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AVA effects of regularized least-squares inversion

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ABSTRACT

As we search for hydrocarbons in areas where the earth's subsurface is too complex to accurately image with migration algorithms, we find ourselves turning to imaging techniques such as least-squares. My version of least-squares inversion imaging uses a downward continuation operator to produce offset ray parameter gathers (equivalent to angle gathers) and a regularization operator that is a derivative along the angle axis. The regularization operator stabilizes the inversion and helps to fill in illumination gaps. Essentially, I assume that any large, sudden changes in amplitude along the reflection angle axis are caused by poor illumination. This methodology is effective for reducing artifacts and helping to compensate for poor illumination. However, there is still the question of how this regularization will affect any real amplitude variation with angle (AVA) that should be seen in the model. In this paper, I address the question of how the derivative type regularization operator affects expected AVA in a simple model with no illumination problems. I experiment with various numbers of iterations and various strengths of regularization. Overall, I find that this operator, as implemented in this paper, can have a minor effect on the true AVA due to edge effects. However, it does not affect all possible AVA information, so I remain hopeful that the derivative-type regularization operator can be modified to allow us to trust AVA information from models produced by my regularized least-squares inversion.

INTRODUCTION

Our ongoing quest for hydrocarbons requires that we improve our ability to image the earth's subsurface. This is particularly true in areas around salt bodies, which can be good hydrocarbon traps but cause poor seismic illumination in the surrounding subsurface. Conventional imaging techniques such as migration cannot provide an adequate picture of these poorly illuminated areas (Muerdter et al., 1996; Prucha et al., 1998). In such areas, random noise and processing artifacts can easily obscure the small amount of signal that exists. A common type of artifact seen in these areas is caused by multipathing. Many authors have reduced these artifacts by generating images through Kirchhoff-type migration that create angle domain common image gathers (Xu et al., 2001). The artifacts are even better handled by downward continuation migration (Prucha et al., 1999a; Stolk and Symes, 2004).

However, reducing multipathing artifacts does not significantly improve the image where

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illumination is poor. If we wish to try to gain information on rock properties through amplitude analysis (de Bruin et al., 1990), we will have to deal with both these artifacts and the poor illumination. To improve the image and potentially recover accurate information on rock properties, we must move beyond migration.

Although migration is not sufficient to image the subsurface in areas with poor illumination, we can use migration as an imaging operator in a least-squares inversion scheme (Nemeth et al., 1999; Duquet and Marfurt, 1999; Ronen and Liner, 2000). In areas with poor illumination, the inversion problem is ill-conditioned, therefore it is wise to regularize the inversion scheme (Tikhonov and Arsenin, 1977). The regularization operator can be designed to exploit knowledge we have about the expected amplitude behavior and dip orientation of events in the image (Prucha and Biondi, 2002).

When using regularized inversion for imaging, the choice of regularization operator is critical. An intelligent and fairly safe choice is to penalize large amplitude changes as the reflection angle varies for a given point in the subsurface (Kuehl and Sacchi, 2001; Prucha and Biondi, 2002). I refer to this as "geophysical" regularization. This process will help to reduce artifacts and improve the image, but its impact on possible amplitude variation with angle (AVA) analysis must be considered. Kuehl and Sacchi (2002) found that a similar regularization scheme used to compensate for incomplete data rather than the problem of poor illumination could still provide accurate AVA information.

In this paper, I examine the effects of geophysically regularized inversion on a simple synthetic dataset with known AVA. I begin by reviewing the theory of Regularized Inversion with model Preconditioning (RIP). Then I explain the regularization operator used for geophysical RIP. I apply this RIP algorithm to the simple synthetic, using varying numbers of iterations and different strengths of regularization to evaluate the impact of the regularization on the AVA.

REVIEW OF REGULARIZED INVERSION

Iterative least-squares inversion can be expressed simply as the conjugate-gradient minimization of this objective function:

$$min\{Q(\mathbf{m}) = ||\mathbf{Lm} - \mathbf{d}||^2\} \tag{1}$$

where $\mathbf{L}$ is a linear modeling operator, $\mathbf{d}$ is the data, and $\mathbf{m}$ is the model. In this paper, $\mathbf{L}$ is the adjoint of the downward continuation migration operator explained by Prucha et al. (1999b). This migration algorithm produces a model with the axes depth (z), common reflection point (CRP), and offset ray parameter (p_h). Offset ray parameter is related to the reflection angle θ (where θ is half of the opening angle between incident and reflected rays) by:

$$p_h = \frac{2\sin\theta\cos\phi}{V(z,CRP)}. \tag{2}$$

where ϕ is the local dip and $V(z,CRP)$ is the local velocity at the reflection point.

The minimization can be expressed more concisely as a fitting goal:

$$\mathbf{0} \approx \mathbf{Lm} - \mathbf{d}. \tag{3}$$

However, many issues exist, including poor illumination, that make this iterative inversion likely to have a large null space. Any noise that exists within that null space will not be constrained during the inversion and can grow with each iteration until the problem becomes unstable. Fortunately, we can stabilize this problem with regularization (Tikhonov and Arsenin, 1977). The regularization adds a second fitting goal that we are minimizing at the same time as we minimize the first one:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{Lm} - \mathbf{d} \\ \mathbf{0} &\approx \epsilon\mathbf{Am}. \end{aligned} \tag{4}$$

The first expression in (4) is the "data fitting goal," meaning that it is responsible for making a model that is consistent with the data. The second expression is the "model styling goal," meaning that it allows us to impose some idea of what the model should look like using the regularization operator $\mathbf{A}$. The strength of the regularization is controlled by the regularization parameter ϵ.

Unfortunately, the inversion process described by fitting goals (4) can take many iterations to produce a satisfactory result. We can reduce the necessary number of iterations by making the problem a preconditioned one. We use the preconditioning transformation $\mathbf{m} = \mathbf{A}^{-1}\mathbf{p}$ (Fomel et al., 1997; Fomel and Claerbout, 2003) to give us these fitting goals:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{LA}^{-1}\mathbf{p} - \mathbf{d} \\ \mathbf{0} &\approx \epsilon\mathbf{p}. \end{aligned} \tag{5}$$

$\mathbf{A}^{-1}$ is obtained by mapping the multi-dimensional regularization operator $\mathbf{A}$ to helical space and applying polynomial division (Claerbout, 1998). This makes our imaging method a Regularized Inversion with model Preconditioning (RIP).

THE REGULARIZATION OPERATOR

The regularization operator $\mathbf{A}$ is designed to shape the model to conform to some expectations of its characteristics, particularly its covariance (Tarantola, 1986; Prucha et al., 2000). When using downward continuation migration as the linear imaging operator in the least-squares inversion problem, it is known that poor illumination will cause the resulting model to have gaps in events along the offset ray parameter (or the equivalent reflection angle) axis (Prucha et al., 2000). When attempting to image the subsurface where we know illumination is poor, it is reasonable to design $\mathbf{A}$ to regularize amplitudes along the offset ray parameter (p_h) axis.

To create a simple regularization operator, I begin with the assumption that the velocity model used by the imaging operator is correct and therefore there is no moveout along the p_h axis. Therefore, $\mathbf{A}$ can act horizontally along the p_h axis, penalizing sudden changes in amplitude. This is called "geophysical" regularization. Essentially, $\mathbf{A}$ is a derivative operator.

It will act most forcefully on very large, sudden amplitude changes along the p_h axis. This should compensate for the sudden holes caused by poor illumination without changing AVA trends that generally vary smoothly (Shuey, 1985).

Previous experiments using this geophysical regularization where illumination is poor have shown that it helps to "heal" the gaps along the p_h axis (Prucha et al., 2000). While this is encouraging, the question of how much the geophysical regularization affects any genuine amplitude variation with offset ray parameter or angle (AVA) remains open. To answer this question, I have designed a simple test with known AVA and no illumination problems.

AMPLITUDE EFFECTS

The synthetic model I used to test the AVA effects of geophysical regularization is a simple subsurface with four flat layers (Figure 1). The velocity within each layer is constant and I assume a constant density throughout the model. I generated synthetic finite difference data from this velocity model. The expected AVA at each of the three interfaces can be calculated from the velocities of the adjacent layers.

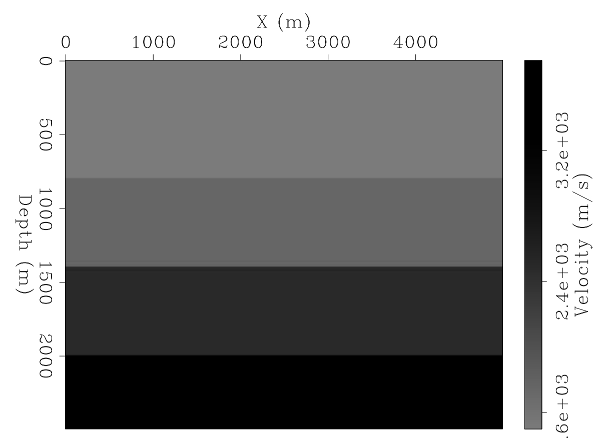

Figure 1: Simple velocity model. marie1-ava.vel [ER]

Effects of migration operator

I first investigated the effects of my downward continuation migration operator on the AVA. I migrated the synthetic data using the correct velocity model and extracted my calculated amplitudes along each of the three interfaces. These results can be seen in Figure 2. In each of the panels, the solid line represents the theoretical AVA values and the dots show the AVA values obtained after migration. The horizontal axis is shown in offset ray parameter (p_h), which is not an intuitively obvious physical unit. Based on the relationship in equation (2), the range of opening angles for the shallowest interface (left panel of Figure 2) is $0^o - 44^o$, the range for the second interface (center panel of Figure 2) is $0^o - 46^o$, and the range for the deepest interface (right panel of Figure 2) is $0^o - 68^o$. Overall, the results for each of the three interfaces are good. The calculated values for the second and third interfaces are not as consistent as the shallowest interface, partly due to the presence of more migration artifacts at depth. In the case of the deepest interface, it is also an effect of the survey geometry: we cannot expect the deep event to have reliable amplitude information at large p_h (larger than an opening angle of 36^o) because the common midpoint and offset ranges of the seismic data are limited, so energy reflecting at large angles from the deep event are lost. However, the overall trends of all of the calculated AVA values are fairly accurate.

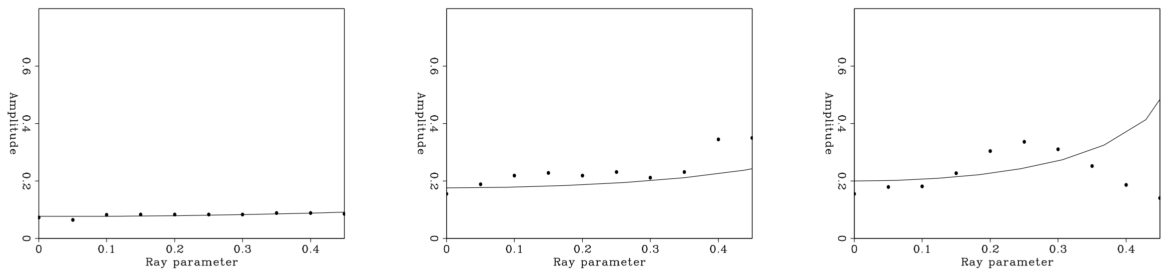

Figure 2: Amplitude variation with p_h. Left panel shows the results for the shallowest interface as seen in Figure 1, center panel for the second interface, and right panel for the deepest interface. The solid line shows the theoretical value and the dots show the values obtained from the image after downward continuation migration. marie1-ava.comp.mig [CR]

Effects of RIP

Having obtained satisfactory AVA results with my downward continuation migration operator, I moved on to test my geophysically Regularized Inversion with model Preconditioning (RIP). For my first test, I set ϵ to zero. Although this essentially sets the model styling goal of fitting goals (5) to zero, I am preconditioning the problem so I am still regularizing the inversion through the $\mathbf{A}^{-1}$ in the data fitting goal. Using this formulation, I ran tests using 5, 10 and 15 iterations. The results after 5 iterations can be seen in Figure 3. The results after 10 iterations are shown in Figure 4 and the results after 15 iterations are shown in Figure 5.

For all three exercises, we notice that the shallowest interface (left panels) has developed an artificial increasing trend. This is partly due to an edge effect at the small p_h, and partially due to the effects of the regularization. It is more pronounced for this interface because the true AVA trend is expected to be almost flat. The edge effect at small p_h is present for the other two interfaces as well.

Looking at the results for the second interface (center panels of Figures 3 through 5), it appears that our results after regularized inversion are more accurate than the result we saw from the migration (center panel of Figure 2). For the second interface, other than the edge effect at small p_h, the AVA trend is quite accurate after 5, 10 and 15 iterations. The best result for the second interface appears to be that after 10 iterations. The result after 15 iterations is deteriorating slightly as the inversion is trying harder to accomodate artifacts that exist in the data.

The results for the deepest interface (right panels of Figures 3 through 5) are also better than the result from the migration (right panel of Figure 2). They have the same problems at large p_h due to survey geometry, and have the edge effect at small p_h seen for the other interfaces, but the AVA trend between these two extremes is close to the expected trend. Due to the known problem at large p_h, I have actually chosen to turn the regularization operator off halfway along the p_h axis to keep the inversion from spending all of its effort trying to correct the sudden decrease in amplitude. Once again, it seems that the AVA trend in the result after 10 iterations is the best.

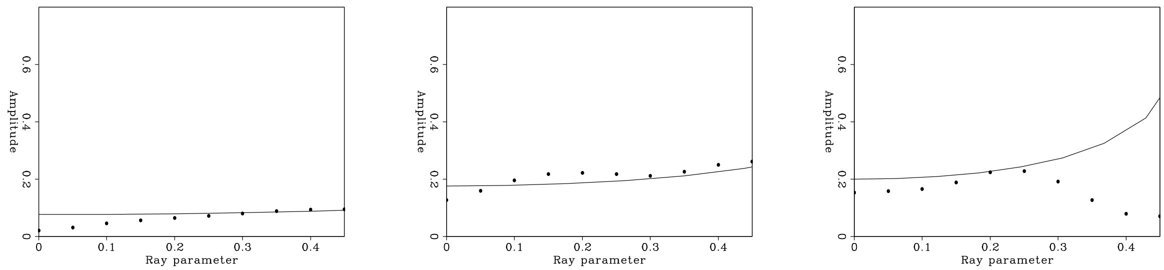

Figure 3: Amplitude variation with p_h for geophysical regularization with 5 iterations and $\epsilon = 0$. Left panel shows the results for the shallowest interface as seen in Figure 1, center panel for the second interface, and right panel for the deepest interface. The solid line shows the theoretical value and the dots show the values obtained after inversion. marie1-ava.comp.5it0eps [CR]

To examine the effect of a stronger regularization, I next set ϵ to .002. This means that the data fitting goal still influences the resulting model more than the regularization, but now the regularization is acting through the model styling goal as well as the data fitting goal. This value for ϵ was selected based on previous trial-and-error experiments in which poor illumination was a problem (Clapp, 2003). The results after 5 iterations of this test can be seen in Figure 6. The results after 10 and 15 iterations are in Figures 7 and 8, respectively. Since there are no sudden, large amplitude changes along the p_h axis, this stronger regularization should not affect the results much more than the inversion results with $\epsilon = 0$. As expected, these results are similar to the results with $\epsilon = 0$. We see the same unfortunate edge effect at small p_h for all of the interfaces in all three experiments (5, 10 and 15 iterations). We also see the effects of the survey geometry on the deepest interface (right panels of Figures 6-8), where I have once again elected to turn off the regularization operator. Overall, the increased strength of regularization has not affected the AVA trends of any of the interfaces any more than the previous inversion experiment.

CONCLUSIONS

The geophysically regularized least-squares inversion described in this paper is designed to minimize large, sudden changes in amplitudes along the offset ray parameter axis (or equivalent reflection angle axis). This behavior is based on the assumption that such changes are due to poor illumination, and true AVA trends should be relatively smooth in comparison. I have demonstrated that the derivative-type operator used for the regularization in the experiments in

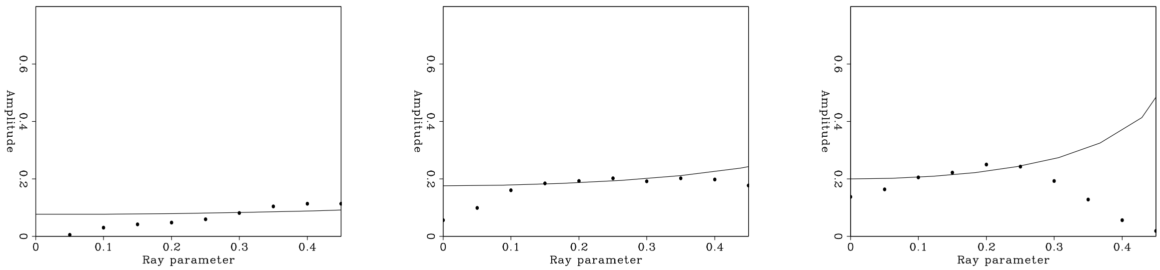

Figure 4: Amplitude variation with p_h for geophysical regularization with 10 iterations and $\epsilon = 0$. Left panel shows the results for the shallowest interface as seen in Figure 1, center panel for the second interface, and right panel for the deepest interface. The solid line shows the theoretical value and the dots show the values obtained after inversion. marie1-ava.comp.10it0eps [CR]

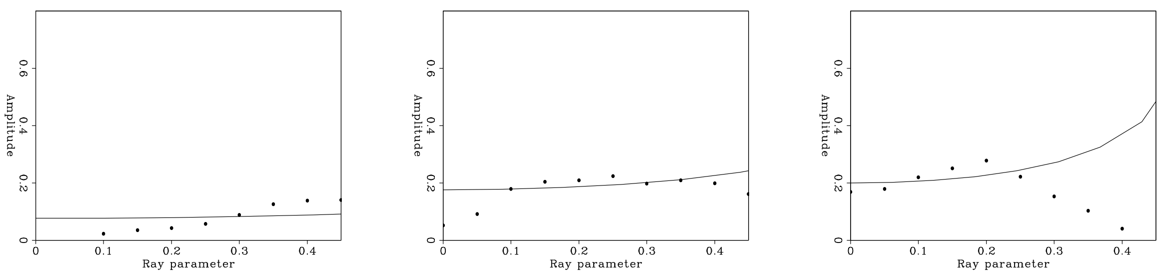

Figure 5: Amplitude variation with p_h for geophysical regularization with 15 iterations and $\epsilon = 0$. Left panel shows the results for the shallowest interface as seen in Figure 1, center panel for the second interface, and right panel for the deepest interface. The solid line shows the theoretical value and the dots show the values obtained after inversion. marie1-ava.comp.15it0eps [CR]

this paper do leave the true AVA trends intact, with some error due to edge effects. This result is encouraging as it indicates that with some further work to reduce the edge effects, I can still extract reliable AVA information from images produced by my geophysically regularized inversion.

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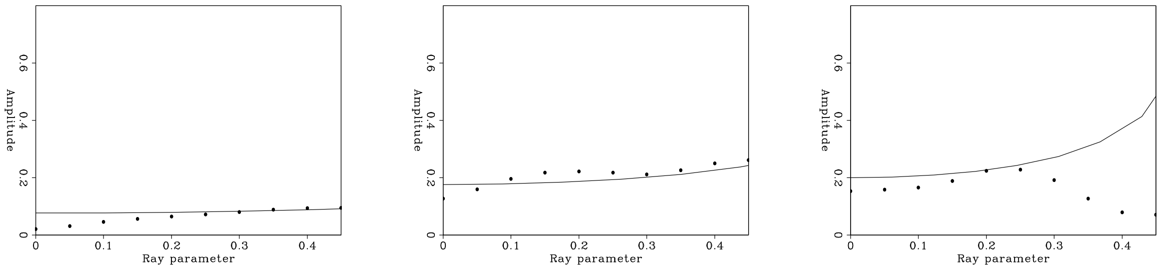

Figure 6: Amplitude variation with p_h for geophysical regularization with 5 iterations and $\epsilon = 0.002$. Left panel shows the results for the shallowest interface as seen in Figure 1, center panel for the second interface, and right panel for the deepest interface. The solid line shows the theoretical value and the dots show the values obtained after inversion. marie1-ava.comp.5it002eps [CR]

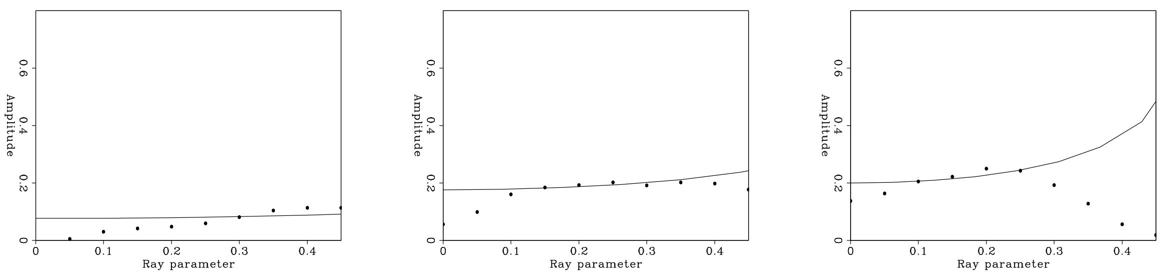

Figure 7: Amplitude variation with p_h for geophysical regularization with 10 iterations and $\epsilon = 0.002$. Left panel shows the results for the shallowest interface as seen in Figure 1, center panel for the second interface, and right panel for the deepest interface. The solid line shows the theoretical value and the dots show the values obtained after inversion. marie1-ava.comp.10it002eps [CR]

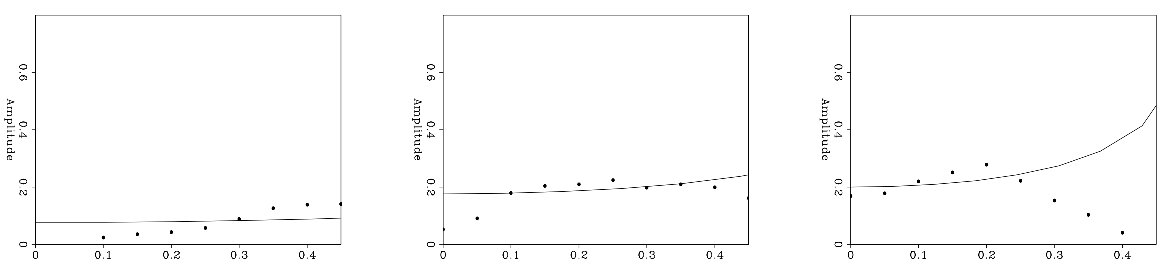

Figure 8: Amplitude variation with p_h for geophysical regularization with 15 iterations and $\epsilon = 0.002$. Left panel shows the results for the shallowest interface as seen in Figure 1, center panel for the second interface, and right panel for the deepest interface. The solid line shows the theoretical value and the dots show the values obtained after inversion. marie1-ava.comp.15it002eps [CR]

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