single linearizion of a complex synthetic model (Figure 1). There were several problems with this approach. The most significant problem was that a single non-linear iteration, was far from sufficient. After one iteration, we still have significant move-out that another non-linear iteration of tomography has a chance of using. When doing multiple non-linear iterations we have two choices to make at each iteration. First, should we use the minimum energy model (no random perturbation) or introduce random perturbations? Second, if we are adding random perturbations, how many models should we create at each non-linear iteration?

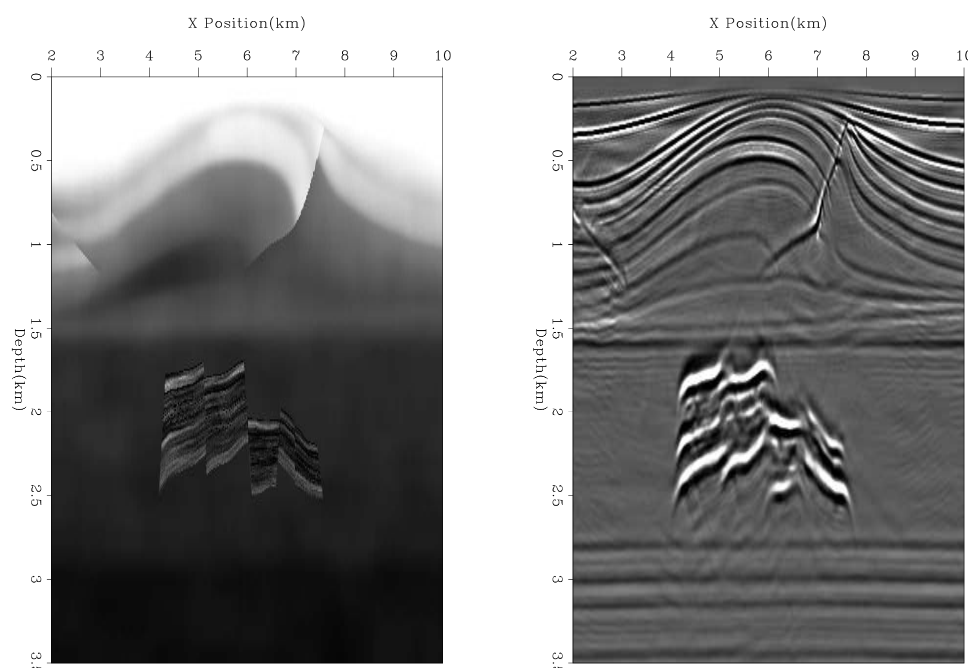

Figure 1: The left panels shows the velocity model used as input to the finite difference scheme used to create the data. The right panel is the resulting migrating the data with the correct velocity. The fine structure seen below 1.6km is the *reservoir*. bob3-model [CR,M]

For this experiment I decided to create five random perturbed models in the first non-linear iteration. From these five models I generated twenty five models during the second non-linear iteration. I then used these twenty-five models in a conventional migration velocity updating scheme. This gives some measure on the effect of the starting guess on the final solution. Each of the twenty-five models were equally reasonable points from which start a tomographic loop. The difference between the final images gives me some measure of the uncertainty in this updating scheme.

The left panel of Figure 2 shows my starting guess for the velocity problem. The right panel shows the resulting image. The velocity was created by applying a strong smoother to the correct velocity field then scaling the resulting model by .9. Figure 3 shows the results after one non-linear iteration. The top panel are the five realizations of γ. The center panels are the resulting five velocity models, and the bottom five panels are the migrated images using these velocity models. The anticline trend is in all of the realizations but we still see significant differences in how the velocity estimate deals with the listric fault.

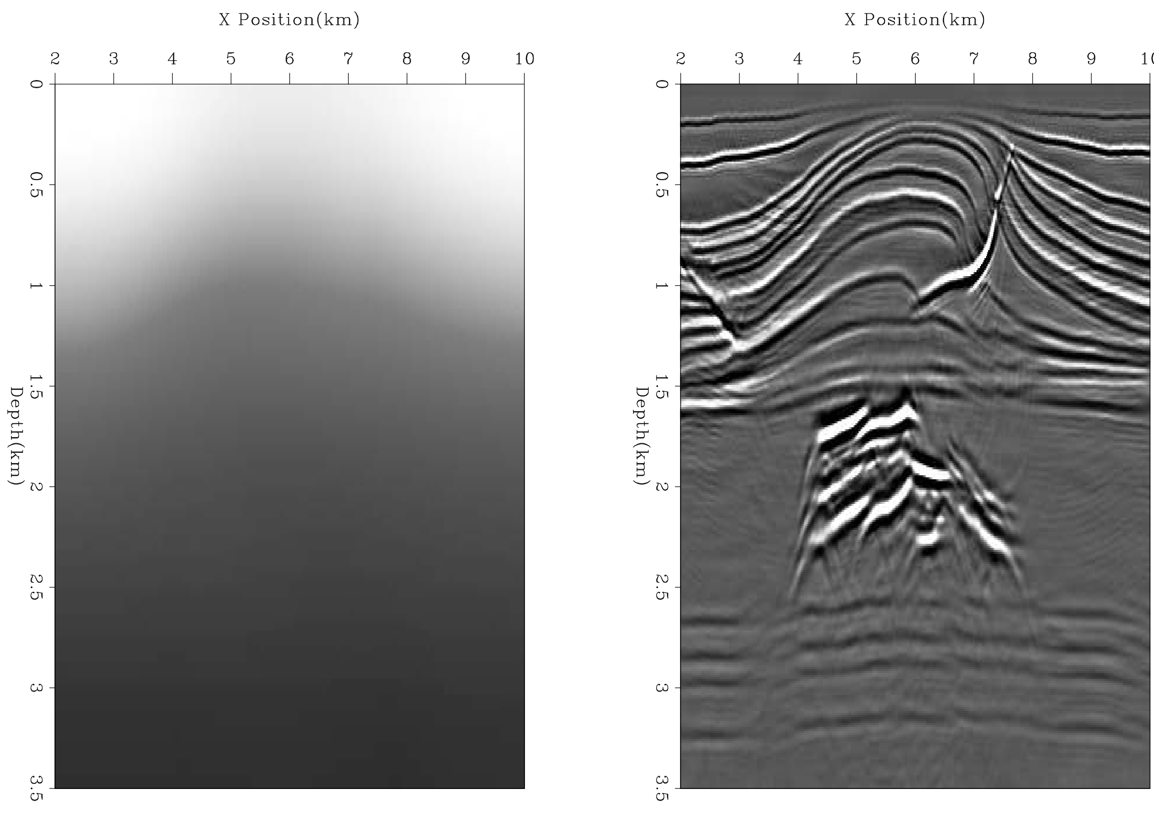

Figure 2: The left panel shows the initial velocity model. The right panel the resulting migration. bob3-iter0 [CR,M]

After four iterations, now with twenty-five different models, the differences are more dramatic. Figure 4 show the twenty-five different gamma panels. We see an overall reduction in the amount move-out (closer to gray), but the realizations still have significantly different character. The twenty-five velocity models (Figure 5) also show significant variation, especially as we go deeper in the model. After four iterations we see significant differences in

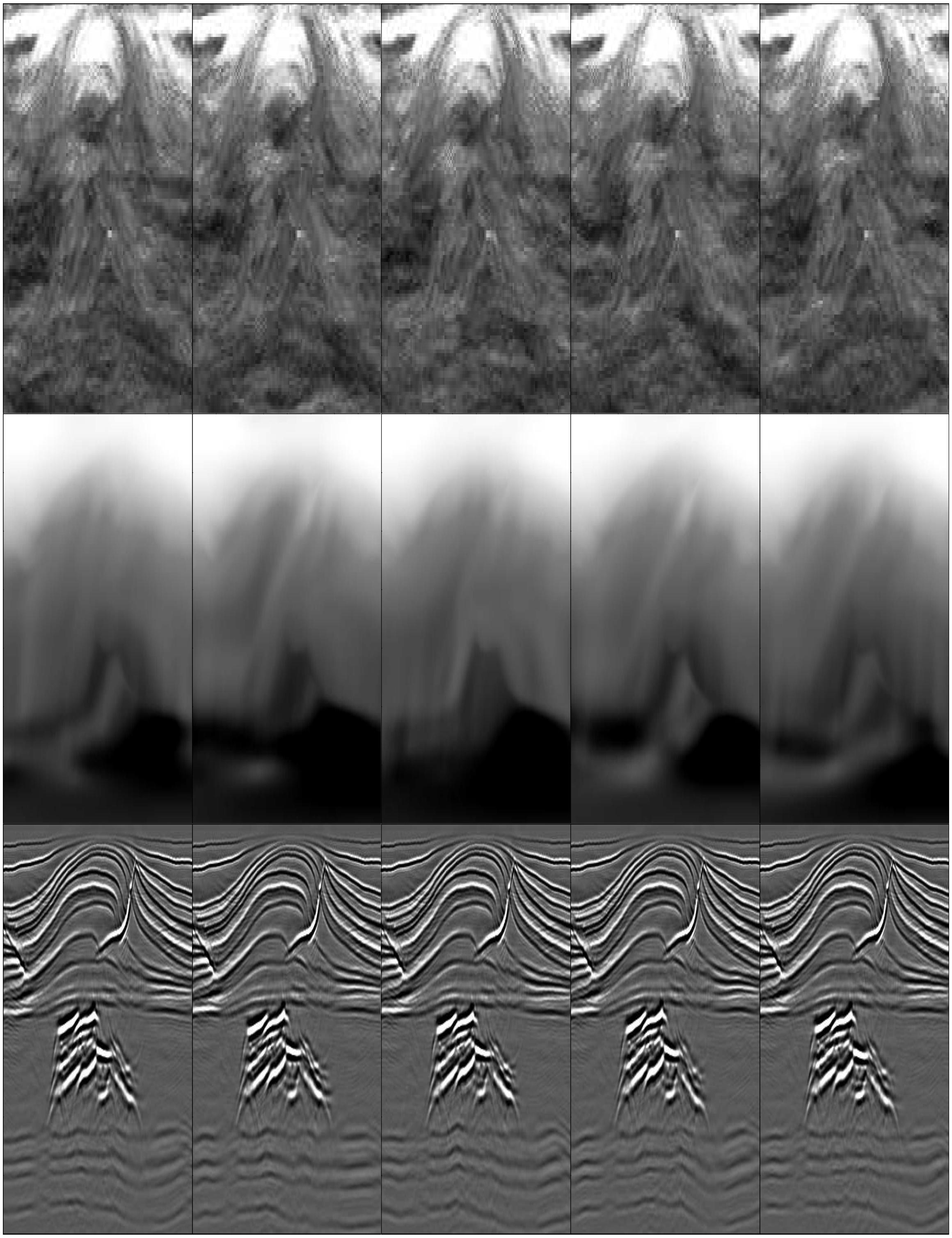

Figure 3: The top panel are the five realizations of γ. The center panels are the resulting five velocity models, and the bottom five panels are the migrated images using these velocity models. bob3-iter1 [CR,M]

the images (Figure 4). In most of the models we have focused the anticline structure, but the images have significant variation below. The basement reflectors are discontinuous in many of the models.

CONCLUSION

Multiple reasonable starting points for migration velocity analysis are generated by adding uncertainity to the moveout analysis procedure. It is demonstrated on a complex synthetic that these different starting points can have large effect on the final velocity model and the resulting image.

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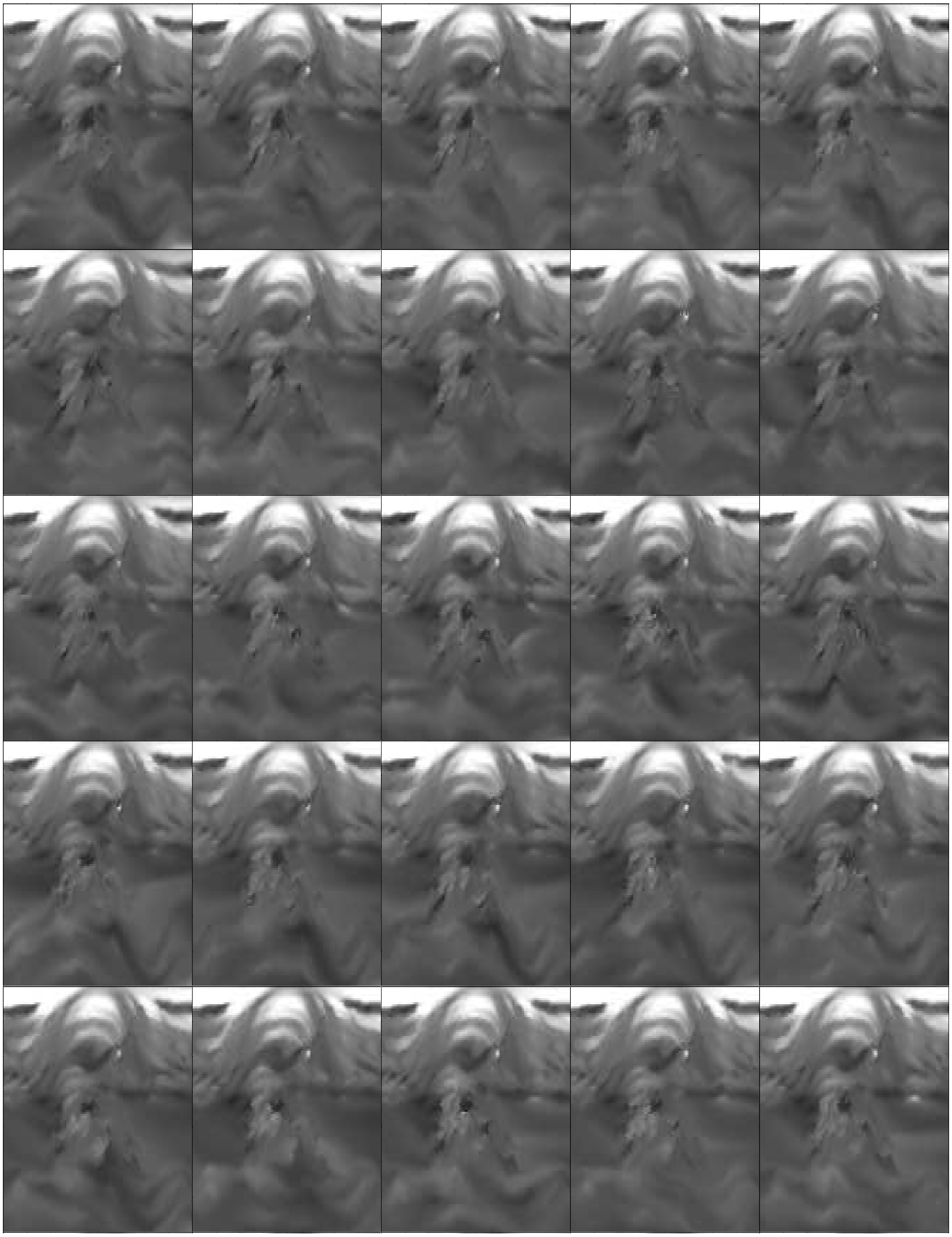

Figure 4: The twenty-five gamma panels after the third non-linear iteration. Note how we are overall closer to 1.0 (gray), but we still see differences in the various panels. bob3-iter3_g [CR]

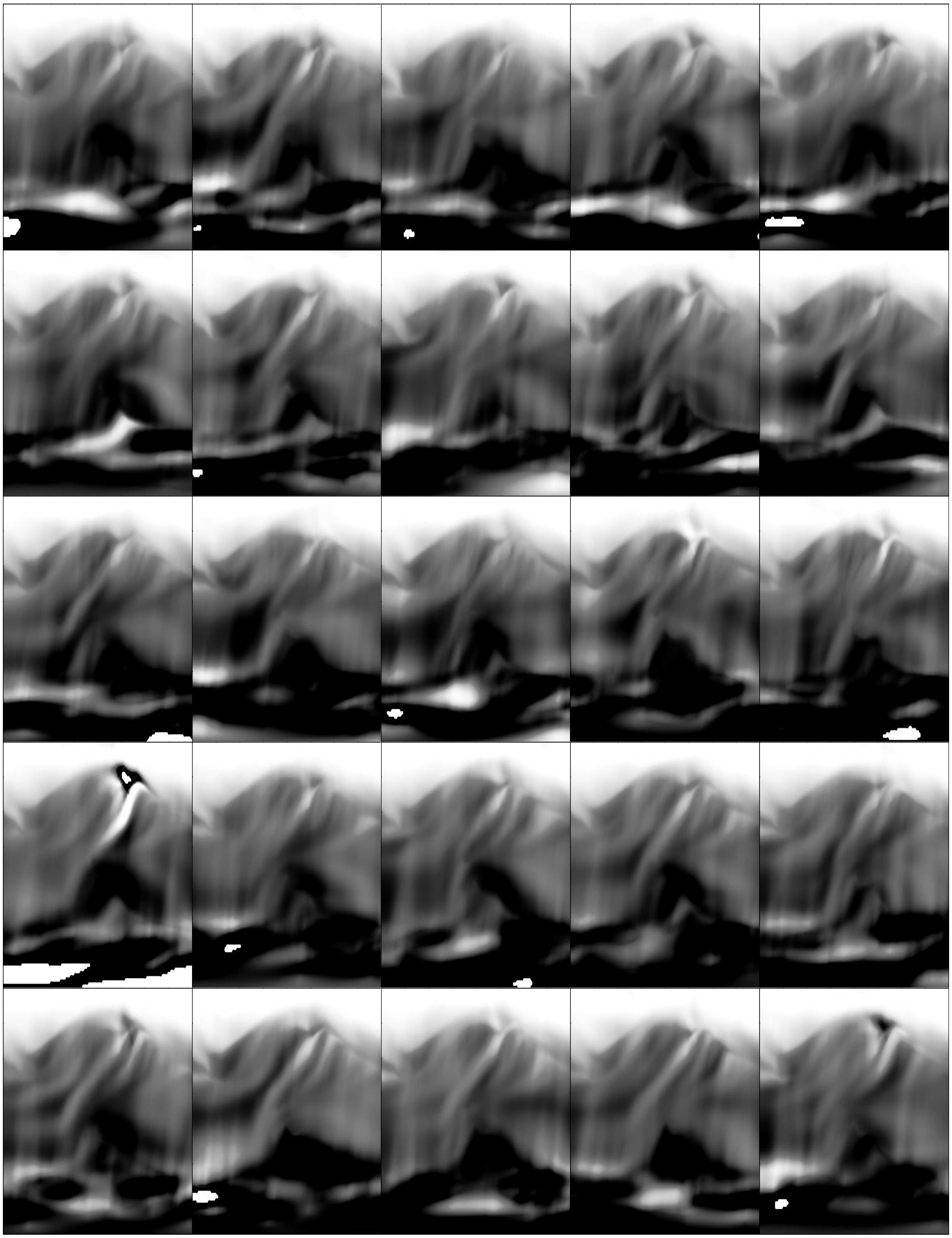

Figure 5: The final twenty-five velocity models. bob3-iter4_v [CR,M]

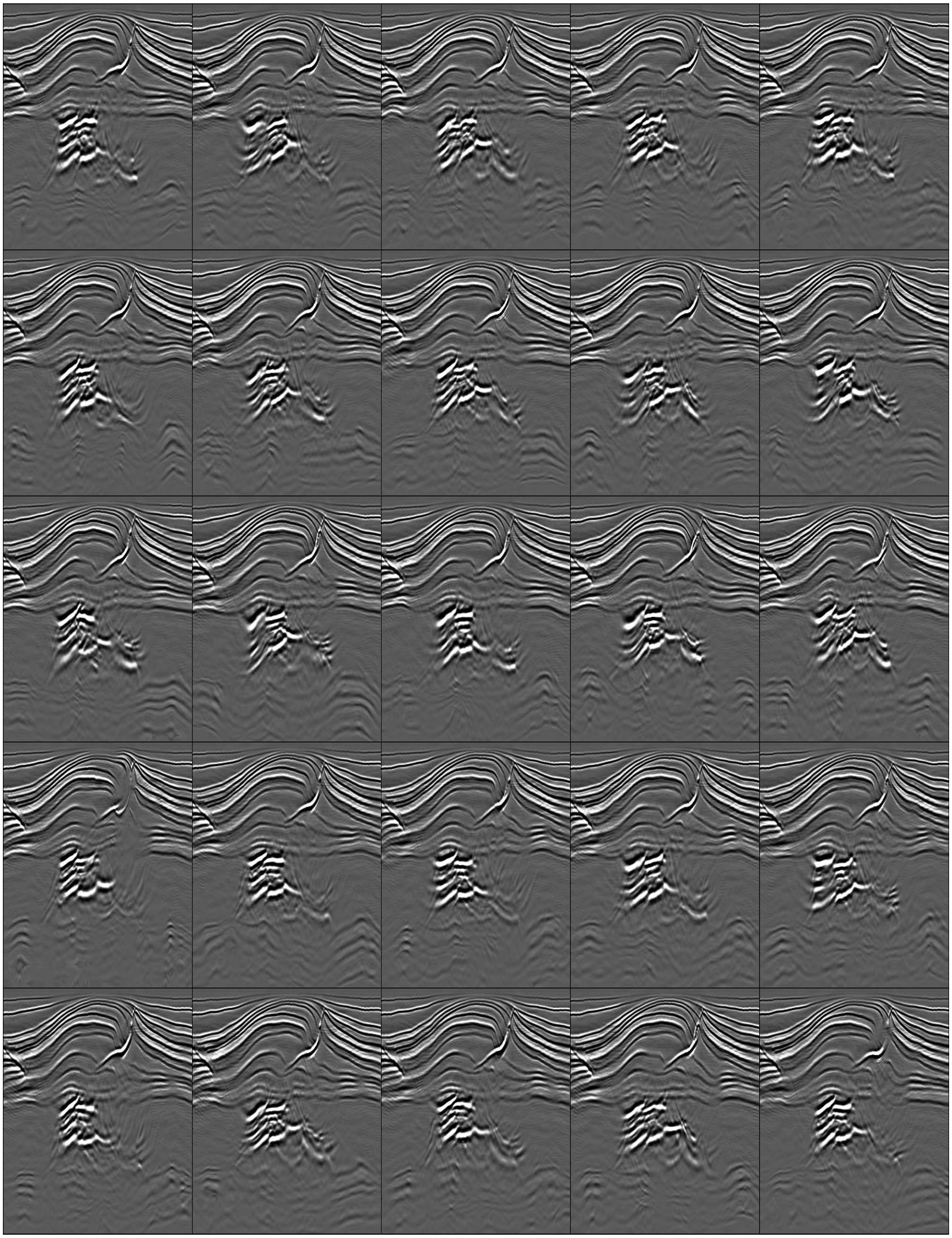

Figure 6: The twenty-five different images. Note the differences, especially in the reservoir. bob3-iter4_i [CR]

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